Portmanteau Test Statistics

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Abstract

In this vignette, we give a brief description about the portmanteau test statistics given in the **portes** package. Some applications, including two examples from Mahdi and McLeod (2011) are given in this vignette as well.

Keywords: ARMA models, Monte-Carlo significance test, Portmanteau test, VARMA models .

1. Box and Pierce portmanteau test

In the univariate time series, Box and Pierce (1970) introduced the portmanteau statistic

$$Q_m = n \sum_{\ell=1}^{m} \hat{r}_\ell^2 \tag{1}$$

where $\hat{r}_\ell = \sum_{t=\ell+1}^{n} \hat{a}_t \hat{a}_{t-\ell} / \sum_{t=1}^{n} \hat{a}_t^2$, and $\hat{a}_1, \ldots, \hat{a}_n$ are the residuals. This test statistic is implemented in the R function `BoxPierce()` and can be used in the multivariate case as well. It has a chi-square distribution with $k^2(m-(p+q))$ degrees of freedom where k represents the dimension of the time series. The usage of this function is extremely simple:

```
BoxPierce(obj, lags = seq(5, 30, 5), order = 0, SquaredQ = FALSE),
```

where `obj` is a univariate or multivariate series with class `"numeric"`, `"matrix"`, `"ts"`, or `("mts" "ts")`. It can be also an object of fitted time-series model with class `"ar"`, `"arima0"`, `"Arima"`, `"varest"`, `"FitAR"`, or `"FitFGN"`. `lags` is a vector of numeric integers represents the lag values, m, at which we need to check the adequacy of the fitted model. The argument `order` is used for degrees of freedom of asymptotic chi-square distribution. If `obj` is a fitted time-series model with class `"ar"`, `"arima0"`, `"Arima"`, `"varest"`, `"FitAR"`, or `"FitFGN"` then no need to enter the value of `order` as it will be automatically determined. In general `order = p + q`, where `p` and `q` are the orders of the autoregressive (or vector autoregressive) and moving average (or vector moving average) models respectively. `order = 0` is used for testing random series, fractional gaussian noise, or generalized autoregressive conditional heteroscedasticity. Finally, when `SquaredQ = TRUE`, then apply the test on the squared values. This checks for Autoregressive Conditional Heteroscedastic, `ARCH`, effects. When `SquaredQ = FALSE`, then apply the test on the usual residuals.

Note that the function `portest()` with the arguments `test = "BoxPierce"`, `MonteCarlo = FALSE`, and `order = 0` will gives the same results of the function `BoxPierce()`. The Monte-Carlo version of this test statistic is implemented in the function `portest()` as an argument `test = "BoxPierce"` provided that `MonteCarlo = TRUE` is selected.

```
portest(obj, lags = seq(5, 30, 5), order = 0, test = "BoxPierce", MonteCarlo = TRUE,
       nslaves = 1, NREP = 1000, InfiniteVarianceQ = FALSE, SquaredQ = FALSE)
```

1.1. Example 1

First a simple univariate example is provided. We fit an AR (2) model to the logarithms of Canadian lynx trappings from 1821 to 1934. Data is available from the R package **datasets** under the name `lynx`. This model was selected using the BIC criterion. The asymptotic distribution and the Monte-Carlo version of Q_m statistic are given in the following R code for lags $m = 5, 10, 15, 20, 25, 30$ with **snow** package using PC with two CPU's.

```
R> library("portes")
R> library("snow")
R> nslaves <- 2
R> lynxData <- log(lynx)
R> p <- SelectModel(lynxData, ARModel = "AR", Criterion = "BIC",
+     Best = 1)
R> Fitlynx <- FitAR(lynxData, p, ARModel = "AR")
R> BoxPierce(Fitlynx)

 Lags Statistic df    p-value
    5  6.748225  3 0.08037069
   10 15.856081  8 0.04448698
   15 22.631444 13 0.04631764
   20 30.304179 18 0.03459211
   25 34.157210 23 0.06291892
   30 37.963103 28 0.09909886

R> portest(Fitlynx, test = "BoxPierce", nslaves = nslaves)

        2 slaves are spawned successfully. 0 failed.
 Lags Statistic df    p-value
    5  6.748225  3 0.08491508
   10 15.856081  8 0.03296703
   15 22.631444 13 0.02597403
   20 30.304179 18 0.02197802
   25 34.157210 23 0.03296703
   30 37.963103 28 0.04395604
```

For lags $m \geq 10$, the Monte-Carlo version of Box and Pierce test is more decisively suggests model inadequacy, whereas the asymptotic chi-square suggests inadequacy at lags 10 to 20 and adequacy otherwise. Fitting a subset autoregressive using the BIC (McLeod and Zhang 2008), the portmanteau test based on both methods, Monte-Carlo and asymptotic distribution suggest model adequacy.

```
R> SelectModel(log(lynx), lag.max = 15, ARModel = "ARp", Criterion = "BIC",
+     Best = 1)
```

```
[1]  1  2  4 10 11
```

```
R> FitsubsetAR <- FitARp(log(lynx), c(1, 2, 4, 10, 11))
R> BoxPierce(FitsubsetAR)
```

```
 Lags Statistic df   p-value
    5  2.382300  0        NA
   10  4.258836  0        NA
   15  6.532786  4 0.1627363
   20  9.887818  9 0.3596432
   25 13.258935 14 0.5062439
   30 16.172499 19 0.6457394
```

```
R> portest(FitsubsetAR, test = "BoxPierce", nslaves = nslaves)
```

```
        2 slaves are spawned successfully. 0 failed.
 Lags Statistic df   p-value
    5  2.382300  0 0.5224775
   10  4.258836  0 0.7742258
   15  6.532786  4 0.8311688
   20  9.887818  9 0.7992008
   25 13.258935 14 0.7852148
   30 16.172499 19 0.7752248
```

1.2. Example 2

In this example we consider the monthly log stock returns of Intel corporation data from January 1973 to December 2003. First we apply the Q_m statistic directly on the returns using the asymptotic distribution and the Monte-Carlo significance test. The results suggest that returns data behaves like white noise series as no significant serial correlations found.

```
R> monthintel <- as.ts(monthintel)
R> BoxPierce(monthintel)
```

```
 Lags Statistic df    p-value
    5  4.666889  5 0.45786938
   10 14.364748 10 0.15699489
   15 23.120348 15 0.08161787
   20 24.000123 20 0.24238680
   25 29.617977 25 0.23891229
   30 31.943703 30 0.37015020
```

```
R> portest(monthintel, test = "BoxPierce", nslaves = nslaves)
```

```
        2 slaves are spawned successfully. 0 failed.
 Lags Statistic df   p-value
```

```
    5  4.666889  5 0.4385614
   10 14.364748 10 0.1518482
   15 23.120348 15 0.0979021
   20 24.000123 20 0.2157842
   25 29.617977 25 0.2307692
   30 31.943703 30 0.3166833
```

After that we apply the Q_m statistic on the squared returns. The results suggest that the monthly returns are not serially independent and the return series may suffers of ARCH effects.

```
R> BoxPierce(monthintel, SquaredQ = TRUE)
```

```
 Lags Statistic df      p-value
    5  40.78073  5 1.039009e-07
   10  49.57872 10 3.189915e-07
   15  81.90133 15 3.131517e-11
   20  86.50575 20 3.006796e-10
   25  87.54737 25 7.161478e-09
   30  88.55017 30 1.087505e-07
```

```
R> portest(monthintel, test = "BoxPierce", nslaves = nslaves, SquaredQ = TRUE)
```

```
        2 slaves are spawned successfully. 0 failed.
 Lags Statistic df     p-value
    5  40.78073  5 0.000999001
   10  49.57872 10 0.000999001
   15  81.90133 15 0.000999001
   20  86.50575 20 0.000999001
   25  87.54737 25 0.000999001
   30  88.55017 30 0.000999001
```

2. Ljung and Box portmanteau test

Ljung and Box (1978) modified Box and Pierce (1970) test statistic by

$$\hat{Q}_m = n(n+2)\sum_{\ell=1}^{m}(n-\ell)^{-1}\hat{r}_\ell^2. \tag{2}$$

This test statistic is is also asymptotically chi-square with degrees of freedom $k^2(m-p-q)$ and implemented in the R function `LjungBox()`,

```
LjungBox(obj, lags = seq(5, 30, 5), order = 0, SquaredQ = FALSE),
```

where the arguments of this function are described as before.

In R, the function `Box.test()` was built to compute the Box and Pierce (1970) and Ljung and Box (1978) test statistics only in the univariate case where we can not use more than one single lag value at a time. The functions `BoxPierce()` and `LjungBox()` are more general than `Box.test()` and can be used in the univariate or multivariate time series at vector of different lag values as well as they can be applied on an output object from a fitted model.

Note that the function `portest()` with the arguments `test = "LjungBox", MonteCarlo = FALSE`, and `order = 0` will gives the same results of the function `LjungBox()`. The Monte-Carlo version of this test statistic is implemented in the function `portest()` as an argument `test = "LjungBox"` provided that `MonteCarlo = TRUE` is selected.

```
portest(obj, lags = seq(5, 30, 5), order = 0, test = "LjungBox", MonteCarlo = TRUE,
       nslaves = 1, NREP = 1000, InfiniteVarianceQ = FALSE, SquaredQ = FALSE)
```

2.1. Example 3

The built in R function `auto.arima()` in the package **forecast** is used to fit the best ARIMA model based on the AIC criterion to the measurements of the annual flow of the river Nile at Aswan from the years 1871 to 1970,

```
R> library("forecast")

R> FitNile <- auto.arima(Nile)
```

Then the `LjungBox` portmanteau test is applied on the residuals of the fitted model at lag values 5, 10, 15, 20, 25, and 30 which yields that the assumption of the adequacy in the fitted model is fail to reject.

```
R> LjungBox(FitNile)

 Lags Statistic df   p-value
    5  1.257698  3 0.7392018
   10  9.705584  8 0.2863011
   15 11.415751 13 0.5760319
   20 12.861450 18 0.7997373
   25 14.437766 23 0.9136466
   30 17.395015 28 0.9403734

R> portest(FitNile, test = "LjungBox", nslaves = nslaves)

        2 slaves are spawned successfully. 0 failed.
 Lags Statistic df   p-value
    5  1.257698  3 0.8521479
   10  9.705584  8 0.3256743
   15 11.415751 13 0.6023976
   20 12.861450 18 0.8211788
   25 14.437766 23 0.9200799
   30 17.395015 28 0.9370629
```

3. Hosking portmanteau test

Hosking (1980) generalized the univariate portmanteau test statistics given in eqns. (1, 2) to the multivariate case. He suggested the modified multivariate portmanteau test statistic

$$\tilde{Q}_m = n^2 \sum_{\ell=1}^{m} (n-\ell)^{-1} \hat{\boldsymbol{r}}'_\ell (\hat{\boldsymbol{R}}_0^{-1} \otimes \hat{\boldsymbol{R}}_0^{-1}) \hat{\boldsymbol{r}}_\ell \tag{3}$$

where $\hat{\boldsymbol{r}}_\ell = \text{vec}\hat{\boldsymbol{R}}'_\ell$ is a $1 \times k^2$ row vector with rows of $\hat{\boldsymbol{R}}_\ell$ stacked one next to the other, and m is the lag order. The $\otimes$ denotes the Kronecker product (http://en.wikipedia.org/wiki/Kronecker_product), $\hat{\boldsymbol{R}}_\ell = \boldsymbol{L}'\hat{\boldsymbol{\Gamma}}_\ell \boldsymbol{L}$, $\boldsymbol{L}\boldsymbol{L}' = \hat{\boldsymbol{\Gamma}}_0^{-1}$ where $\hat{\boldsymbol{\Gamma}}_\ell = n^{-1}\sum_{t=\ell+1}^{n} \hat{\boldsymbol{a}}_t \hat{\boldsymbol{a}}'_{t-\ell}$ is the lag ℓ residual autocovariance matrix.

The asymptotic distributions of $\tilde{Q}_m$ is chi-squared with $k^2(m-p-q)$ degrees of freedom. In **portest** package, this statistic is implemented in the function `Hosking()`:

```
Hosking(obj, lags = seq(5, 30, 5), order = 0, SquaredQ = FALSE),
```

where the arguments of this function is described as before. Note that the function `portest()` with the arguments `test = "Hosking"`, `MonteCarlo = FALSE`, and `order = 0` will gives the same results of the function `Hosking()`. The Monte-Carlo version of this test statistic is implemented in the function `portest()` as an argument `test = "Hosking"` provided that `MonteCarlo = TRUE` is selected.

```
portest(obj, lags = seq(5, 30, 5), order = 0, test = "Hosking", MonteCarlo = TRUE,
       nslaves = 1, NREP = 1000, InfiniteVarianceQ = FALSE, SquaredQ = FALSE)
```

3.1. Example 4

In this example, we consider fitting a VAR $(k), k = 1, 2, 3$ model to the monthly log returns of the IBM stock and the S&P 500 index from January 1926 to December 1999 with 888 observations (Tsay 2005, p. 356). The p-values for the modified portmanteau test of Hosking (1980), $\tilde{Q}_m$, are computed using the Monte-Carlo test procedure with 10^3 replications. For additional comparisons, the p-values for $\tilde{Q}_m$ are also evaluated using asymptotic approximations.

```
R> IBMSP500 <- monthibmspln
R> FitIBMSP5001 <- ar.ols(IBMSP500, aic = TRUE, intercept = F, order.max = 1)
R> Hosking(FitIBMSP5001)

 Lags Statistic  df      p-value
    5  38.33044  16 0.0013574550
   10  61.42150  36 0.0051949240
   15  72.97170  56 0.0633819777
   20 118.87159  76 0.0012179623
   25 152.37966  96 0.0002208340
   30 171.72563 116 0.0006001655
```

```
R> portest(FitIBMSP5001, test = "Hosking", nslaves = nslaves)

        2 slaves are spawned successfully. 0 failed.
 Lags Statistic  df     p-value
    5  38.33044  16 0.002997003
   10  61.42150  36 0.003996004
   15  72.97170  56 0.072927073
   20 118.87159  76 0.003996004
   25 152.37966  96 0.000999001
   30 171.72563 116 0.002997003

R> FitIBMSP5002 <- ar.ols(IBMSP500, aic = TRUE, intercept = F, order.max = 2)
R> Hosking(FitIBMSP5002)

 Lags Statistic  df     p-value
    5  28.12271  12 0.005307838
   10  50.23144  32 0.021174563
   15  61.53279  52 0.171676954
   20 104.28887  72 0.007697842
   25 138.24856  92 0.001303988
   30 156.56512 112 0.003487092

R> portest(FitIBMSP5002, test = "Hosking", nslaves = nslaves)

        2 slaves are spawned successfully. 0 failed.
 Lags Statistic  df     p-value
    5  28.12271  12 0.003996004
   10  50.23144  32 0.020979021
   15  61.53279  52 0.145854146
   20 104.28887  72 0.013986014
   25 138.24856  92 0.001998002
   30 156.56512 112 0.005994006

R> FitIBMSP5003 <- ar.ols(IBMSP500, aic = TRUE, intercept = F, order.max = 3)
R> Hosking(FitIBMSP5003)

 Lags Statistic  df     p-value
    5  18.08797   8 0.020576519
   10  40.78971  28 0.056135837
   15  52.21967  48 0.313383239
   20  93.82650  68 0.020716599
   25 124.25318  88 0.006631765
   30 142.81916 108 0.013972657

R> portest(FitIBMSP5003, test = "Hosking", nslaves = nslaves)
```

```
       2 slaves are spawned successfully. 0 failed.
 Lags Statistic  df     p-value
    5  18.08797   8 0.029970030
   10  40.78971  28 0.053946054
   15  52.21967  48 0.307692308
   20  93.82650  68 0.023976024
   25 124.25318  88 0.004995005
   30 142.81916 108 0.018981019
```

All results reject the fitted VAR (1), VAR (2) and VAR (3) models.

3.2. Example 5

The trivariate quarterly time series, 1960–1982, of West German investment, income, and consumption was discussed by Lütkepohl (2005, §3.23). So $n = 92$ and $k = 3$ for this series. As in Lütkepohl (2005, §4.24) we model the logarithms of the first differences. Using the AIC and FPE , Lütkepohl (2005, Table 4.25) selected a VAR (2) for this data. All diagnostic tests reject simple randomness, VAR (0). The asymptotic distribution and the Monte-Carlo tests for VAR (1) suggests model inadequacy supports the choice of the VAR (2) model.

```
R> data("WestGerman")
R> DiffData <- matrix(numeric(3 * 91), ncol = 3)
R> for (i in 1:3) DiffData[, i] <- diff(log(WestGerman[, i]), lag = 1)
R> FitWestGerman <- ar.ols(DiffData, aic = FALSE, order.max = 2,
+     intercept = FALSE)
R> Hosking(FitWestGerman)
```

```
 Lags Statistic  df   p-value
    5  30.36128  27 0.2981674
   10  71.94191  72 0.4797610
   15 122.49894 117 0.3455266
   20 171.96132 162 0.2811881
   25 209.45688 207 0.4391932
   30 254.48482 252 0.4443308
```

```
R> portest(FitWestGerman, test = "Hosking", nslaves = nslaves)
```

```
       2 slaves are spawned successfully. 0 failed.
 Lags Statistic  df   p-value
    5  30.36128  27 0.3796204
   10  71.94191  72 0.5064935
   15 122.49894 117 0.3546454
   20 171.96132 162 0.2667333
   25 209.45688 207 0.4265734
   30 254.48482 252 0.4235764
```

4. Li and McLeod portmanteau test

Li and McLeod (1981) suggested the multivariate modified portmanteau test statistic

$$\tilde{Q}_m^{(L)} = n \sum_{\ell=1}^{m} \hat{\boldsymbol{r}}'_\ell (\hat{\boldsymbol{R}}_0^{-1} \otimes \hat{\boldsymbol{R}}_0^{-1}) \hat{\boldsymbol{r}}_\ell + \frac{k^2 m(m+1)}{2n} \tag{4}$$

which is distributed as chi-squared with $k^2(m-p-q)$ degrees of freedom. In **portes** package, the test statistic $\tilde{Q}_m^{(L)}$ is implemented in the function `LiMcLeod()`,

```
LiMcLeod(obj, lags = seq(5, 30, 5), order = 0, SquaredQ = FALSE),
```

where the arguments of this function is described as before.

Note that the function `portest()` with the arguments `test = "LiMcLeod"`, `MonteCarlo = FALSE`, and `order = 0` will gives the same results of the function `LiMcLeod()`. The Monte-Carlo version of this test statistic is implemented in the function `portest()` as an argument `test = "LiMcLeod"` provided that `MonteCarlo = TRUE` is selected.

```
portest(obj, lags = seq(5, 30, 5), order = 0, test = "LiMcLeod", MonteCarlo = TRUE,
       nslaves = 1, NREP = 1000, InfiniteVarianceQ = FALSE, SquaredQ = FALSE)
```

5. Generalized variance portmanteau test

Peña and Rodriguez (2002) proposed a univariate portmanteau test of goodness-of-fit test based on the m-th root of the determinant of the m-th Toeplitz residual autocorrelation matrix

$$\hat{\mathcal{R}}_m = \begin{pmatrix} \hat{r}_0 & \hat{r}_1 & \dots & \hat{r}_m \\ \hat{r}_{-1} & \hat{r}_0 & \dots & \hat{r}_{m-1} \\ \vdots & \dots & \ddots & \vdots \\ \hat{r}_{-m} & \hat{r}_{-m+1} & \dots & \hat{r}_0 \end{pmatrix} \tag{5}$$

where $\hat{r}_0 = 1$ and $\hat{r}_{-\ell} = \hat{r}_\ell$, for all ℓ. They approximated the distribution of their proposed test statistic by the gamma distribution and provided simulation experiments to demonstrate the improvement of their statistic in comparison with the one that is given in Eq. (2).

Peña and Rodriguez (2006) suggested to modify this test by taking the log of the $(m+1)$-th root of the determinant in Eq. (5). They proposed two approximations by using the Gamma and Normal distributions to the asymptotic distribution of this test and indicated that the performance of both approximations for checking the goodness-of-fit in linear models is similar and more powerful for small sample size than the previous one. Lin and McLeod (2006) introduced the Monte-Carlo version of this test as they noted that it is quite often that the generalized variance portmanteau test does not agree with the suggested Gamma approximation. Mahdi and McLeod (2011) generalized both methods to the multivariate time series,

$$\mathfrak{D}_m = \frac{-3n}{2m+1} \log |\, \hat{\mathfrak{R}}_m \,|, \tag{6}$$

where

$$\hat{\mathfrak{R}}_m = \begin{pmatrix} \mathbb{I}_k & \hat{\boldsymbol{R}}_1 & \dots & \hat{\boldsymbol{R}}_m \\ \hat{\boldsymbol{R}}_{-1} & \mathbb{I}_k & \dots & \hat{\boldsymbol{R}}_{m-1} \\ \vdots & \dots & \ddots & \vdots \\ \hat{\boldsymbol{R}}_{-m} & \hat{\boldsymbol{R}}_{-m+1} & \dots & \mathbb{I}_k \end{pmatrix}. \tag{7}$$

The null distribution is approximately χ^2 with $k^2(1.5m(m+1)(2m+1)^{-1} - p - q)$ degrees of freedom and it is implemented in the R function `gvtest()`,

```
gvtest(obj, lags = seq(5, 30, 5), order = 0, SquaredQ = FALSE),
```

where the arguments of this function are described as before.

Note that the function `portest()` with the arguments `test = "gvtest", MonteCarlo = FALSE`, and `order = 0` will gives the same results of the function `gvtest()`. The Monte-Carlo version of this test statistic is implemented in the function `portest()` as an argument `test = "gvtest"` provided that `MonteCarlo = TRUE` is selected.

```
portest(obj, lags = seq(5, 30, 5), order = 0, test = "gvtest", MonteCarlo = TRUE,
       nslaves = 1, NREP = 1000, InfiniteVarianceQ = FALSE, SquaredQ = FALSE)
```

5.1. Example 6

Consider again the log numbers of Canadian lynx trappings univariate series from 1821 to 1934, where the AR(2) model is selected based on the BIC criterion using the function `SelectModel` in the R package **FitAR** as a first step in the analysis. Now, we apply the statistic $\mathfrak{D}_m$ on the fitted model based on the asymptotic distribution and the Monte-Carlo significance test,

```
R> gvtest(Fitlynx)

 Lags Statistic        df     p-value
    5  5.984989  2.090909 0.054687987
   10 10.036630  5.857143 0.115222212
   15 21.447021  9.612903 0.014964682
   20 31.810564 13.365854 0.003100578
   25 38.761595 17.117647 0.002040281
   30 43.936953 20.868852 0.002252062

R> portest(Fitlynx, test = "gvtest", nslaves = nslaves)

       2 slaves are spawned successfully. 0 failed.
 Lags Statistic        df     p-value
    5  5.984989  2.090909 0.063936064
   10 10.036630  5.857143 0.080919081
   15 21.447021  9.612903 0.006993007
   20 31.810564 13.365854 0.002997003
   25 38.761595 17.117647 0.000999001
   30 43.936953 20.868852 0.000999001
```

After that, we fit the subset autoregressive AR $_{(1,2,4,10,11)}$ using the BIC and then we apply $\mathfrak{D}_m$ as before,

```
R> gvtest(FitsubsetAR)

 Lags Statistic          df     p-value
    5  2.374225  0.0000000          NA
   10  3.598248  0.0000000          NA
   15  5.661285  0.6129032 0.008190694
   20  8.590962  4.3658537 0.090004731
   25 11.462473  8.1176471 0.184353957
   30 13.900470 11.8688525 0.297764350

R> portest(FitsubsetAR, test = "gvtest", nslaves = nslaves)

        2 slaves are spawned successfully. 0 failed.
 Lags Statistic          df   p-value
    5  2.374225  0.0000000 0.3476523
   10  3.598248  0.0000000 0.6703297
   15  5.661285  0.6129032 0.7152847
   20  8.590962  4.3658537 0.6953047
   25 11.462473  8.1176471 0.6663337
   30 13.900470 11.8688525 0.6793207
```

However the approximation asymptotic distribution of the statistic $\mathfrak{D}_m$ suggests that the subset AR model is an adequate model for lags $m \geq 20$, the Monte-Carlo portmanteau test is clearly suggest that the subset AR model is an adequate model.

5.2. Example 7

consider again fitting a VAR $(k), k = 1, 2, 3$ model to the monthly log returns of the IBM stock and the S&P 500 index from January 1926 to December 1999 with 888 observations (Tsay 2005, p. 356).

```
R> gvtest(FitIBMSP5001)

 Lags Statistic       df     p-value
    5  26.73298 12.36364 0.010069145
   10  50.16580 27.42857 0.005076554
   15  66.95921 42.45161 0.009606334
   20  87.59443 57.46341 0.006384252
   25 108.82328 72.47059 0.003699716
   30 128.30068 87.47541 0.002940363

R> portest(FitIBMSP5001, test = "gvtest", nslaves = nslaves)
```

```
        2 slaves are spawned successfully. 0 failed.
 Lags Statistic       df     p-value
    5  26.73298 12.36364 0.002997003
   10  50.16580 27.42857 0.003996004
   15  66.95921 42.45161 0.006993007
   20  87.59443 57.46341 0.004995005
   25 108.82328 72.47059 0.005994006
   30 128.30068 87.47541 0.003996004
```

```
R> gvtest(FitIBMSP5002)
```

```
 Lags Statistic        df    p-value
    5  16.24518  8.363636 0.04647938
   10  38.00564 23.428571 0.02910435
   15  54.26122 38.451613 0.04688480
   20  74.35787 53.463415 0.03091912
   25  95.55057 68.470588 0.01701104
   30 114.96754 83.475410 0.01272103
```

```
R> portest(FitIBMSP5002, test = "gvtest", nslaves = nslaves)
```

```
        2 slaves are spawned successfully. 0 failed.
 Lags Statistic        df     p-value
    5  16.24518  8.363636 0.006993007
   10  38.00564 23.428571 0.007992008
   15  54.26122 38.451613 0.016983017
   20  74.35787 53.463415 0.017982018
   25  95.55057 68.470588 0.008991009
   30 114.96754 83.475410 0.008991009
```

```
R> gvtest(FitIBMSP5003)
```

```
 Lags Statistic        df    p-value
    5  6.914649  4.363636 0.16977954
   10 24.655501 19.428571 0.18989321
   15 39.324113 34.451613 0.26078729
   20 58.297021 49.463415 0.18238250
   25 79.102500 64.470588 0.10384117
   30 98.361967 79.475410 0.07416598
```

```
R> portest(FitIBMSP5003, test = "gvtest", nslaves = nslaves)
```

```
        2 slaves are spawned successfully. 0 failed.
 Lags Statistic        df    p-value
    5  6.914649  4.363636 0.05394605
   10 24.655501 19.428571 0.06493506
```

```
15 39.324113 34.451613 0.11488511
20 58.297021 49.463415 0.08891109
25 79.102500 64.470588 0.06093906
30 98.361967 79.475410 0.04995005
```

While the fitted VAR (1) and VAR (2) models are rejected, the $\mathfrak{D}_m$ diagnostic suggests that the fitted VAR (3) maybe consider to be an adequate model.

5.3. Example 8

In this example, we consider the quarterly time series, 1960–1982, of West German investment, income, and consumption studied before.

We apply the statistic $\mathfrak{D}_m$ on the fitted VAR (2) model based on the asymptotic distribution and the Monte-Carlo significance test,

```
R> gvtest(FitWestGerman)

 Lags Statistic        df      p-value
    5  20.90960  18.81818 0.3310522834
   10  52.17337  52.71429 0.4951413528
   15  91.80348  86.51613 0.3283404972
   20 135.40962 120.29268 0.1637652676
   25 195.17389 154.05882 0.0139543395
   30 257.76048 187.81967 0.0005343724

R> portest(FitWestGerman, test = "gvtest", nslaves = nslaves)

        2 slaves are spawned successfully. 0 failed.
 Lags Statistic        df   p-value
    5  20.90960  18.81818 0.3116883
   10  52.17337  52.71429 0.5424575
   15  91.80348  86.51613 0.5624376
   20 135.40962 120.29268 0.5954046
   25 195.17389 154.05882 0.4005994
   30 257.76048 187.81967 0.3486513
```

Using the asymptotic distribution, results suggest that the VAR (2) model is adequate at lags $m < 25$ and inadequate at lags $m \geq 25$, whereas Monte-Carlo test supports the choice of the VAR (2) model.

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