Robust Generalized Linear Models

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The CC-family contains functions of composite of concave and convex functions. The CC-estimators are derived from minimizing loss functions in the CC-family via composite optimization by conjugation operator (COCO). The CC-estimators include robust generalized linear models. See Wang (2020).

Robust logistic regression

In a UK hospital, 135 expectant mothers were surveyed on the decision of breastfeeding their babies or not, along with two-level predictive factors. Description and references can be found in Heritier et al. (2009).

```r
require("mpath")
data(breastfeed)
```

Remove rows with missing values.

```r
breastfeed=na.omit(breastfeed)
```

We compute binomial-induced CC-estimators, i.e., robust logistic regression, and display the robust weights for each model.

```r
sval <- c(1.5, 1.5, 5, 2.5, 3.5, 2.5, 2.2, 7)
cfun <- c("hcave", "acave", "bcave", "ccave", "dcave", "gcave", "tcave", "ecave")
id <- 1:8
for(i in c(1:5,8,6,7)){
fitnew <- ccglm(breast~., data=breastfeed, s=sval[i], cfun=i, dfun=binomial(),
                trace=FALSE)
goodid <- sort.list(fitnew$weights_update)[id]
plot(fitnew$weights_update, type="n", ylab="weight",
     main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")")),
                           list(cfun=cfun[i], s = sval[i]))))
points(fitnew$weights_update[-goodid], ylab="weight",
       main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")")),
                             list(cfun=cfun[i], s = sval[i]))))
points(sort.list(fitnew$weights_update)[id], sort(fitnew$weights_update)[id], pch=3,
       col="red")
}
```

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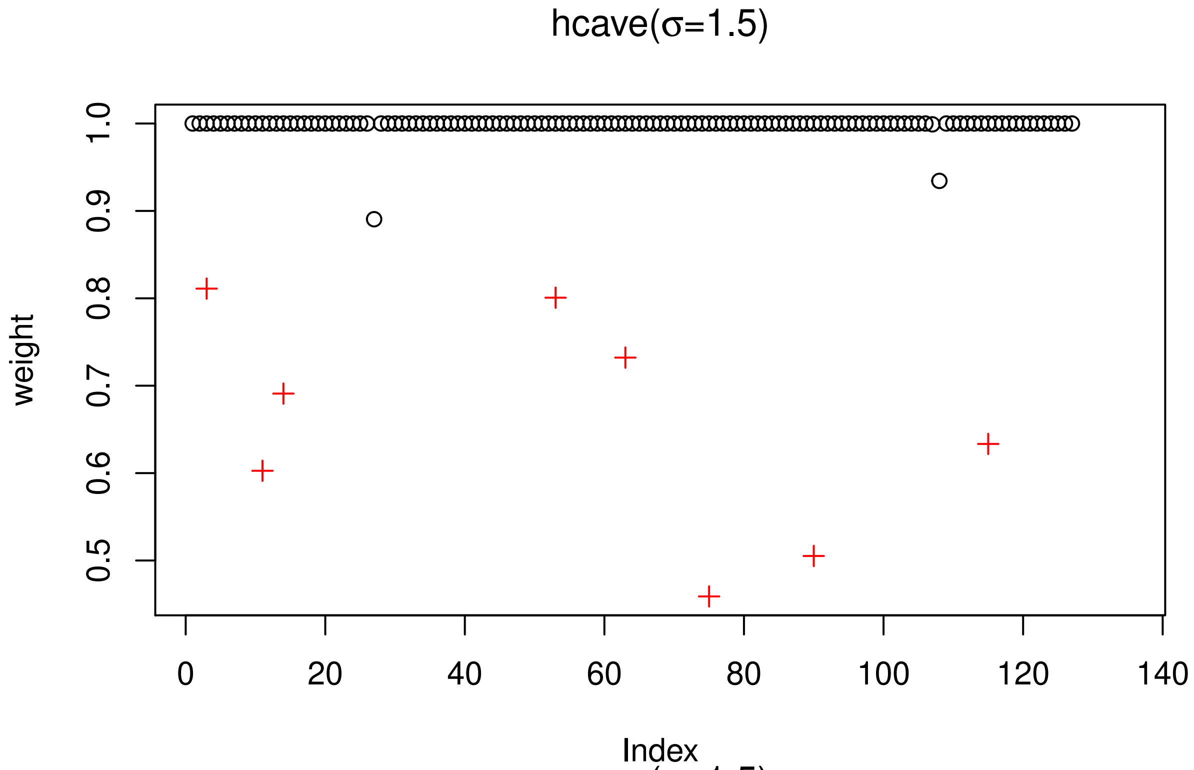
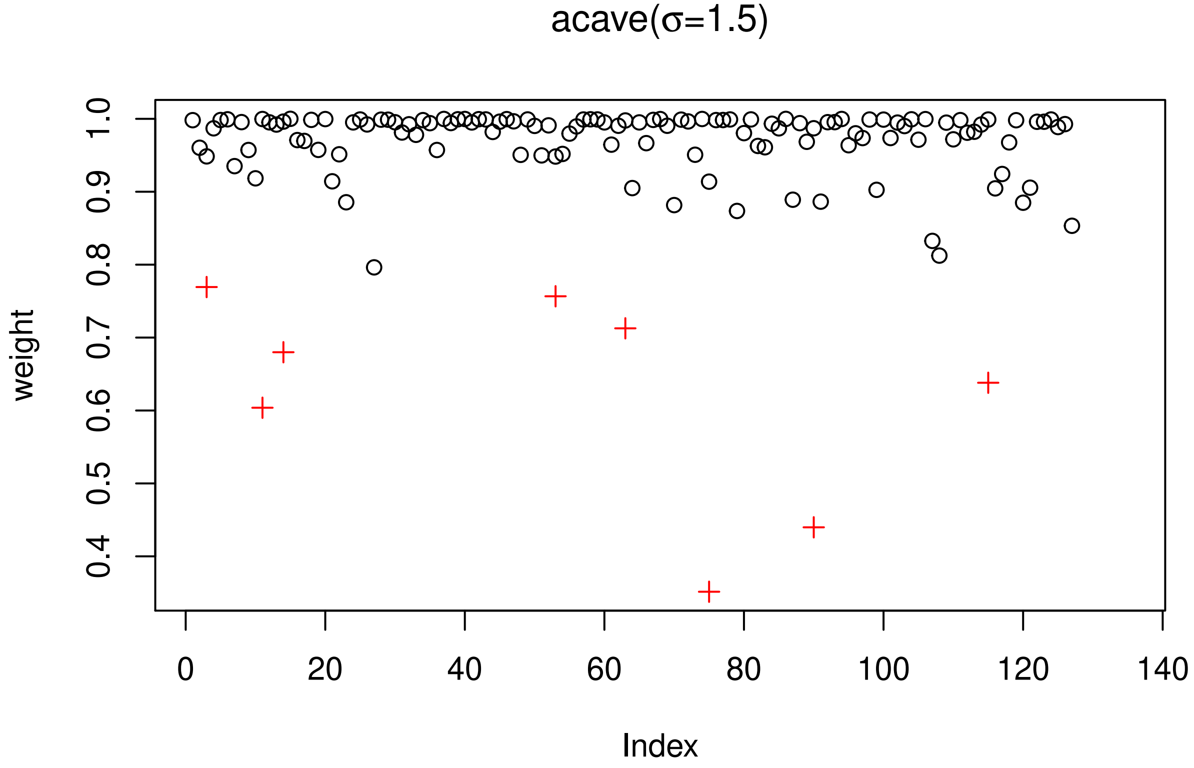
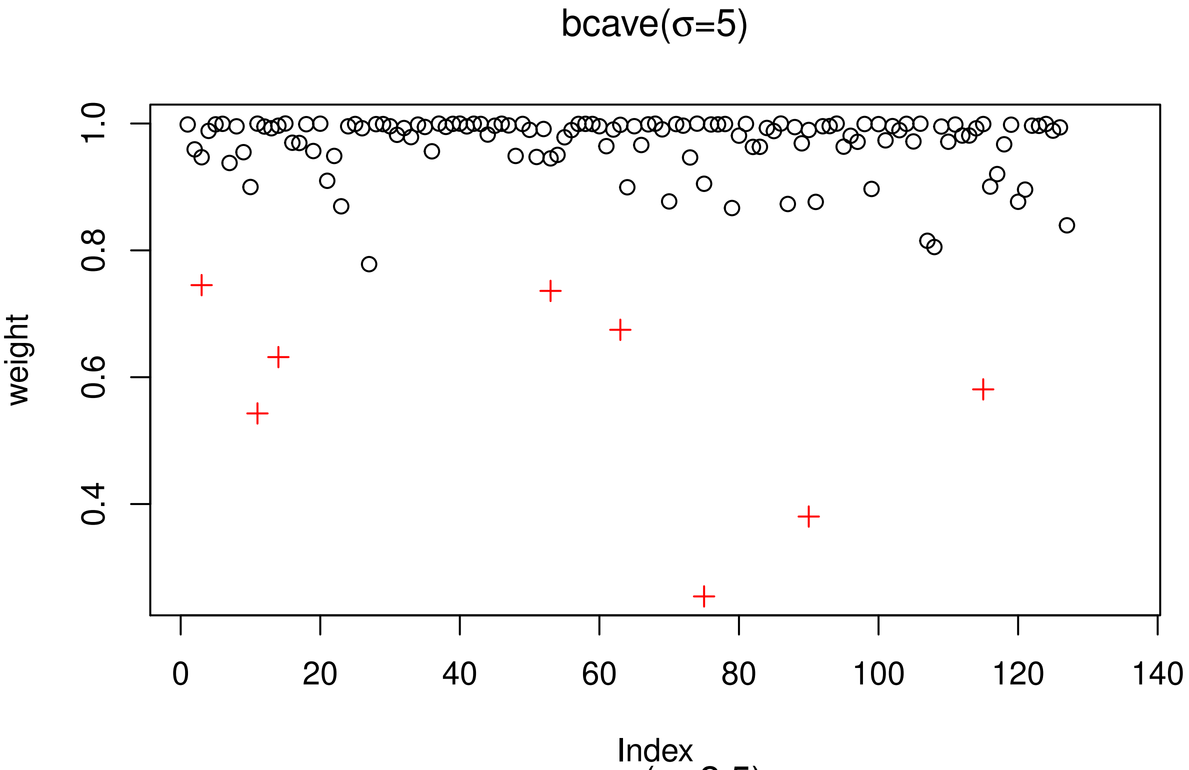
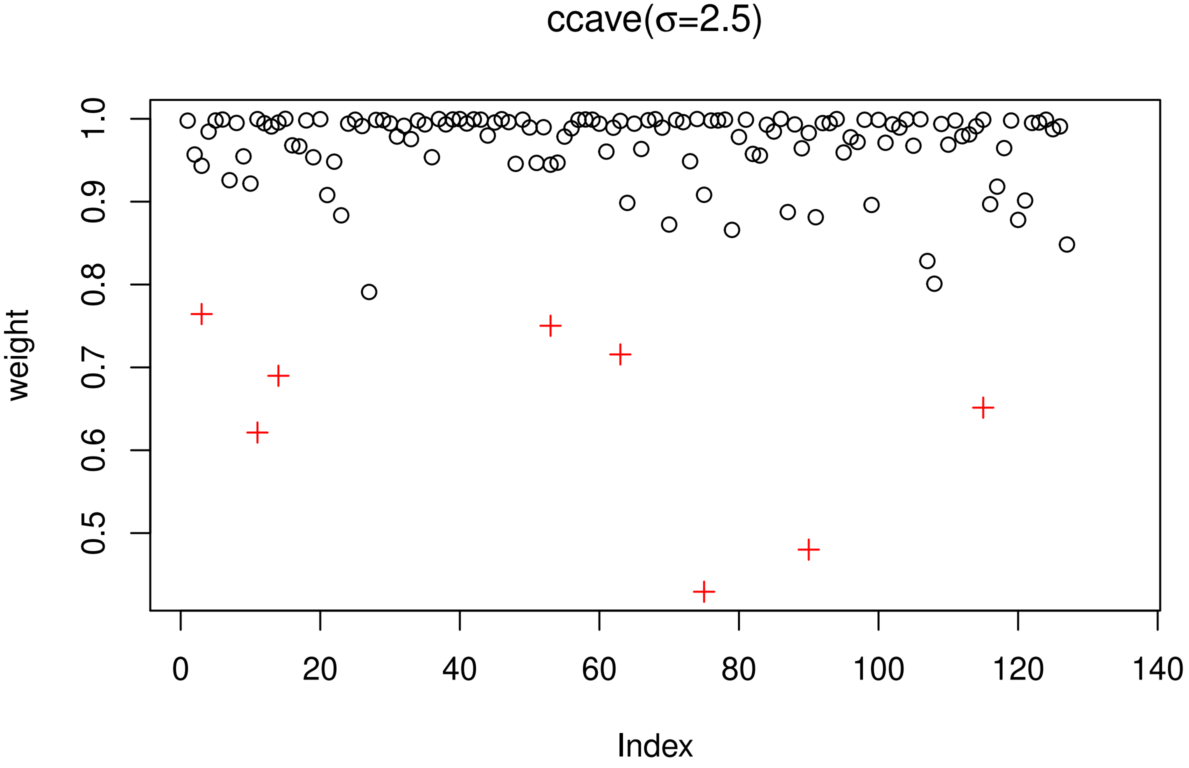
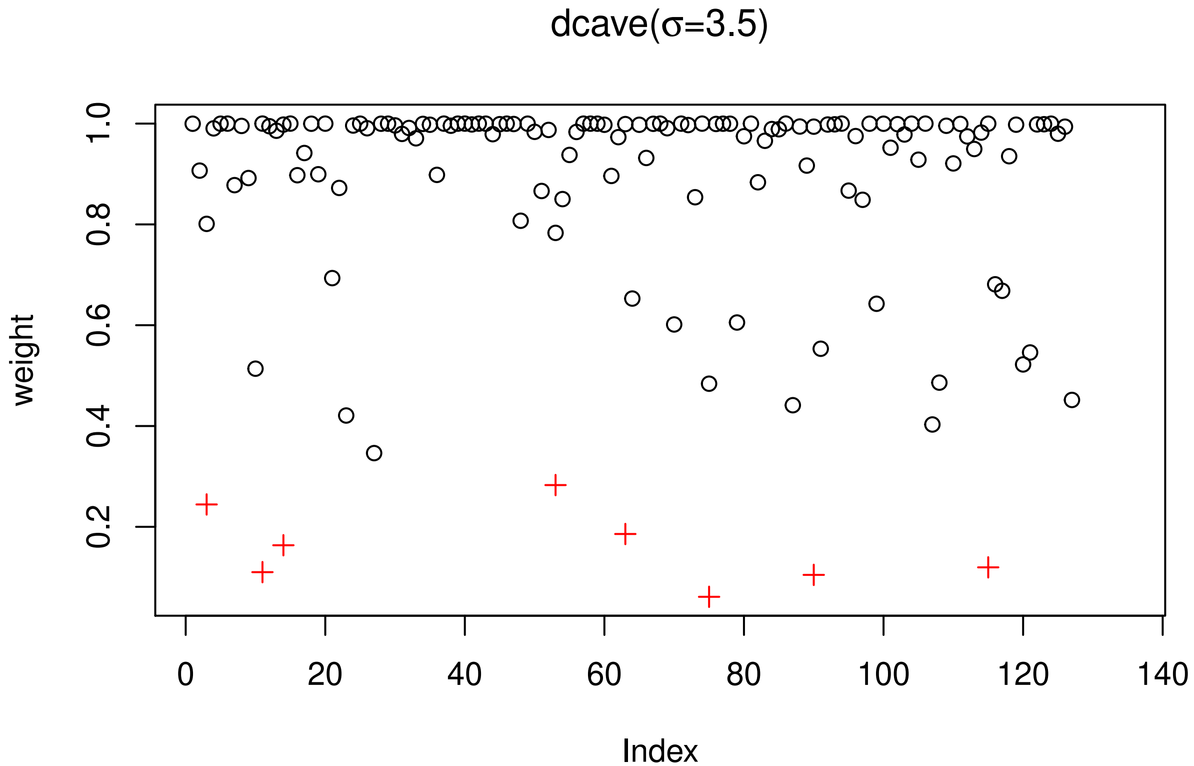
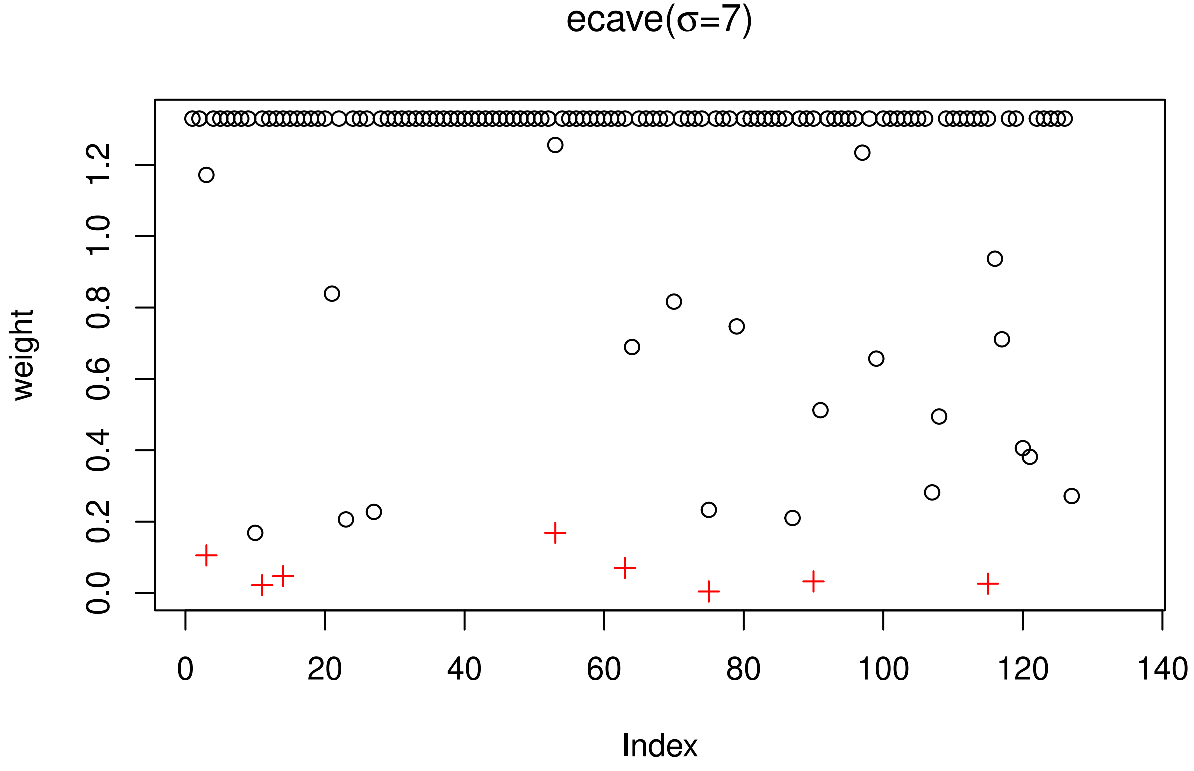

hcave(σ=1.5)
weight
1.0
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100
120
140
Index

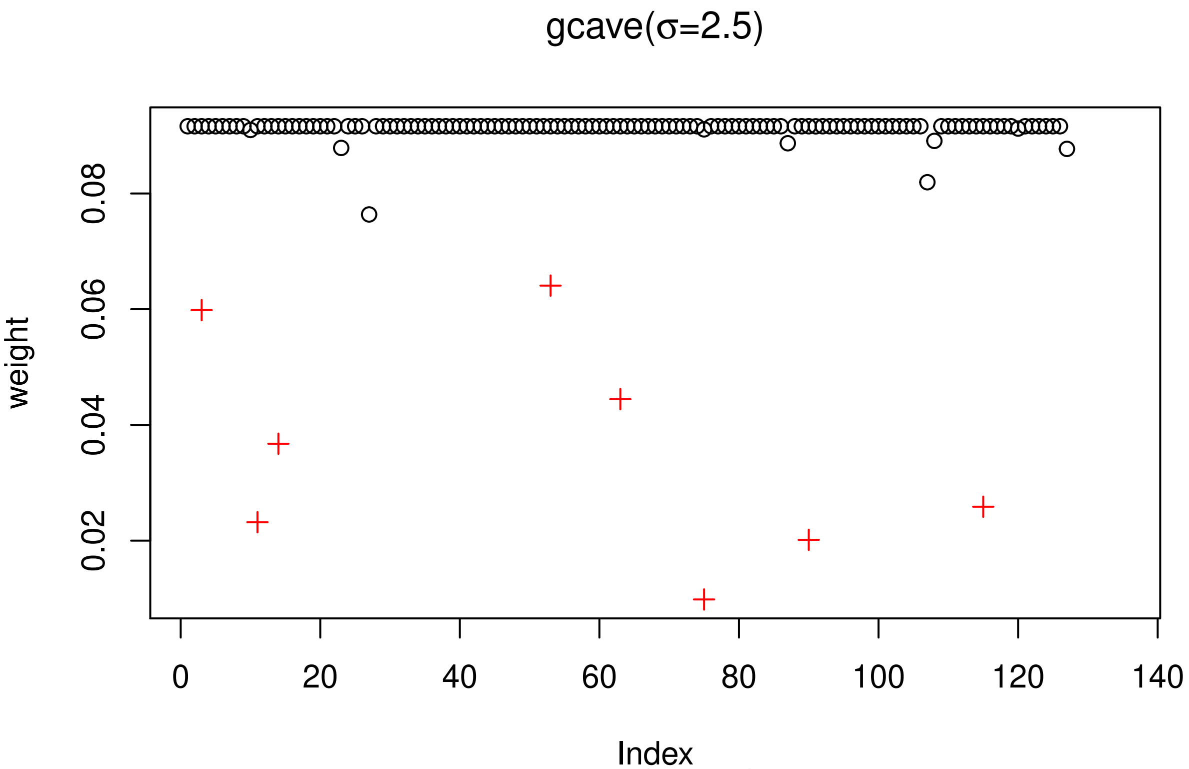

acave(σ=1.5)
weight
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0
20
40
60
80
100
120
140
Index

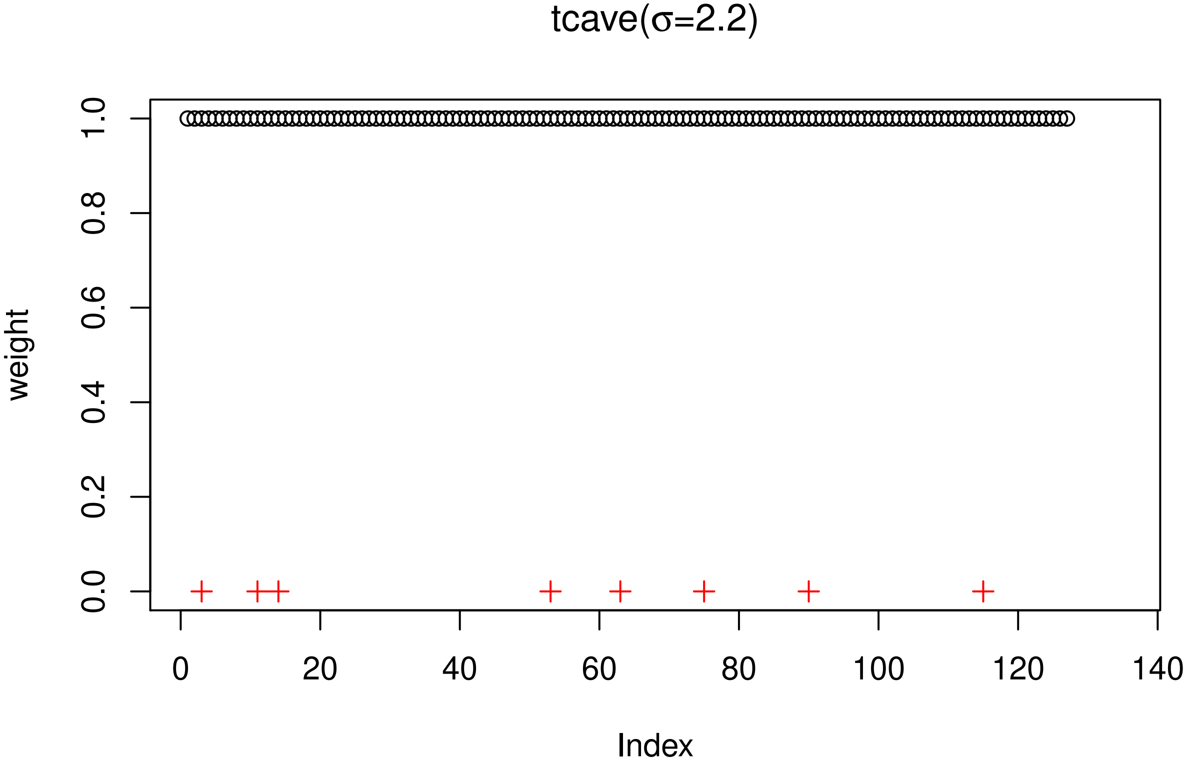

bcave(σ=5)
weight
1.0
0.8
0.6
0.4
0
20
40
60
80
100
120
140
Index


ccave(σ=2.5)
weight
1.0
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100
120
140
Index


dcave(σ=3.5)
weight
1.0
0.8
0.6
0.4
0.2
0
20
40
60
80
100
120
140
Index


ecave(σ=7)
weight
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
140
Index





Despite large estimated probability ≥ 0.8 of trying to breastfeed or not in a logistic regression, these individuals took the opposite decisions.

```
fit.glm <- glm(as.integer(breast)-1~., data=breastfeed, family=binomial())
id <- c(3, 11, 14, 53,63, 75,90, 115)
pred <- predict(fit.glm, type="response") ### predicted probabilities
```

```r
cbind(breastfeed, pred)[id,]
```

```
##     breast pregnancy howfed howfedfr partner smokenow smokebf age educat
## 3   Bottle Beginning Breast   Breast Partner       No      No  39     16
## 12  Bottle Beginning Breast   Breast  Single       No      No  29     18
## 15  Bottle Beginning Bottle   Breast Partner       No      No  33     21
## 56  Bottle       End Bottle   Breast Partner       No      No  25     16
## 66  Breast       End Bottle   Bottle Partner      Yes     Yes  27     16
## 78  Bottle Beginning Breast   Bottle Partner       No      No  28     28
## 93  Breast Beginning Bottle   Bottle  Single      Yes     Yes  19     16
## 118 Bottle Beginning Breast   Breast  Single       No      No  20     18
##        ethnic       pred
## 3       White 0.77424035
## 12  Non-white 0.89871707
## 15      White 0.83665383
## 56      White 0.82204044
## 66      White 0.18544970
## 78  Non-white 0.97026037
## 93      White 0.02275785
## 118 Non-white 0.87454933
```

For variable selection, we develop a usual penalized LASSO logistic regression, where the optimal penalty parameter λ is chosen by a 10-fold cross-validation.

```r
n.cores <- 2
set.seed(195)
fitcv.glm <- cv.glmreg(as.integer(breast)-1~., data=breastfeed, penalty="enet",
                       family="binomial", type="loss",plot.it=FALSE, parallel=TRUE,
                       n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized logistic regression

```r
min(fitcv.glm$cv)
```

```
## [1] -7.815975
```

Penalized logistic regression with penalty LASSO

```r
coef(fit)[,fitcv.glm$lambda.which]
```

```
##         (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##        -2.492768348       -0.552721156        0.257557827        1.220755264
##      partnerPartner        smokenowYes         smokebfYes                age
##         0.862448995       -2.252414341        0.623626979        0.002488979
##              educat    ethnicNon-white
##         0.119557901        1.541320581
```

Compute the SCAD logistic regression, where the optimal penalty parameter λ is chosen by a 10-fold cross-validation. The SCAD logistic regression is more sparse than the LASSO estimator.

```r
set.seed(195)
fitcv.glm <- cv.glmreg(as.integer(breast)-1~., data=breastfeed, penalty="snet",
                       family="binomial", type="loss", plot.it=FALSE, parallel=TRUE,
                       n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized logistic regression

```r
min(fitcv.glm$cv)
```

```
## [1] -7.815975
```

Penalized logistic regression with penalty SCAD

```r
coef(fit)[,fitcv.glm$lambda.which]
```

```
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##         0.09874844         0.00000000         0.00000000         1.04702265
##     partnerPartner        smokenowYes         smokebfYes                age
##         0.47532632        -2.00125709         0.00000000         0.00000000
##             educat    ethnicNon-white
##         0.00000000         1.94414156
```

The λ value in SCAD is then utilized to compute binomial-induced SCAD CC-estimators for various concave components.

```r
for(i in c(1:5,8,6,7)){
  cat("\ncfun=", cfun[i], "\n")
    fit.ccglmreg <- ccglmreg(breast~., data=breastfeed, s=sval[i], cfun=i, penalty="snet",
                       lambda=fitcv.glm$lambda.optim, dfun=binomial(), parallel=FALSE,
                       type.path="nonactive", standardize=TRUE)
    print(coef(fit.ccglmreg))
}
```

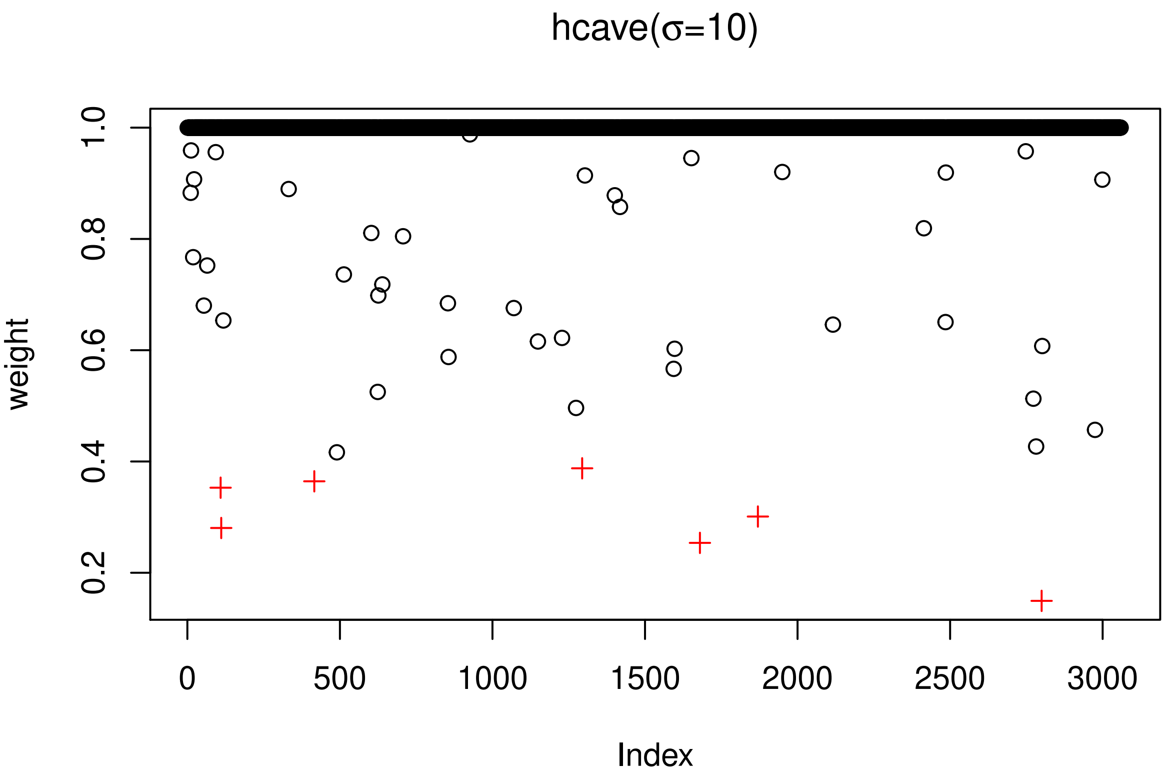
```
## 
## cfun= hcave 
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##        -0.20262257         0.00000000         0.00000000         1.41623162
##     partnerPartner        smokenowYes         smokebfYes                age
##         0.24121875        -2.31220066         0.00000000         0.00000000
##             educat    ethnicNon-white
##         0.02524874         2.48775264
## 
## cfun= acave 
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##        0.323505087        0.000000000        0.000000000        1.194139787
##     partnerPartner        smokenowYes         smokebfYes                age
##        0.197984927       -2.383687907        0.000000000        0.000000000
##             educat    ethnicNon-white
##        0.008521202        2.523000073
## 
## cfun= bcave 
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##         0.32721541         0.00000000         0.00000000         1.20998505
##     partnerPartner        smokenowYes         smokebfYes                age
##         0.12916071        -2.44460015         0.00000000         0.00000000
##             educat    ethnicNon-white
##         0.01285265         2.63969594
## 
## cfun= ccave 
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##        0.354230145        0.000000000        0.000000000        1.177822530
##     partnerPartner        smokenowYes         smokebfYes                age
##        0.224698662       -2.376636566        0.000000000        0.000000000
##             educat    ethnicNon-white
##        0.006084647        2.483034951
##
## cfun= dcave
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##         2.71145780         0.00000000         0.12034058         0.02718298
##     partnerPartner        smokenowYes         smokebfYes                age
##         0.00000000        -3.89297984         0.00000000         0.00000000
##             educat    ethnicNon-white
##         0.00000000         1.15836471
##
## cfun= ecave
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##         3.26756873         0.00000000         0.00000000         0.05330153
##     partnerPartner        smokenowYes         smokebfYes                age
##         0.00000000        -4.24833062         0.00000000         0.00000000
##             educat    ethnicNon-white
##         0.00000000         2.44864324
##
## cfun= gcave
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##        -0.70232531         0.00000000         0.00000000         1.75532959
##     partnerPartner        smokenowYes         smokebfYes                age
##         0.00000000        -2.68637620         0.00000000         0.00000000
##             educat    ethnicNon-white
##         0.06456509         3.24729928
##
## cfun= tcave
##        (Intercept) pregnancyBeginning       howfedBreast     howfedfrBreast
##         -2.2679907          0.0000000          0.0000000          1.2678561
##     partnerPartner        smokenowYes         smokebfYes                age
##          0.0000000         -2.4827295          0.0000000          0.0000000
##             educat    ethnicNon-white
##          0.1648061          3.5883907
```

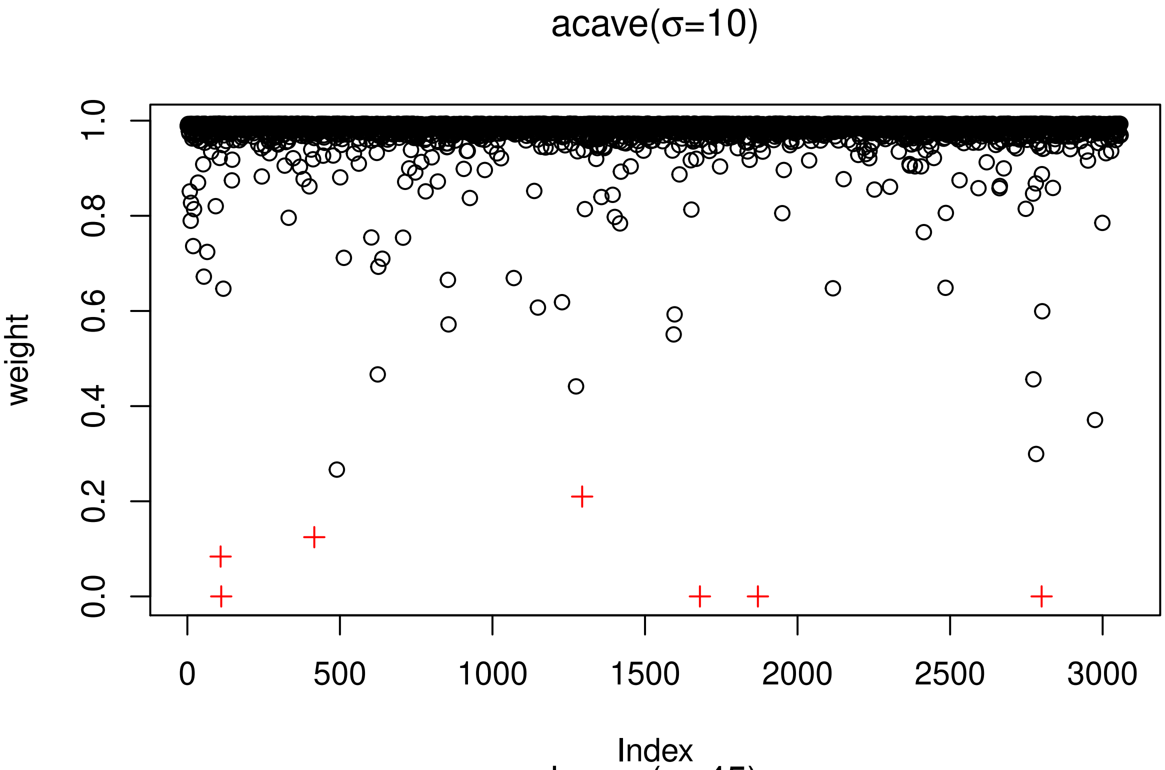
Robust Poisson regression

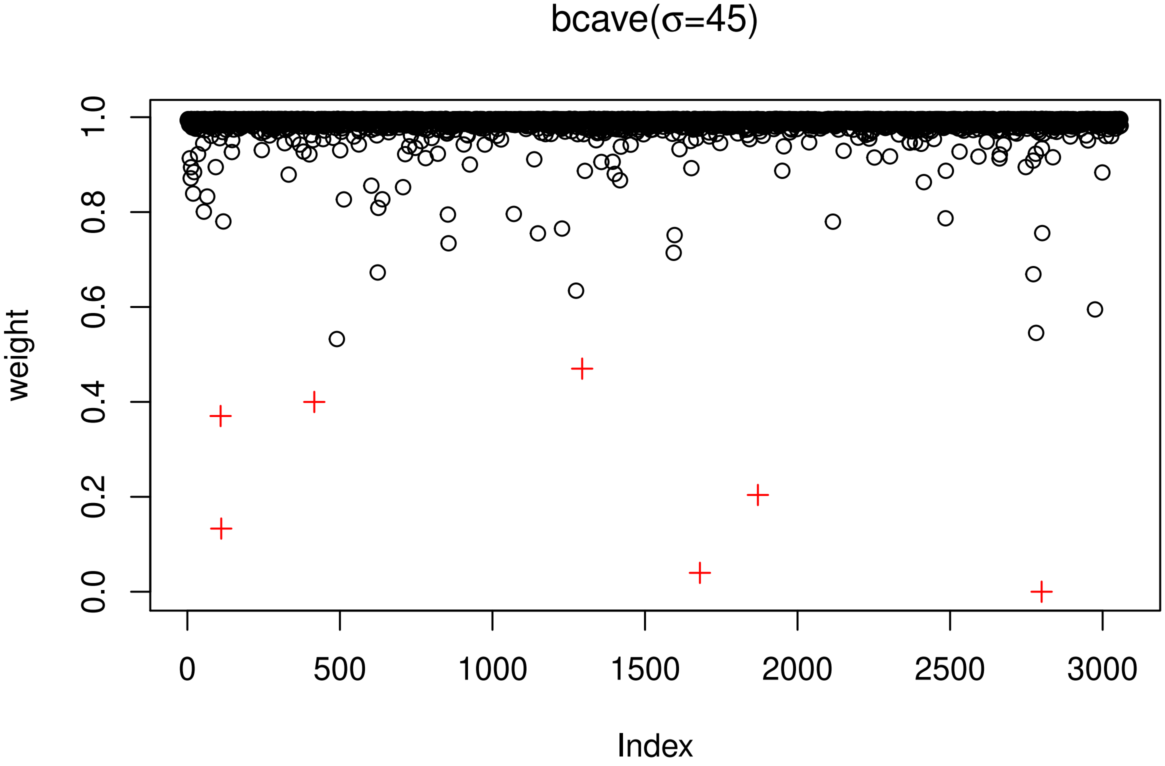
A cohort of 3066 Americans over the age of 50 were studied on health care utilization, doctor office visits Heritier et al. (2009). The survey also contained 24 predictors in demographic, health needs and economic access. We compute Poisson-induced CC-estimators, i.e., robust Poisson regressions. The seven smallest weights occur to the subjects with 200, 208, 224, 260, 300, 365 and 750 doctor visits in two years.

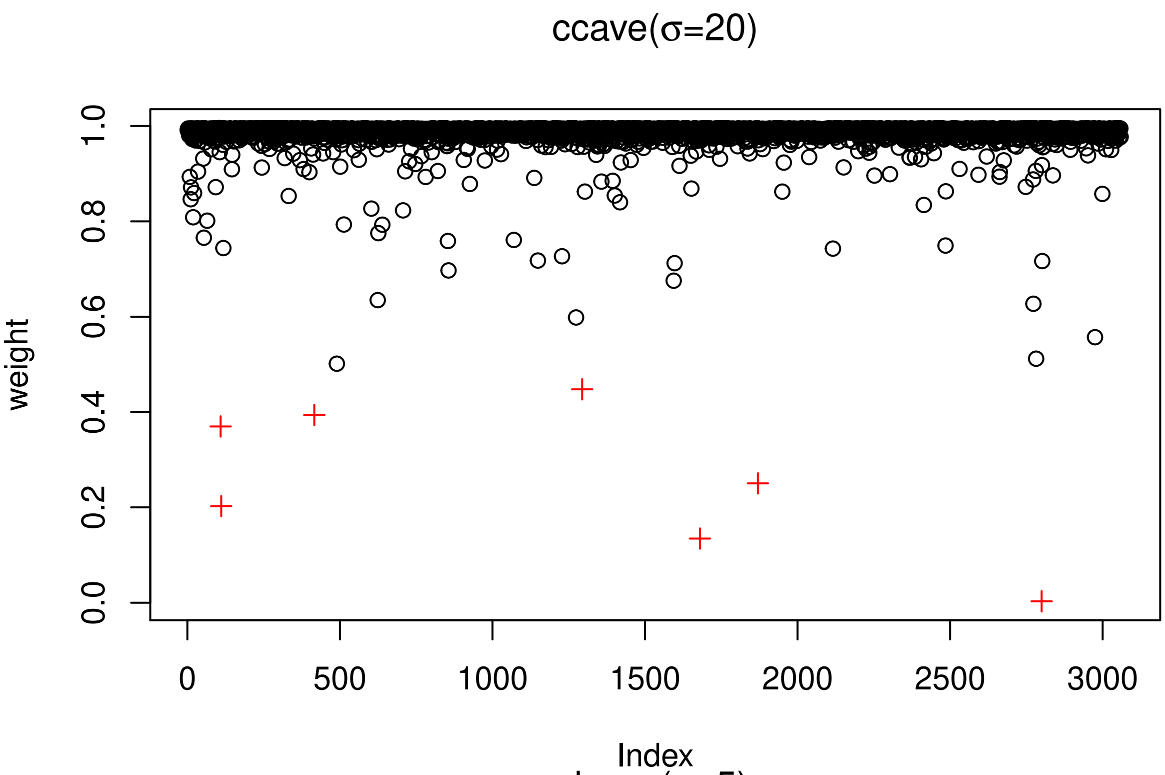
```r
data(docvisits)
sval <- c(10, 10, 45, 20, 5, 5, 280, 200)
cfun <- c("hcave", "acave", "bcave", "ccave", "dcave", "gcave", "tcave", "ecave")
id <- 1:7
for(i in c(1:5,8,6,7)){
     fitnew <-ccglm(visits~age+factor(gender)+factor(race)+factor(hispan)
                    +factor(marital)+factor(arthri)+factor(cancer)
                    +factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
                    +factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
                    +edyears+feduc+meduc+log(income+1)+factor(insur),
                    data=docvisits,cfun=i,s=sval[i],dfun=poisson(),trace=FALSE)
```

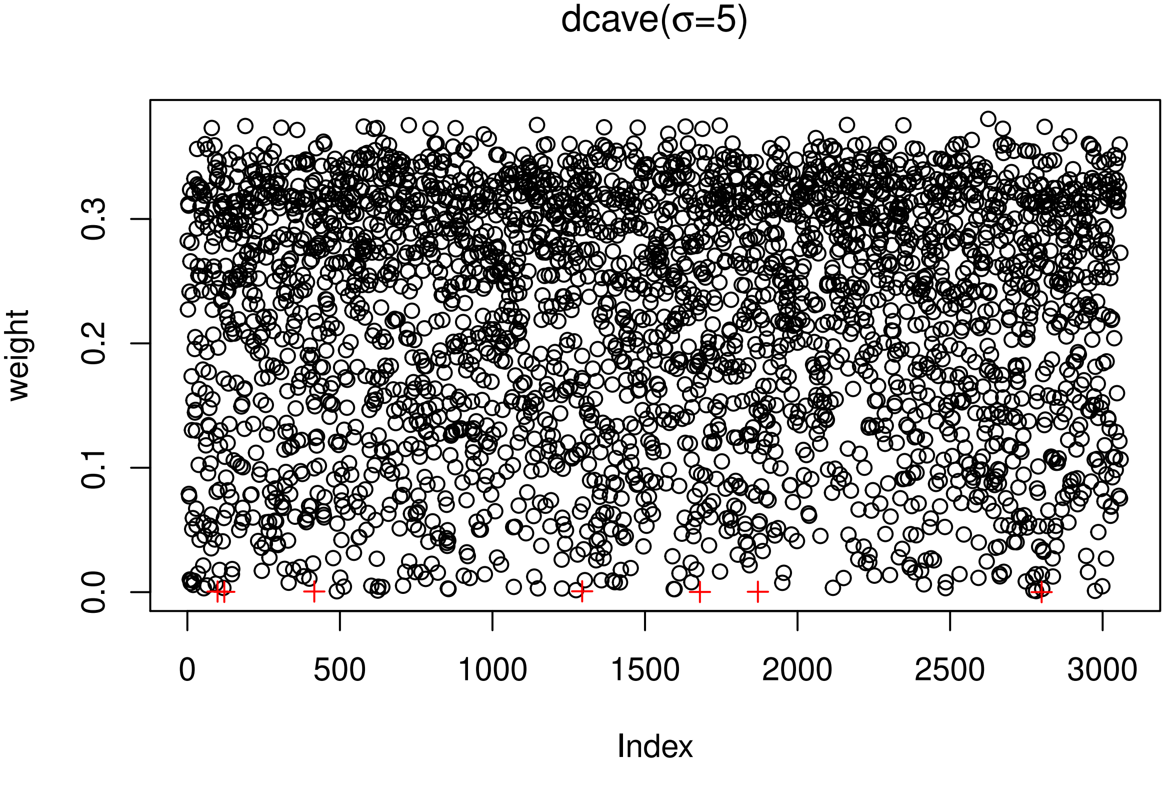
```
    goodid <- sort.list(fitnew$weights_update)[id]
    plot(fitnew$weights_update, type="n", ylab="weight",
         main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")")),
                                list(cfun=cfun[i], s = sval[i]))))
    points(fitnew$weights_update[-goodid], ylab="weight",
           main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")")),
                                  list(cfun=cfun[i], s = sval[i]))))
if(i > 4){
         ### deal with overlapped points: obs 109, 111
         x <- sort.list(fitnew$weights_update)[id]
         y <- sort(fitnew$weights_update)[id]
         xnew <- sort(x)
         ynew <- y[sort.list(x)]
         points(xnew[1]-10, ynew[1], pch=3,  col="red")
         points(xnew[2]+10, ynew[2], pch=3,  col="red")
         points(xnew[3:7], ynew[3:7], pch=3,  col="red")
}
else points(sort.list(fitnew$weights_update)[id], sort(fitnew$weights_update)[id],
            pch=3, col="red")
}
```

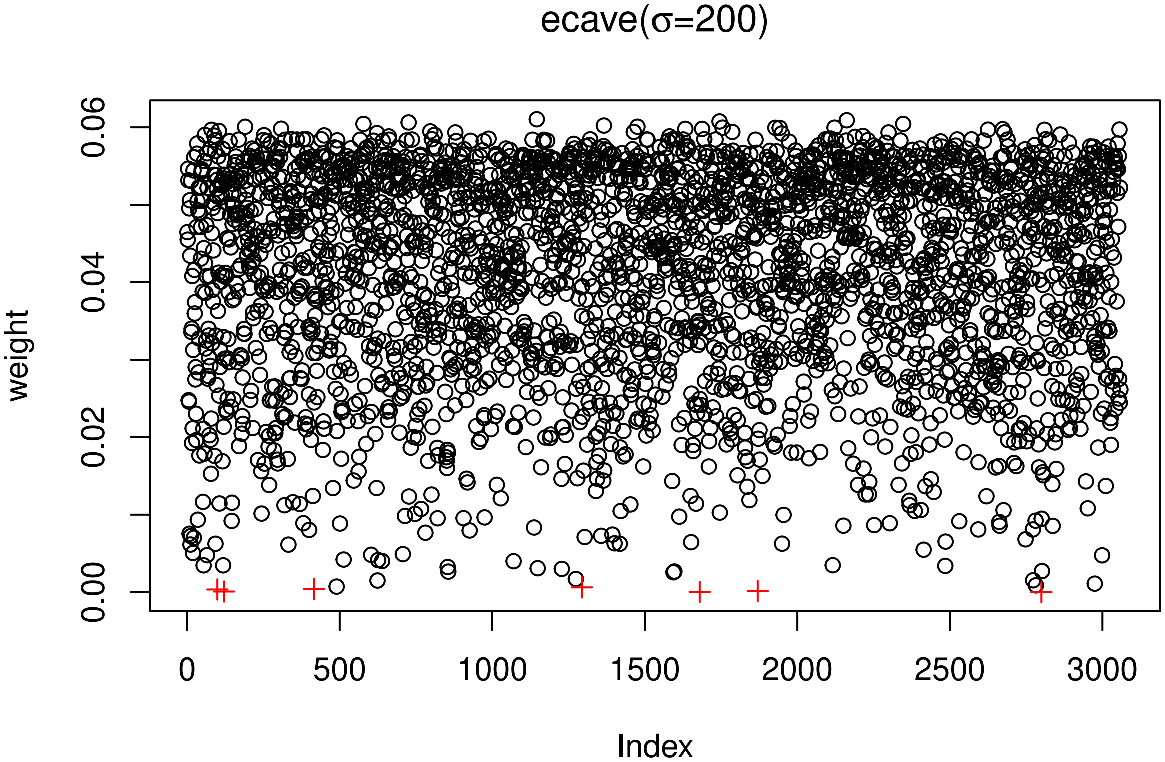


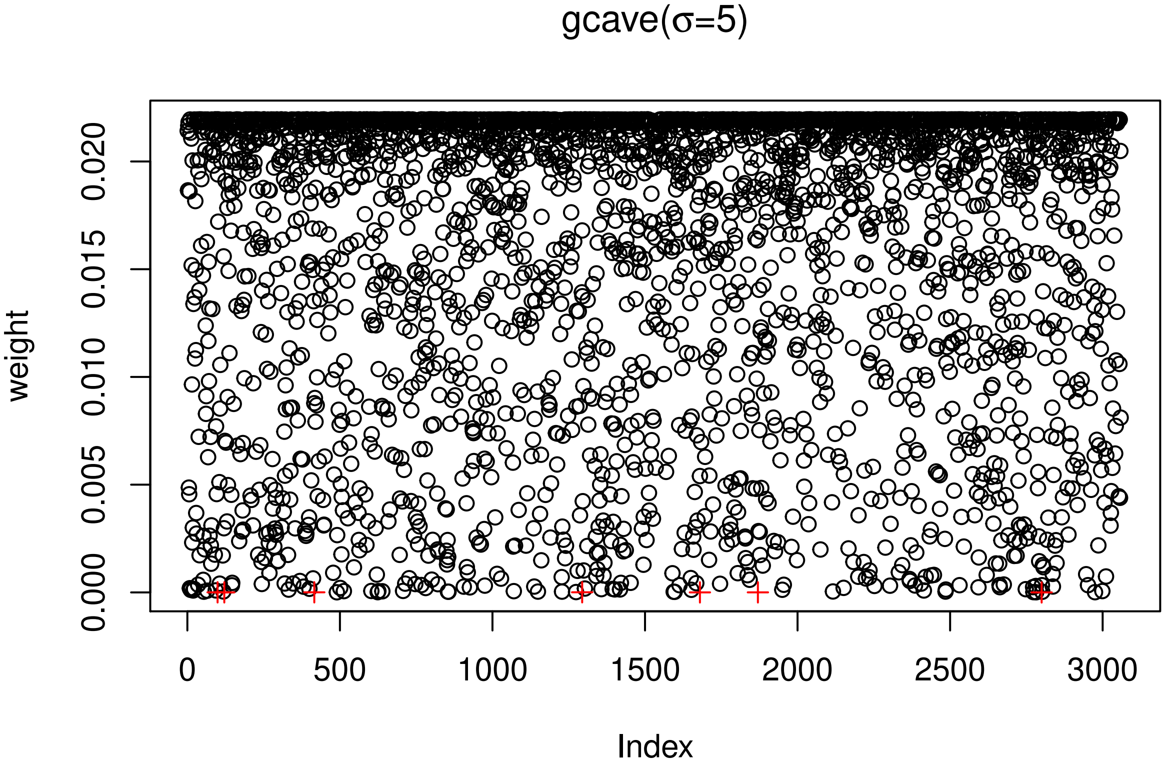

acave(σ=10)
weight
1.0
0.8
0.6
0.4
0.2
0.0
0
500
1000
1500
2000
2500
3000
Index

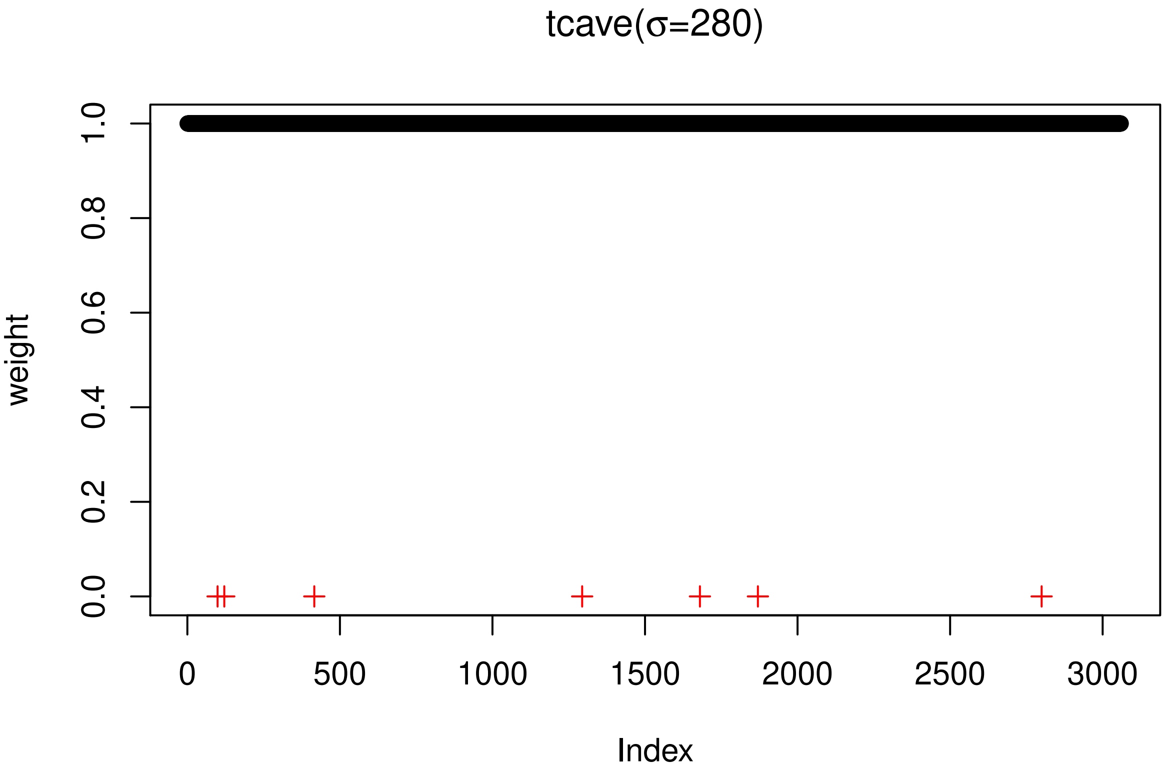

bcave(σ=45)
weight
1.0
0.8
0.6
0.4
0.2
0.0
0
500
1000
1500
2000
2500
3000
Index


ccave(σ=20)
weight
0.0
0.2
0.4
0.6
0.8
1.0
0
500
1000
1500
2000
2500
3000
Index


dcave(σ=5)
weight
0.0
0.1
0.2
0.3
0
500
1000
1500
2000
2500
3000
Index


ecave(σ=200)
weight
0.00
0.02
0.04
0.06
0
500
1000
1500
2000
2500
3000
Index


gcave(σ=5)
weight
0.000
0.005
0.010
0.015
0.020
0
500
1000
1500
2000
2500
3000
Index



Outliers of office visits

```
newid <- sort(sort.list(fitnew$weights_update)[id])
docvisits$visits[newid]
```

```
## [1] 224 300 208 200 365 260 750
```

Penalized Poisson regression with LASSO penalty. The tuning parameter λ value is chosen by cross-validation.

```
set.seed(195)
fitcv.glm <- cv.glmreg(visits~age+factor(gender)+factor(race)+factor(hispan)
                       +factor(marital)+factor(arthri)+factor(cancer)
                       +factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
                       +factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
                       +edyears+feduc+meduc+log(income+1)+factor(insur),
                       data=docvisits,family="poisson", penalty="enet", type="loss",
                       plot.it=FALSE, parallel=TRUE, n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized Poisson regression

```
min(fitcv.glm$cv)
```

```
## [1] -2894.529
```

Penalized Poisson regression with penalty LASSO

```
coef(fit)[,fitcv.glm$lambda.which]
```

```
##      (Intercept)               age   factor(gender)1    factor(race)1
##      1.958525362      -0.006577549       0.058079225     -0.016603770
##  factor(hispan)1 factor(marital)1   factor(arthri)1  factor(cancer)1
##      0.005810267       0.000000000       0.087497840      0.115530929
## factor(hipress)1  factor(diabet)1     factor(lung)1  factor(hearth)1
##      0.139162108      0.272014643      0.071727840      0.264918089
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##      0.069391756      0.220222838      0.068125763      0.043447857
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##      0.068932457      0.312374326      0.616403088      0.545308893
##          edyears            feduc            meduc  log(income + 1)
##      0.009212421     -0.023840699      0.000000000      0.058544656
##   factor(insur)1
##      0.072032891
```

Penalized Poisson regression with SCAD penalty. The tuning parameter λ value is chosen by cross-validation.

```r
set.seed(195)
fitcv.glm <- cv.glmreg(visits~age+factor(gender)+factor(race)+factor(hispan)
                      +factor(marital)+factor(arthri)+factor(cancer)
                      +factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
                      +factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
                      +edyears+feduc+meduc+log(income+1)+factor(insur),
                      data=docvisits, family="poisson", penalty="snet", type="loss",
                      plot.it=FALSE, parallel=TRUE, n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized Poisson regression

```r
min(fitcv.glm$cv)
```

```
## [1] -2894.529
```

Penalized Poisson regression with penalty SCAD

```r
coef(fit)[,fitcv.glm$lambda.which]
```

```
##      (Intercept)              age  factor(gender)1    factor(race)1
##      1.858069526     -0.003786934      0.000000000      0.000000000
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##      0.000000000      0.000000000      0.029059308      0.067717808
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##      0.120234297      0.296950385      0.000000000      0.291999839
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##      0.001233415      0.252816298      0.000000000      0.000000000
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##      0.000000000      0.368482223      0.681643963      0.641110056
##          edyears            feduc            meduc  log(income + 1)
##      0.004754850      0.000000000      0.000000000      0.040644933
##   factor(insur)1
##      0.017165257
```

The λ value in SCAD is then utilized to compute robust Poisson SCAD CC-estimators for various concave components.

```r
for(i in c(1:5,8,6,7)){
  cat("\ncfun=", cfun[i], "\n")
  fit.ccglmreg <- ccglmreg(visits~age+factor(gender)+factor(race)+factor(hispan)
                     +factor(marital)+factor(arthri)+factor(cancer)
                     +factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
                     +factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
                     +edyears+feduc+meduc+log(income+1)+factor(insur),
                     data=docvisits, s=sval[i], cfun=i, penalty="snet",
                     lambda=fitcv.glm$lambda.optim, dfun=poisson(), parallel=FALSE,
                     type.path="nonactive", standardize=TRUE)
 print(coef(fit.ccglmreg))
}
```

```
##
## cfun= hcave
##      (Intercept)              age  factor(gender)1    factor(race)1
##       1.98635667       0.00000000       0.00000000       0.00000000
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##       0.00000000       0.00000000       0.03808422       0.03052763
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##       0.10857169       0.22472945       0.01353496       0.32244446
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##       0.04586816       0.26510976       0.00000000       0.00000000
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##       0.00000000       0.25088902       0.43649856       0.53724280
##          edyears            feduc            meduc  log(income + 1)
##       0.00000000       0.00000000       0.00000000       0.00000000
##   factor(insur)1
##       0.00000000
##
## cfun= acave
##      (Intercept)              age  factor(gender)1    factor(race)1
##       1.98189657       0.00000000       0.00000000       0.00000000
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##       0.00000000       0.00000000       0.05203613       0.03289539
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##       0.08280116       0.19573875       0.02543506       0.33252481
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##       0.06802881       0.28272718       0.00000000       0.00000000
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##       0.00000000       0.14459462       0.36550972       0.48845297
##          edyears            feduc            meduc  log(income + 1)
##       0.00000000       0.00000000       0.00000000       0.00000000
##   factor(insur)1
##       0.00000000
##
## cfun= bcave
##      (Intercept)              age  factor(gender)1    factor(race)1
##     1.978829e+00    -5.216532e-05     0.000000e+00     0.000000e+00
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##     0.000000e+00     0.000000e+00     3.802145e-02     1.872132e-02
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##     1.222130e-01     1.954667e-01     2.796854e-02     3.279856e-01
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##     7.044509e-02     2.903266e-01     0.000000e+00     0.000000e+00
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##     0.000000e+00     2.729839e-01     3.864905e-01     5.060047e-01
##          edyears            feduc            meduc  log(income + 1)
##     0.000000e+00     0.000000e+00     0.000000e+00     0.000000e+00
##   factor(insur)1
##     0.000000e+00
##
```

```
## cfun= ccave
##      (Intercept)              age  factor(gender)1    factor(race)1
##     1.976567e+00    -4.375527e-05     0.000000e+00     0.000000e+00
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##     0.000000e+00     0.000000e+00     3.463168e-02     1.715012e-02
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##     1.290332e-01     1.881076e-01     2.466065e-02     3.285646e-01
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##     6.225529e-02     2.828722e-01     0.000000e+00     0.000000e+00
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##     0.000000e+00     2.736037e-01     3.977317e-01     5.175466e-01
##          edyears            feduc            meduc  log(income + 1)
##     0.000000e+00     0.000000e+00     0.000000e+00     0.000000e+00
##   factor(insur)1
##     0.000000e+00
##
## cfun= dcave
##      (Intercept)              age  factor(gender)1    factor(race)1
##       1.82551207       0.00000000       0.00000000       0.00000000
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##       0.00000000       0.00000000       0.02685254       0.00000000
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##       0.05332835       0.02872780       0.00000000       0.35662745
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##       0.00000000       0.03407841       0.00000000       0.00000000
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##       0.00000000       0.00000000       0.00000000       0.60245837
##          edyears            feduc            meduc  log(income + 1)
##       0.00000000       0.00000000       0.00000000       0.00000000
##   factor(insur)1
##       0.00000000
##
## cfun= ecave
##      (Intercept)              age  factor(gender)1    factor(race)1
##      1.881486009      0.000000000      0.000000000      0.000000000
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##      0.000000000      0.000000000      0.034132803      0.006426225
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##      0.066216272      0.067616141      0.000000000      0.346514697
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##      0.000000000      0.080270550      0.000000000      0.000000000
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##      0.000000000      0.046662355      0.359270807      0.592414299
##          edyears            feduc            meduc  log(income + 1)
##      0.000000000      0.000000000      0.000000000      0.000000000
##   factor(insur)1
##      0.000000000
##
## cfun= gcave
##      (Intercept)              age  factor(gender)1    factor(race)1
##       1.78471352       0.00000000       0.00000000       0.00000000
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##       0.00000000       0.00000000       0.02635143       0.00000000
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##       0.06526733       0.01393760       0.00000000       0.34003347
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##       0.00000000       0.02149100       0.00000000       0.00000000
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##       0.00000000       0.00000000       0.00000000       0.65357731
##          edyears            feduc            meduc  log(income + 1)
##       0.00000000       0.00000000       0.00000000       0.00000000
##   factor(insur)1
##       0.00000000
##
## cfun= tcave
##      (Intercept)              age  factor(gender)1    factor(race)1
##      1.969315265      0.000000000      0.000000000      0.000000000
##  factor(hispan)1 factor(marital)1  factor(arthri)1  factor(cancer)1
##      0.000000000      0.000000000      0.064288784      0.030497991
## factor(hipress)1  factor(diabet)1    factor(lung)1  factor(hearth)1
##      0.076827493      0.244339304      0.030909720      0.325954228
##  factor(stroke)1   factor(psych)1   factor(iadla)1   factor(iadla)2
##      0.130262217      0.306230170      0.001956634      0.000000000
##   factor(iadla)3   factor(adlwa)1   factor(adlwa)2   factor(adlwa)3
##      0.000000000      0.200614987      0.372065923      0.460492140
##          edyears            feduc            meduc  log(income + 1)
##      0.001441250      0.000000000      0.000000000      0.000000000
##   factor(insur)1
##      0.000000000
```

References

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