shows that the average area under the LLM is 0.96. Progress indicators are suppressed with `verb=0`. Alternatively, the probability that input location `xx` = 0.5 is under the LLM is given by

```
> 1 - mean(lin.gpllm.tr$trace$XX[[1]]$b1)
```

```
[1] 0.96
```

This is the same value as the area under the LLM since the process is stationary (i.e., there is no treed partitioning).

3.2 1-d Synthetic Sine Data

Consider 1-dimensional simulated data which is partly a mixture of sines and cosines, and partly linear.

$$z(x) = \begin{cases} \sin\left(\frac{\pi x}{5}\right) + \frac{1}{5}\cos\left(\frac{4\pi x}{5}\right) & x < 10 \\ x/10 - 1 & \text{otherwise} \end{cases} \tag{16}$$

The `R` code below obtains $N = 100$ evenly spaced samples from this data in the domain $[0, 20]$, with noise added to keep things interesting. Some evenly spaced predictive locations `XX` are also created.

```
> X <- seq(0, 20, length = 100)
> XX <- seq(0, 20, length = 99)
> Z <- (sin(pi * X/5) + 0.2 * cos(4 * pi * X/5)) *
+     (X <= 9.6)
> lin <- X > 9.6
> Z[lin] <- -1 + X[lin]/10
> Z <- Z + rnorm(length(Z), sd = 0.1)
```

By design, the data is clearly nonstationary. Perhaps not knowing this, a good first model choice for this data might be a GP.

```
> sin.bgp <- bgp(X = X, Z = Z, XX = XX, verb = 0)
```

Figure 5 shows the resulting posterior predictive surface under the GP. Notice how the (stationary) GP gets the wiggliness of the sinusoidal region, but fails to capture the smoothness of the linear region. This is because the data comes from a process that is nonstationary.

So one might consider a Bayesian treed linear model (LM) instead.

```
> sin.btlm <- btlm(X = X, Z = Z, XX = XX)
```

```
burn in:
**GROW** @depth 0: [0,0.424242], n=(43,57)
**GROW** @depth 1: [0,0.252525], n=(26,19)
**GROW** @depth 2: [0,0.131313], n=(14,13)
r=1000 d=[0] [0] [0] [0]; n=(11,19,17,53)
```

```
> plot(sin.bgp, main = "GP,", layout = "surf")
```

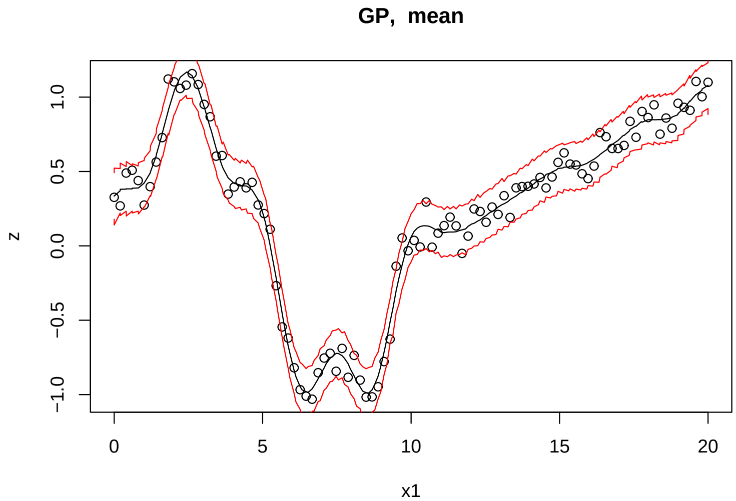


Figure 5: Posterior predictive distribution using `bgp` on synthetic sinusoidal data: mean and 90% credible interval

```
r=2000 d=[0] [0] [0] [0]; n=(11,17,19,53)

Sampling @ nn=99 pred locs:
r=1000 d=[0] [0] [0] [0]; mh=3 n=(15,14,18,53)
r=2000 d=[0] [0] [0] [0]; mh=4 n=(14,14,19,53)
r=3000 d=[0] [0] [0] [0]; mh=4 n=(12,16,19,53)
r=4000 d=[0] [0] [0] [0]; mh=4 n=(13,16,18,53)
r=5000 d=[0] [0] [0] [0]; mh=4 n=(13,15,19,53)
Grow: 0.8403%, Prune: 0%, Change: 36.3%, Swap: 84%
```

MCMC progress indicators show successful *grow* and *prune* operations as they happen, and region sizes n every 1,000 rounds. Specifying `verb=3`, or higher will show echo more successful tree operations, i.e., *change*, *swap*, and *rotate*.

Figure 6 shows the resulting posterior predictive surface (*top*) and trees (*bottom*). The MAP partition ($\hat{\mathcal{T}}$) is also drawn onto the surface plot (*top*) in the form of vertical lines. The treed LM captures the smoothness of the linear region just fine, but comes up short in the sinusoidal region—doing the best it can with piecewise linear models.

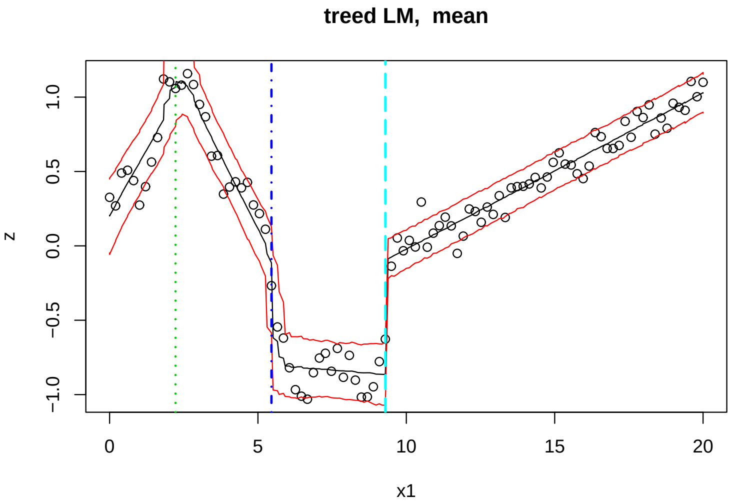
The ideal model for this data is the Bayesian treed GP because it can be both smooth and wiggly.

```
> sin.btgp <- btgp(X = X, Z = Z, XX = XX, verb = 0)
```

Figure 7 shows the resulting posterior predictive surface (*top*) and MAP $\hat{\mathcal{T}}$ with height=2.

Finally, speedups can be obtained if the GP is allowed to jump to the LLM [13], since half of the response surface is *very* smooth, or linear. This is not

```
> plot(sin.btlm, main = "treed LM,", layout = "surf")
```




```
> tgp.trees(sin.btlm)
```

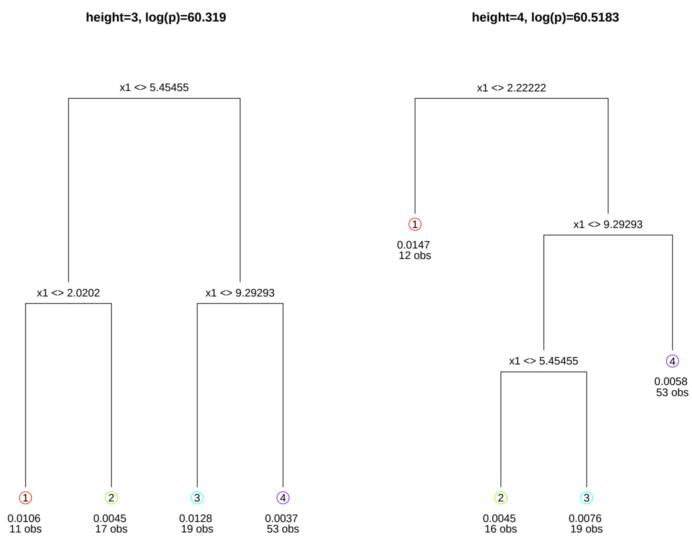


Figure 6: *Top:* Posterior predictive distribution using `btlm` on synthetic sinusoidal data: mean and 90% credible interval, and MAP partition ($\hat{\mathcal{T}}$); *Bottom* MAP trees for each height encountered in the Markov chain showing $\hat{\sigma}^2$ and the number of observation n, at each leaf.

shown here since the results are very similar to those above, replacing `btgp` with `btgpllm`. The example in the next subsection offers a comparison for 2-d data.

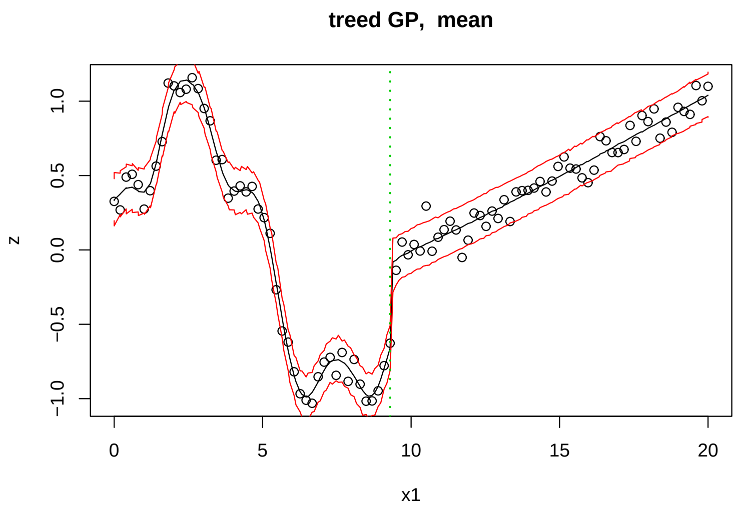


Figure 7: Posterior predictive distribution using `btgp` on synthetic sinusoidal data: mean and 90% credible interval, and MAP partition ($\hat{\mathcal{T}}$)

3.3 Synthetic 2-d Exponential Data

The next example involves a two-dimensional input space in $[-2, 6] \times [-2, 6]$. The true response is given by

$$z(\mathbf{x}) = x_1 \exp(-x_1^2 - x_2^2). \tag{17}$$

A small amount of Gaussian noise (with sd $= 0.001$) is added. Besides its dimensionality, a key difference between this data set and the last one is that it is not defined using step functions; this smooth function does not have any artificial breaks between regions. The `tgp` package provides a function for data subsampled from a grid of inputs and outputs described by (17) which concentrates inputs (`X`) more heavily in the first quadrant where the response is more interesting. Predictive locations (`XX`) are the remaining grid locations.

```
> exp2d.data <- exp2d.rand()
> X <- exp2d.data$X
> Z <- exp2d.data$Z
> XX <- exp2d.data$XX
```

The treed LM is clearly just as inappropriate for this data as it was for the sinusoidal data in the previous section. However, a stationary GP fits this data just fine. After all, the process is quite well behaved. In two dimensions one has a choice between the isotropic and separable correlation functions. Separable is the default in the `tgp` package. For illustrative purposes here, I shall use the isotropic power family.

```
> exp.bgp <- bgp(X = X, Z = Z, XX = XX, corr = "exp",
+     verb = 0)
```