Package `kcirt`

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1 Intro: What's "KCIRT"?

k-Cube Item Response Theory. The model upon which this `kcirt` package was built — k-Cube IRT, is really only a slight generalization of the "Forced Choice Thurstonian IRT" model developed through the excellent work of Anna Brown and Alberto Maydeu-Olivares [1]. The generalization was motivated by a desire to gain insight into how the presence or absence of items or entire blocks might affect the loadings of other items presented within a forced choice assessment. Mathematically, this generalization is manifest by an almost incidental consequence of writing the model in full matrix form.

In the formulation given in (1), the *loadings* matrix, $\mathbf{\Lambda}$, is square. The Forced Choice Thurstonian IRT model assumes $\mathbf{\Lambda}$ is diagnonal — but it needn't be.

For example, the loading present as the first element in the first row of $\mathbf{\Lambda}$ maps the state (scale) to which the first item points into the observation space. If the second loading in the first row of $\mathbf{\Lambda}$ is not zero, then the state to which the second item points will *moderate* the relationship between the first item and its manifestation in the observation space. By permitting non-zeroness of off-diagonal elements in $\mathbf{\Lambda}$, one might — so the reasoning goes — catch a glimpse of how possible interplay between items might affect an instrument's performance.

1.1 The System

Have d be the number of latent constructs, p be the number of response blocks (or questions), n be the number of items to be assigned rank, and $\tilde{n} = (n-1)n/2$ be the number of possible one-sided pairings between the n items.

For each observational unit,

$$\boldsymbol{y}^{\star} = \boldsymbol{\Delta}\,\boldsymbol{\mu} + \boldsymbol{\Delta}\,\boldsymbol{\Lambda}\,\mathbf{S}\,\boldsymbol{\eta} + \boldsymbol{\Delta}\,\boldsymbol{\varepsilon} \tag{1}$$

$$\boldsymbol{y} = \mathbf{1}_{\boldsymbol{y}^{\star}>0} \tag{2}$$

where $\boldsymbol{\Delta}$, the "delta" function, is $(\tilde{n}\,p) \times (n\,p)$; $\boldsymbol{\Lambda}$, the system hyperparameter, is $(n\,p) \times (n\,p)$; $\mathbf{S}$, the "slot" function, is $(n\,p) \times d$; $\boldsymbol{\eta}$, the latent state, is $d \times 1$; and $\boldsymbol{\varepsilon} \sim \mathcal{N}[\mathbf{0}, \mathbf{I}\,\sigma_{\varepsilon}^{2}]$ describe the system shocks.

The latent state is assumed to arise through $\boldsymbol{\eta} \sim \mathcal{N}[\mathbf{0}, \Sigma_{\eta}]$ — furthermore, we assume that this random variable, as well as the shocks, $\boldsymbol{\varepsilon}$, are independently realized across observational units.

The system observational-space is occupied by $\boldsymbol{y}$; the value $\boldsymbol{y}^{\star}$ is unobserved.

Generally, the objective is predicting the system states, $\boldsymbol{\eta}$, through concomitant estimation of the hyperparameters, $\boldsymbol{\Lambda}$, given realizations of $\boldsymbol{y}$. $\mathbf{S}$ and $\boldsymbol{\Delta}$ are defined through the mappings between items and states, and are hence known; they, along with $\boldsymbol{\Lambda}$ and the item utilities, $\boldsymbol{\mu}$, are assumed to be invariant across observational units.

References

[1] A. Brown and A. Maydeu-Olivares. How IRT Can Solve Problems of Ipsative Data in Forced-Choice Questionnaires. *Psychological Methods*, November 2012.