

Package ‘PTLENKies’

September 3, 2026

Type Package

Title Computation of PTLENKies Distribution Properties

Version 0.1.0

Author Mintodê Nicodème Atchadé [aut],
Théophile Otodji [aut, cre]

Maintainer Théophile Otodji <otodjitheodule@gmail.com>

Description Implements statistical tools for analyzing, simulating, and computing properties of the Power Topp Leone Exponential Negative Kies Burr (PTLENKBurr). See Atchadé M, Otodji T, and Djibril A (2024) <doi:10.1063/5.0179458> and Atchadé M, Otodji T, Djibril A, and N'bouké M (2023) <doi:10.1515/phys-2023-0151> for details.

Depends R (>= 4.1.0)

License GPL-2

Encoding UTF-8

LazyData true

Suggests knitr, rmarkdown, testthat (>= 3.0.0)

Language en

Config/roxygen2/version 8.0.0

Imports graphics, stats

Config/testthat/edition 3

NeedsCompilation no

Repository CRAN

Date/Publication 2026-09-03 11:50:47 UTC

Contents

C_PTLENKBurr	2
E_PTLENKBurr	3
GlassFibers	4
Plot_CPTLENKBurr	4

Plot_PPTLENKBurr	6
P_PTLENKBurr	7
Q_PTLENKBurr	8
R_PTLENKBurr	9
Sim_PTLENKBurr	10

Index	12
--------------	-----------

C_PTLENKBurr	<i>Cumulative Distribution Function (CDF) of the PTLENKBurr Distribution</i>
--------------	--

Description

This function calculates the Cumulative Distribution Function (CDF) of the PTLENKBurr distribution.

Usage

C_PTLENKBurr(x, m, n, p, lambda, beta, c, k)

Arguments

x	Value or vector of positive values up to which to calculate the CDF.
m	Parameter m of the distribution (m > 0).
n	Parameter n of the distribution (n > 0).
p	Parameter p of the distribution (p > 0).
lambda	Parameter lambda of the distribution (lambda > 0).
beta	Parameter beta of the distribution (beta > 0).
c	Parameter c of the Burr XII baseline component (c > 0).
k	Parameter k of the Burr XII baseline component (k > 0).

Details

It takes parameters x, m, n, p, lambda, beta, c, and k, and returns the CDF value at x based on these parameters.

The Cumulative Distribution Function (CDF) of the PTLENKBurr distribution is defined as:

$$F(x; m, n, p, \lambda, \beta, c, k) = \exp \left(mn \left(1 - \frac{1}{K(x)^p} \right) \right) \left[2 - \exp \left(n \left(1 - \frac{1}{K(x)^p} \right) \right) \right]^m$$

where

$$K(x) = 1 - \exp \left(-\lambda \left(\frac{1 - (1 + x^c)^{-k}}{(1 + x^c)^{-k}} \right)^\beta \right)$$

and $m, n, p, \lambda, \beta, c, k > 0, x > 0$.

Value

A numeric vector containing the values of the CDF evaluated at x.

Examples

```
C_PTLENKBurr(
  x = c(0.5, 1.0, 1.5),
  m = 1, n = 1, p = 1,
  lambda = 0.5, beta = 1.5, c = 2, k = 1
)
```

E_PTLENKBurr	<i>Maximum Likelihood Estimation (MLE) of the PTLENKBurr Distribution</i>
--------------	---

Description

Estimates the parameters of the Power Transformed Lindley Extended Modified Kies Burr XII (PTLENKBurr) distribution using maximum likelihood estimation (MLE) via constrained optimization with [nlminb](#).

Usage

```
E_PTLENKBurr(data, start_params = NULL, lower = 1e-05)
```

Arguments

data	Numeric vector of positive observations.
start_params	Optional numeric vector of length 7 containing initial parameter values for c(m, n, p, lambda, beta, c, k). Default is c(1, 1, 1, 1, 1, 1, 1).
lower	Lower bounds for parameters. Default is 1e-5 for all 7 parameters.

Details

Optimization is performed using [nlminb](#). Infeasible iterations returning non-positive densities, NA, or NaN values are assigned a high penalty.

Value

A named numeric vector of estimated parameters: m, n, p, lambda, beta, c, and k.

Examples

```
data_sample <- c(0.5, 1.2, 0.8, 2.1, 0.3, 1.5, 0.9)
E_PTLENKBurr(data_sample)
```

GlassFibers	<i>Dataset: GlassFibers</i>
-------------	-----------------------------

Description

The dataset contains observations collected at the UK National Physical Laboratory regarding the breaking strengths of 1.5 cm glass fibers, previously reported in Atchade (2023).

Usage

```
data(GlassFibers)
```

Format

A numeric vector of breaking strengths.

Details

This dataset contains measurements corresponding to the breaking strengths of 1.5 cm glass fibers. The data are presented as a numeric vector.

References

Atchade, N. (2023). Statistical modeling of lifetime data and probability distributions.

Plot_CPTLENKBurr	<i>Plot the Cumulative Distribution Function (CDF) of the PTLENKBurr Distribution</i>
------------------	---

Description

Generates an enhanced plot of the Cumulative Distribution Function (CDF) of the PTLENKBurr distribution over $[0, +\infty)$, automatically calculating and displaying the numerical median.

Usage

```
Plot_CPTLENKBurr(  
  m,  
  n,  
  p,  
  lambda,  
  beta,  
  c,  
  k,  
  max_x = NULL,  
  show_median = TRUE,
```

```

    col_line = "deepskyblue4",
    col_median = "firebrick3",
    legend_pos = "right",
    ...
)

```

Arguments

<code>m</code>	Parameter m of the distribution ($m > 0$).
<code>n</code>	Parameter n of the distribution ($n > 0$).
<code>p</code>	Parameter p of the distribution ($p > 0$).
<code>lambda</code>	Parameter λ of the distribution ($\lambda > 0$).
<code>beta</code>	Parameter β of the distribution ($\beta > 0$).
<code>c</code>	Parameter c of the Burr XII baseline component ($c > 0$).
<code>k</code>	Parameter k of the Burr XII baseline component ($k > 0$).
<code>max_x</code>	Optional maximum x-axis value. If NULL (default), it is automatically computed using the CDF.
<code>show_median</code>	Logical. If TRUE (default), calculates and plots the median.
<code>col_line</code>	Color of the CDF curve. Default is "deepskyblue4".
<code>col_median</code>	Color of the median lines and point. Default is "firebrick3".
<code>legend_pos</code>	Position of the legend. Default is "right".
<code>...</code>	Additional graphical parameters passed to plot .

Details

The median is numerically calculated by finding the root of $F(x) - 0.5 = 0$ using [uniroot](#).

Value

A plot of the CDF and an invisible `list` containing the evaluation grid data frame and the numerical value of the median.

Examples

```

Plot_CPTLENKBurr(
  m = 1, n = 1, p = 1, lambda = 0.5,
  beta = 1.5, c = 2, k = 1
)

```

Plot_PPTLENKBurr	<i>Plot the Probability Density Function (PDF) of the PTLENKBurr Distribution</i>
------------------	---

Description

Generates an enhanced plot of the Probability Density Function (PDF) of the PTLENKBurr distribution over $[0, +\infty)$, automatically calculating and displaying the numerical mode and median.

Usage

```
Plot_PPTLENKBurr(
  m,
  n,
  p,
  lambda,
  beta,
  c,
  k,
  max_x = NULL,
  show_mode = TRUE,
  show_median = TRUE,
  col_line = "deepskyblue4",
  col_mode = "darkorange2",
  col_median = "firebrick3",
  legend_pos = "topright",
  ...
)
```

Arguments

m	Parameter m of the distribution ($m > 0$).
n	Parameter n of the distribution ($n > 0$).
p	Parameter p of the distribution ($p > 0$).
lambda	Parameter lambda of the distribution ($\lambda > 0$).
beta	Parameter beta of the distribution ($\beta > 0$).
c	Parameter c of the Burr XII baseline component ($c > 0$).
k	Parameter k of the Burr XII baseline component ($k > 0$).
max_x	Optional maximum x-axis value. If NULL (default), it is automatically computed using the CDF.
show_mode	Logical. If TRUE (default), calculates and plots the mode.
show_median	Logical. If TRUE (default), calculates and plots the median.
col_line	Color of the PDF curve. Default is "deepskyblue4".
col_mode	Color of the mode marker. Default is "darkorange2".

col_median	Color of the median marker. Default is "firebrick3".
legend_pos	Position of the legend. Default is "topright".
...	Additional graphical parameters passed to plot .

Details

The upper bound `max_x` is determined automatically by finding where $F(x) \geq 0.999$. The mode is calculated numerically by maximizing the PDF via [optimize](#), and the median is found via [uniroot](#) on the CDF ([C_PTLENKBurr](#)).

Value

A plot of the PDF and an invisible `list` containing the evaluation grid data frame, calculated mode, and median.

Examples

```
Plot_PPTLENKBurr(  
  m = 1, n = 1, p = 1, lambda = 0.5,  
  beta = 1.5, c = 2, k = 1  
)
```

P_PTLENKBurr	<i>Probability Density Function (PDF) of the PTLENKBurr Distribution</i>
--------------	--

Description

This function calculates the Probability Density Function (PDF) of the PTLENKBurr distribution.

Usage

```
P_PTLENKBurr(x, m, n, p, lambda, beta, c, k)
```

Arguments

x	Value or vector of positive values to evaluate the PDF at.
m	Parameter m of the distribution ($m > 0$).
n	Parameter n of the distribution ($n > 0$).
p	Parameter p of the distribution ($p > 0$).
lambda	Parameter lambda of the distribution ($\text{lambda} > 0$).
beta	Parameter beta of the distribution ($\text{beta} > 0$).
c	Parameter c of the Burr XII baseline component ($c > 0$).
k	Parameter k of the Burr XII baseline component ($k > 0$).

Details

$$f(x; m, n, p, \lambda, \beta, c, k) = 2mnp\lambda\beta ckx^{c-1}(1+x^c)^{-(k+1)} \frac{(1 - (1+x^c)^{-k})^{\beta-1}}{((1+x^c)^{-k})^{\beta+1}} \frac{1 - K(x)}{K(x)^{p+1}} \exp\left(mn\left(1 - \frac{1}{K(x)^p}\right)\right) \left(1 - \frac{1}{K(x)^p}\right)$$

where

$$K(x) = 1 - \exp\left(-\lambda\left(\frac{1 - (1+x^c)^{-k}}{(1+x^c)^{-k}}\right)^\beta\right)$$

and $m, n, p, \lambda, \beta, c, k > 0, x > 0$.

Value

A numeric vector containing the values of the PDF evaluated at x.

Examples

```
P_PTLENKBurr(
  x = c(0.5, 1.0, 1.5),
  m = 1, n = 1, p = 1,
  lambda = 0.5, beta = 1.5, c = 2, k = 1
)
```

Q_PTLENKBurr

Quantile Function of the PTLENKBurr Distribution

Description

Calculates the quantile value (inverse CDF) of the PTLENKBurr distribution for a given probability u or a vector of probabilities.

Usage

```
Q_PTLENKBurr(u, m, n, p, lambda, beta, c, k, tol = 1e-08)
```

Arguments

u	Numeric vector of probabilities ($0 \leq u \leq 1$).
m	Parameter m of the distribution ($m > 0$).
n	Parameter n of the distribution ($n > 0$).
p	Parameter p of the distribution ($p > 0$).
lambda	Parameter lambda of the distribution ($\lambda > 0$).
beta	Parameter beta of the distribution ($\beta > 0$).
c	Parameter c of the Burr XII baseline component ($c > 0$).
k	Parameter k of the Burr XII baseline component ($k > 0$).
tol	Tolerance level for numerical root finding via uniroot . Default is 1e-8.

Details

The quantile function $Q(u)$ is obtained by numerically solving $F(x) - u = 0$ using [uniroot](#). The probability argument is named `u` to avoid name collision with the distribution parameter `p`.

Value

A numeric vector of quantiles corresponding to the probabilities in `u`.

Examples

```
# Calculate quartiles (25%, 50%, 75%)
Q_PTLENKBurr(
  u = c(0.25, 0.5, 0.75),
  m = 1, n = 1, p = 1,
  lambda = 0.5, beta = 1.5, c = 2, k = 1
)
```

R_PTLENKBurr

*Random Generation for the PTLENKBurr Distribution***Description**

Generates random samples from the PTLENKBurr distribution using inverse transform sampling (quantile method).

Usage

```
R_PTLENKBurr(n_samples, m, n, p, lambda, beta, c, k)
```

Arguments

<code>n_samples</code>	Number of random samples to generate (must be a positive integer).
<code>m</code>	Parameter <code>m</code> of the distribution ($m > 0$).
<code>n</code>	Parameter <code>n</code> of the distribution ($n > 0$).
<code>p</code>	Parameter <code>p</code> of the distribution ($p > 0$).
<code>lambda</code>	Parameter <code>lambda</code> of the distribution ($\text{lambda} > 0$).
<code>beta</code>	Parameter <code>beta</code> of the distribution ($\text{beta} > 0$).
<code>c</code>	Parameter <code>c</code> of the Burr XII baseline component ($c > 0$).
<code>k</code>	Parameter <code>k</code> of the Burr XII baseline component ($k > 0$).

Details

The parameter for the sample size is named `n_samples` to avoid naming conflicts with the distribution parameter `n`. Random variates are generated by applying the quantile function [Q_PTLENKBurr](#) to uniform random variables: $X = Q(U)$ where $U \sim \text{Uniform}(0, 1)$.

Value

A numeric vector of `n_samples` random values generated from the PTLENKBurr distribution.

Examples

```
set.seed(123)
sim_data <- R_PTLENKBurr(
  n_samples = 100,
  m = 1, n = 1, p = 1,
  lambda = 0.5, beta = 1.5, c = 2, k = 1
)
head(sim_data)
```

Sim_PTLENKBurr	<i>Histogram and Fitted Density Function (PDF) of the PTLENKBurr Distribution</i>
----------------	---

Description

Generates a histogram of the provided dataset and overlays the fitted Probability Density Function (PDF) of the PTLENKBurr distribution with parameters estimated via Maximum Likelihood Estimation using [E_PTLENKBurr](#).

Usage

```
Sim_PTLENKBurr(
  data,
  breaks = 15,
  col_hist = "lightblue",
  col_line = "firebrick3",
  ...
)
```

Arguments

<code>data</code>	Numeric vector of positive data values.
<code>breaks</code>	Number of breaks or break specification for the histogram. Default is 15.
<code>col_hist</code>	Fill color for the histogram bars. Default is "lightblue".
<code>col_line</code>	Color for the fitted PDF curve. Default is "firebrick3".
<code>...</code>	Additional graphical parameters passed to hist .

Details

The function uses [E_PTLENKBurr](#) to estimate distribution parameters and evaluates [P_PTLENKBurr](#) over a sequence from near zero to 1.2 times the maximum data value.

Value

An invisible named numeric vector containing the 7 estimated parameters $c(m, n, p, \lambda, \beta, c, k)$.

Examples

```
sample_data <- c(0.5, 1.2, 0.8, 2.1, 0.3, 1.5, 0.9)
Sim_PTLENKBurr(sample_data)
```

Index

* **datasets**

GlassFibers, [4](#)

C_PTLENKBurr, [2](#), [7](#)

E_PTLENKBurr, [3](#), [10](#)

GlassFibers, [4](#)

hist, [10](#)

nlminb, [3](#)

optimize, [7](#)

P_PTLENKBurr, [7](#), [10](#)

plot, [5](#), [7](#)

Plot_CPTLENKBurr, [4](#)

Plot_PPTLENKBurr, [6](#)

Q_PTLENKBurr, [8](#), [9](#)

R_PTLENKBurr, [9](#)

Sim_PTLENKBurr, [10](#)

uniroot, [5](#), [7–9](#)