Crash Introduction to markovchain R package

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Intro

- The markovchain package (Spedicato 2017) will be introduced.
- The package is intended to provide S4 classes to perform probabilistic and statistical analysis of Discrete Time Markov Chains (DTMC). See (Brémaud 1999) for a theoretical review of the mathematics underlying the DTMC models.
- The vignette will show: how to load the package and create a DTMC, how to manage a DTMC, how to perform basic probabilistic analysis, how to fit a DTMC.

- The package is on Cran since Summer 2013.
- It requires a recent version of R (>=3.0). Since version 0.2 parts of code have been moved to Rcpp (Eddelbuettel 2013).
- The package won a slot in Google Summer of Code 2015 for optimizing internals and expanding functionalities.

First moves into the markovchain package

Loading the package

- The package is loaded using

```
#load the package
library(markovchain)
```

Creating a DTMC

- DTMC can be easily create following standard S4 classes syntax. The show method displays it.

```
tmA <- matrix(c(0,0.5,0.5,.5,0,.5,.5,.5,0),nrow = 3,
              byrow = TRUE) #define the transition matrix
dtmcA <- new("markovchain",transitionMatrix=tmA,
             states=c("a","b","c"),
             name="MarkovChain A") #create the DTMC
dtmcA
```

```
## MarkovChain A
##  A  3 - dimensional discrete Markov Chain defined by the
##  a, b, c
##  The transition matrix  (by rows)  is defined as follows
##     a   b   c
## a 0.0 0.5 0.5
## b 0.5 0.0 0.5
## c 0.5 0.5 0.0
```

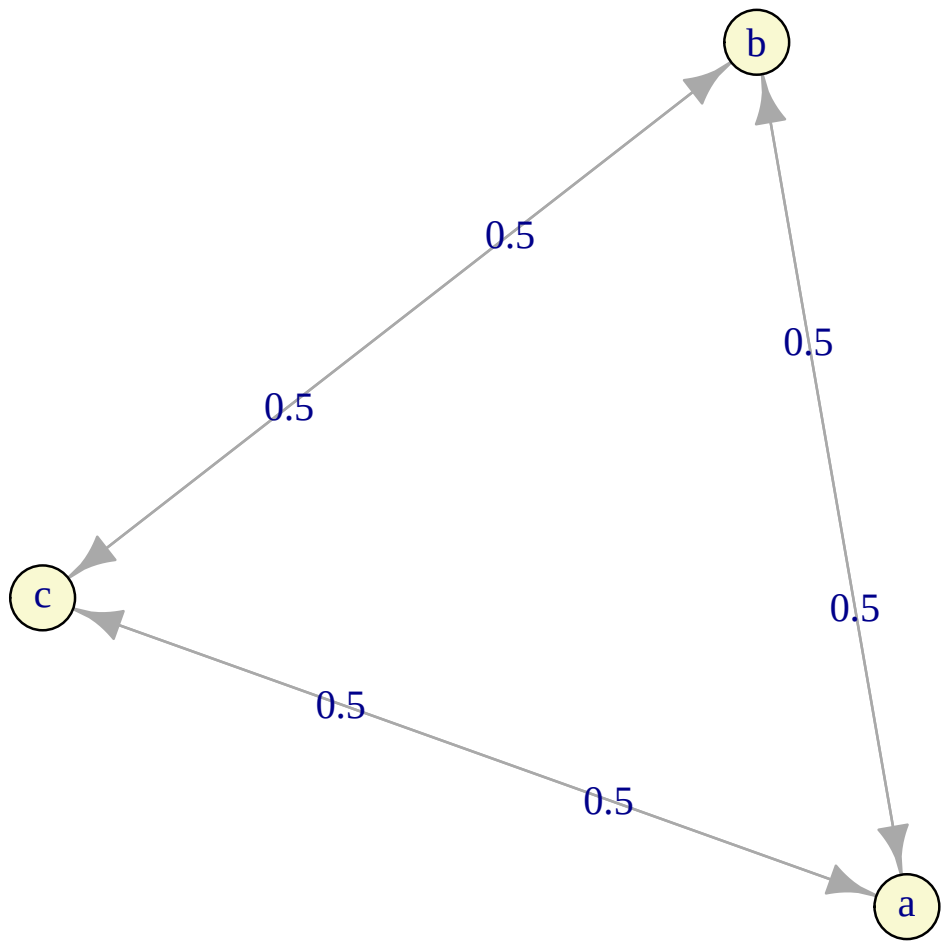
- Otherwise, it can also be created directly coercing a matrix.

```
dtmcA2<-as(tmA, "markovchain") #using coerce from matrix
states(dtmcA2) #note default names assigned to states
```

```
## [1] "s1" "s2" "s3"
```

- It is also possible to display a DTMC, using igraph package (Csardi and Nepusz 2006) capabilities

```
plot(dtmcA)
```



Probabilistic analysis

The basic

- It is possible to access transition probabilities and to perform basic operations.
- Similarly, it is possible to access the conditional distribution of states, $Pr\left(X_{t+1}|X_t = s\right)$

```
dtmcA[2,3] #using [ method
```

```
## [1] 0.5
```

```
transitionProbability(dtmcA,
                      "b","c") #using specific S4 method
```

```
## [1] 0.5
```

```
conditionalDistribution(dtmcA,"b")
```

```
##   a   b   c
## 0.5 0.0 0.5
```

- It is possible to simulate states distribution after n-steps

```
initialState<-c(0,1,0)
steps<-4
finalState<-initialState*dtmcA^steps #using power operator
finalState
```

```
##              a     b      c
## [1,] 0.3125 0.375 0.3125
```

- As well as steady states distribution

```
steadyStates(dtmcA) #S4 method
```

```
##                a         b         c
## [1,] 0.3333333 0.3333333 0.3333333
```

Advanced

- We use an example found on Mathematica Web page, (Wolfram Research 2013)

```
E <- matrix(0, nrow = 4, ncol = 4)
E[1, 2] <- 1;E[2, 1] <- 1/3; E[2, 3] <- 2/3
E[3,2] <- 1/4; E[3, 4] <- 3/4; E[4, 3] <- 1
mcMathematica <- new("markovchain", states = c("a", "b", "c
                     transitionMatrix = E,name = "Mathemati
```

- The summary method shows the proprieties of the DTCM

```
summary(mcMathematica)
```

```
## Mathematica  Markov chain that is composed by:
## Closed classes:
## a b c d
## Recurrent classes:
## {a,b,c,d}
## Transient classes:
## NONE
## The Markov chain is irreducible
## The absorbing states are: NONE
```

Estimation and simulation

The package permits to fit a DTMC estimating the transition matrix from a sequence of data. - createSequenceMatrix returns a function showing previous vs actual states from the pairs in a given sequence.

```r
#using Alofi rainfall dataset
data(rain)
mysequence<-rain$rain
createSequenceMatrix(mysequence)
```

```
##         0 1-5  6+
## 0     362 126  60
## 1-5   136  90  68
## 6+     50  79 124
```

- markovchainFit function allows to obtain the estimated transition matric and the confidence levels (using elliptic MLE hyphotesis).

```
myFit<-markovchainFit(data=mysequence,confidencelevel = .9
myFit
```

```
## $estimate
## MLE Fit
##  A  3 - dimensional discrete Markov Chain defined by the
##  0, 1-5, 6+
##  The transition matrix  (by rows)  is defined as follows
##              0       1-5        6+
## 0   0.6605839 0.2299270 0.1094891
## 1-5 0.4625850 0.3061224 0.2312925
## 6+  0.1976285 0.3122530 0.4901186
##
##
## $standardError
##               0         1-5          6+
## 0   0.03471952 0.02048353 0.01413498
```

- See the vignettes for further fitting methods as well as for functionalities targeted on non - homogeneous Markov chains.

```
alofiMc<-myFit$estimate
alofiMc
```

```
## MLE Fit
##  A  3 - dimensional discrete Markov Chain defined by the
##  0, 1-5, 6+
##  The transition matrix  (by rows)  is defined as follows
##               0       1-5        6+
## 0   0.6605839 0.2299270 0.1094891
## 1-5 0.4625850 0.3061224 0.2312925
## 6+  0.1976285 0.3122530 0.4901186
```

Bibliography I

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