Using stepreg

Walter K. Kremers, Mayo Clinic, Rochester MN

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The Package

The stepreg() and cv.stepreg() funcitons in the *glmnetr* package were written for convenience and stability as opposed to speed or broad applicability. When fitting lasso models we wanted to compare these to standard stepwise regression models. Keeping a more modern approach we tune by either number of terms included in the model (James, Witten, Hastie and Tibshirani, An Introduction to Statistical Learning with applications in R, 2nd ed., Springer, New York, 2021) or by the p critical value for model inclusion, as this too is a common tuning parameter when fitting stepwise models.

When fitting lasso models we often use one-hot coding for predictor factors when setting up the design matrix. This allows lasso to identify and add to the model a term for any one group that might be particularly different from the others. By the penalty lasso stabilizes the model coefficients and keeps them from going to infinity, while ridge will generally uniquely identify coefficients despite any strict collinearities.

Before writing this program we tried different available packages to fit stepwise models for the Cox repression framework but all we tried had difficulties with numerical stability for the large and wide clinical datasets we were working with, and which involved one-hot coding. There may well be a package that would be stable for the data we were analyzing but we decided to write this small function to be able to tune for stability.

This program is slow but our goal was not for routine usage but to use the stepwise procedure on occasion as a reference for the lasso models. For many clinical datasets the lasso clearly outperformed the stepwise procedure, and ran much faster. For many simulated data sets with simplified covariance structures, i.e. independence of the underlying predictors, the lasso did not appear to do much better than the stepwise procedure tuned by number of model terms or p.

Data requirements

The data requirements for stepreg() and cv.stepreg() are similar to those of cv.glmnetr() and we refer to the *Using glmnetr* vignette for a description.

An example dataset

To demonstrate usage of cv.stepreg we first generate a data set for analysis, run an analysis and evaluate. Following the $Using\ glmnetr$ vignette, the code

```
# Simulate data for use in an example survival model fit
# first, optionally, assign a seed for random number generation to get applicable results
set.seed(116291950)
simdata=glmnetr.simdata(nrows=1000, ncols=100, beta=NULL)
```

generates simulated data for analysis. We extract data in the format required for input to the cv.stepreg (and glmnetr) programs.

```
# Extract simulated survival data
xs = simdata$xs  # matrix of predictors
y_ = simdata$yt  # vector of survival times
event = simdata$event  # indicator of event vs. censoring
```

Inspecting the predictor matrix we see

```
# Check the sample size and number of predictors
print(dim(xs))

## [1] 1000 100

# Check the rank of the design matrix, i.e. the degrees of freedom in the predictors
Matrix::rankMatrix(xs)[[1]]

## [1] 94

# Inspect the first few rows and some select columns
print(round(xs[1:10,c(1:12,18:20)],digits=6))
```

```
##
          X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12
                                                            X18
                                                                       X19
                                                                                  X20
##
                                               0
                                                   0 -1.208898
                                                                 0.056971 -0.565631
    [1,]
              0
                 0
                     0
                        1
                           0
                               1
                                  0
                                     0
                                          0
    [2,]
                        0
                           0
                                               0
                                                                 0.427313
                                                                             0.185235
##
           1
              1
                 0
                     0
                               0
                                  0
                                      1
                                          0
                                                      0.395354
##
    [3,]
           1
              0
                 0
                     1
                        0
                           1
                               0
                                  0
                                     0
                                          0
                                               0
                                                      1.044608 -0.746960
                                                                             0.964274
           1
              1
                     0
                        0
                           0
                                                   0 0.028859 -1.277651
                                                                             0.203243
                        0
                               0
                                  0
                                     0
                                                   0 -1.205172 -1.287454 -1.698229
##
    [5,]
              0
                 0
                     1
                           1
                                          0
                                              0
           1
                           0
                                                   0 -1.158210 -0.068841
##
    [6,]
           1
              0
                 0
                     0
                        1
                               1
                                  0
                                     0
                                          0
                                              0
                                                                             1.458800
##
              0
                 0
                     0
                        1
                           0
                               0
                                  0
                                     1
                                          0
                                               0
                                                                 1.095396
                                                                            1.476831
    [7,]
           1
                                                   0 0.151713
    [8,]
           1
              0
                 0
                     1
                        0
                           0
                                               0
                                                   0 -0.139246 -0.424550
                                                                            0.073340
    [9,]
           1
              1
                 0
                     0
                        0
                           0
                               0
                                  1
                                     0
                                          0
                                               0
                                                   0 -0.069326
                                                                 0.172792
                                                                            1.039656
## [10,]
          1
              0
                 0
                    1
                        0
                           0
                                               0
                                                      0.677420
                                                                 1.185946 -1.473551
```

Cross validation (CV) informed stepwise model fit

To fit stepwise regression models where the number of model terms are informed by cross validation to select df, the number of model terms, and p, the entry threshold, we can use the function cv.stepreg() function.

```
# Fit a relaxed lasso model informed by cross validation
cv.stepwise.fit = cv.stepreg(xs,NULL,y_,event,family="cox",folds_n=5,steps_n=30,track=0)
```

Note, in the derivation of the stepwise regression models, individual coefficients may be unstable even when the model may be stable which elicits warning messages. Thus we "wrapped" the call to cv.stepreg() within the suppressWarnings() function to suppress excessive warning messages in this vignette. The first term in the call to cv.stepreg(), xs, is the design matrix for predictors. The second input term, here NULL, is for the start time in case (start, stop) time data setup is used in a Cox survival model. The third term is the outcome variable for the linear regression or logistic regression model and the time of event or censoring in case of the Cox model, and finally the forth term is the event indicator variable for the Cox model taking

the value 1 in case of an event or 0 in case of censoring at time y_. The forth term would be NULL for either linear or logistic regression. Currently the options for family are "guassian" for linear regression, "binomial" for logistic regression (both using the stats glm() function) and "cox" for the Cox proportional hazards regression model using the coxph() function of the R survival package. If one sets track=1 the program will update progress in the R console. For track=0 it will not. To summarize the model fit and inspect the coefficient estimates we use the summary() function.

```
# summarize model fit ...
summary(cv.stepwise.fit)
```

```
##
##
   CV best df = 16, CV best p enter = 0.01 for 16 predictors
##
        in the full data model, from 100 candidate predictors
##
##
     df loglik.null
                                                                           X2
                       loglik
                                    pvalue concordance
                                                                std
          -3709.825 -3705.723 0.004178366
                                             0.8796415 0.005219351 -2.544254
          -3709.825 -3705.723 0.004178366
                                             0.8796415 0.005219351 -2.544254
## 2 16
                        Х7
                                             X11
                                                       X12
                                                                 X14
## 1 -0.4123862 -0.5812514 0.6538633 -0.4939628 0.4246715 -1.387424 -1.647604
## 2 -0.4123862 -0.5812514 0.6538633 -0.4939628 0.4246715 -1.387424 -1.647604
           X18
                     X19
                                X20
##
                                            X21
                                                     X23
                                                               X24
## 1 0.7966722 -1.150425 -0.4928893 -0.1818494 1.075441 0.7174526 -0.4877742
## 2 0.7966722 -1.150425 -0.4928893 -0.1818494 1.075441 0.7174526 -0.4877742
##
            X62
## 1 -0.1259569
## 2 -0.1259569
```

To extract beta's or calculate predicteds we use the predict() function.

```
# get betas ...
betas = predict(cv.stepwise.fit)
t( betas[1:20,] )
##
      X1
                Х2
                            X3 X4 X5 X6
                                                 X7 X8 X9
                                                                X10
                                                                            X11
                                                        0 0.6538633 -0.4939628
      0 -2.544254 -0.4123862
                                0
                                   0
                                      0 -0.5812514
                                                     0
       0 -2.544254 -0.4123862
                                0
                                   0
                                      0 -0.5812514
                                                     0
                                                        0 0.6538633 -0.4939628
##
            X12 X13
                           X14 X15
                                         X16 X17
                                                        X18
                                                                  X19
                                                                              X20
## df 0.4246715
                  0 -1.387424
                                 0 -1.647604
                                                0 0.7966722 -1.150425 -0.4928893
## p 0.4246715
                  0 -1.387424
                                 0 -1.647604
                                               0 0.7966722 -1.150425 -0.4928893
# predicteds ...
preds = predict(cv.stepwise.fit, xs)
t(preds[1:14,])
##
           [,1]
                      [,2]
                                [,3]
                                           [,4]
                                                     [,5]
                                                                [,6]
                                                                          [,7]
```

[,11]

[,12]

[,13]

df -4.652185 -2.777916 -1.515435 -0.979273 0.3337369 -5.318352 -1.121909 ## p -4.652185 -2.777916 -1.515435 -0.979273 0.3337369 -5.318352 -1.121909

df -2.543347 -2.617922 -4.385983 -0.4020953 -4.200559 5.43046 -3.462096 ## p -2.543347 -2.617922 -4.385983 -0.4020953 -4.200559 5.43046 -3.462096

[,10]

[,9]

##

Nested cross validation

Because the values choice for df (number of model terms) or p (significnae level for inclusion) informed by CV are specifically chosen to give a best fit, model fit statistics for the CV derived model will be biased. To address this one can perform a CV on the CV derived estimates, that is a nested cross validation as argued for in SRDM (Simon R, Radmacher MD, Dobbin K, McShane LM. Pitfalls in the Use of DNA Microarray Data for Diagnostic and Prognostic Classification. J Natl Cancer Inst (2003) 95 (1): 14-18. https://academic.oup.com/jnci/article/95/1/14/2520188). This is done here by the nested.glmnetr() function.

For this example we use 3 folds. We would generally using between 5 or 10 folds in practice, to get reasonable run times and to better allow variability in variable selection.

```
#names(nested.gau.fit)
summary(nested.gau.fit)
```

```
Sample information including number of records, number of columns in
##
##
     design (predictor, X) matrix, and df (rank) of design matrix:
##
       family
                       n xs.columns
                                          xs.df null.dev/n
     gaussian
                    1000
                                             94
                                                       7.96
##
                                 100
##
##
    For LASSO, Stepwise regression tuned by df and p, and AIC, average (Ave) model
    performance measures from the 3-fold (NESTED) Cross Validation are given together
##
    with naive summaries calculated using all data without cross validation
##
##
##
                          Ave DevRat Ave Int Ave Slope Ave R-square Ave Non Zero
## lasso
                              0.8705 -0.0407
                                                 1.0317
                                                              0.8715
                                                                           53.0000
## lassoR
                              0.8687 -0.0107
                                                 1.0157
                                                              0.8696
                                                                           29.3333
## lassoR0
                              0.8704 0.0051
                                                0.9989
                                                              0.8707
                                                                           14.0000
## ridge
                              0.8624 -0.0149
                                                1.0166
                                                              0.8626
                                                                           99.0000
##
                          Naive DevRat Naive R-square Non Zero
## lasso
                                0.8863
                                                0.9420
## lassoR
                                0.8769
                                                0.9364
                                                             14
## lassoR0
                                0.8769
                                                0.9364
                                                             14
## ridge
                                0.8919
                                                0.9448
                                                             99
##
##
                          Ave DevRat Ave Int Ave Slope Ave R-square Ave Non Zero
## Stepwise df tuned
                              0.8671 0.0052
                                                0.9940
                                                              0.8674
                                                                           16.6667
                                                0.9839
                                                                           24.0000
## Stepwise p tuned
                              0.8627
                                      0.0282
                                                              0.8631
## Stepwise AIC
                              0.8629 0.0293
                                                0.9803
                                                              0.8632
                                                                           30,0000
##
                          Naive DevRat Naive R-square Non Zero
                                0.8827
                                                0.9395
## Stepwise df tuned
                                                             19
## Stepwise p tuned
                                0.8833
                                                0.9399
                                                             20
## Stepwise AIC
                                0.8878
                                                             30
                                                0.9422
```

Before providing analysis results the output first reports sample information line sample size, the number of predictors and the df (degrees of freedom) of the design matrix.

Next are the nested cross validation results. First are the per record (or per event in case of the Cox model) log-likelihoods which reflect the amount of information in each observation. Since we are not using large

sample theory to base inferences we feel the per record are more intuitive, and they allow comparisons between datasets with unequal sample sizes. Next are the average number of model terms which reflect the complexity of the different models, even if in a naive sense, followed by the agreement statistics, concordance or r-square. These nested cross validated concordances should be essentially unbiased for the given design, unlike the naive concordances where the same data are used to derive the model and calculate the concordances (see SRDM). In this output we are also able to compare the performance of the stepwise regression models with those of the lasso models.

In addition to evaluating the CV informed model fits using another layer of CV, the nested glmnetr() function does the CV fits based upon the whole data set. Here we see, not unexpectedly, that the model fit measures from the nested CV are somewhat smaller than those naively calculated using the original dataset. Depending on the data the nested CV and naive agreement measures can be very similar or disparate.

Fit information for the CV fit can be gotten by extracting the object\$cv.stepreg.fit object and calling the summary() and predict() functions.

```
# Summary of a CV model fit from a nested CV output object
summary(nested.gau.fit$cv.stepreg.fit)
```

```
##
##
   CV best df = 19, CV best p enter = 0.03 for 20 predictors
       in the full data model, from 100 candidate predictors
##
##
##
    df loglik.null
                      loglik
                                 pvalue
                                          rsquare rsquareadj
                                                                  Int
                                                                             X2
## 1 19
          -2456.327 -1384.781 0.01532871 0.8827085 0.8804345 2.541036 -2.374707
## 2 20
          -2456.327 -1382.101 0.02060680 0.8833355
                                                  0.8809522 2.541435 -2.373546
##
            ХЗ
                      Х4
                                Х6
                                          Х8
                                                   X10
                                                             X12
                                                                       X14
## 1 -0.2896507 0.3961826 0.5041838 0.2439999 0.7493202 0.4179345 -1.623983
  2 -0.2901617 0.3946920 0.5111328 0.2397952 0.7451425 0.4111942 -1.624389
##
          X16
                    X18
                              X19
                                         X20
                                                    X21
                                                             X23
                                                                       X24
## 1 -1.747628 0.8906858 -1.102188 -0.5406626 -0.1252505 1.090738 0.6988531
## 2 -1.746315 0.8919730 -1.105330 -0.5425721 -0.1270239 1.090795 0.6985010
##
           X25
                      X28
                                 X43
                                             X62
## 1 -0.4341470 0.07509635 0.00000000 -0.08267686 -0.07654123
## 2 -0.4287512 0.07668351 0.07283277 -0.08163437 -0.07438254
# get betas ...
betas = predict(nested.gau.fit$cv.stepreg.fit)
t( betas[1:10,] )
          Int X1
                        X2
                                   ХЗ
                                             X4 X5
                                                          X6 X7
                                                                       X8 X9
0 0.5041838
                                                              0 0.2439999
## p 2.541435 0 -2.373546 -0.2901617 0.3946920 0 0.5111328 0 0.2397952
# get predicteds ...
preds = predict(nested.gau.fit$cv.stepreg.fit,xs)
t( preds[1:8,] )
           [,1]
                      [,2]
                               [,3]
                                       [,4]
                                                [,5]
                                                         [,6]
                                                                  [,7]
                                                                            [8,]
```