

# Package ‘fungible’

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**Title** Fungible Coefficients and Monte Carlo Functions

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**Depends** R (>= 3.0)

**Imports** e1071, lattice, MASS, mvtnorm, R2Cuba, stringr, nleqslv,  
methods

**Description** Functions for computing fungible coefficients and Monte Carlo data.

**License** GPL (>=2)

**NeedsCompilation** no

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<i>adfCor</i>	<i>Asymptotic Distribution-Free Covariance Matrix of Correlations</i>
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**Description**

Function for computing an asymptotic distribution-free covariance matrix of correlations.

**Usage**

```
adfCor(X, y = NULL)
```

**Arguments**

- |   |                                      |
|---|--------------------------------------|
| X | Data matrix.                         |
| y | Optional vector of criterion scores. |

**Value**

*adfCorMat* Asymptotic distribution-free estimate of the covariance matrix of correlations.

**Author(s)**

Jeff Jones and Niels Waller

**References**

- Browne, M. W. (1984). Asymptotically distribution-free methods for the analysis of covariance structures. *British Journal of Mathematical and Statistical Psychology*, 37, 62–83.
- Steiger, J. H. and Hakstian, A. R. (1982). The asymptotic distribution of elements of a correlation matrix: Theory and application. *British Journal of Mathematical and Statistical Psychology*, 35, 208–215.

**Examples**

```
## Generate non-normal data using monte1
set.seed(123)
## we will simulate data for 1000 subjects
N <- 1000

## R = the desired population correlation matrix among predictors
R <- matrix(c(1, .5, .5, 1), 2, 2)
```

```

## Consider a regression model with coefficient of determination (Rsq):
Rsq <- .50

## and vector of standardized regression coefficients
Beta <- sqrt(Rsq/t(sqrt(c(.5, .5)))) %*% R %*% sqrt(c(.5, .5)) * sqrt(c(.5, .5))

## generate non-normal data for the predictors (X)
## x1 has expected skew = 1 and kurtosis = 3
## x2 has expected skew = 2 and kurtosis = 5
X <- monte1(seed = 123, nvar = 2, nsub = N, cormat = R, skewvec = c(1, 2),
kurtvec = c(3, 5))$data

## generate criterion scores
y <- X %*% Beta + sqrt(1-Rsq)*rnorm(N)

## Create ADF Covariance Matrix of Correlations
adfCor(X, y)

#>          12          13          23
#> 12 0.0012078454 0.0005331086 0.0004821594
#> 13 0.0005331086 0.0004980130 0.0002712080
#> 23 0.0004821594 0.0002712080 0.0005415301

```

**adfCov***Asymptotic Distribution-Free Covariance Matrix of Covariances***Description**

Function for computing an asymptotic distribution-free covariance matrix of covariances.

**Usage**

```
adfCov(X, y = NULL)
```

**Arguments**

- |   |                                      |
|---|--------------------------------------|
| X | Data matrix.                         |
| y | Optional vector of criterion scores. |

**Value**

adfCovMat      Asymptotic distribution-free estimate of the covariance matrix of covariances

**Author(s)**

Jeff Jones and Niels Waller

**References**

Browne, M. W. (1984). Asymptotically distribution-free methods for the analysis of covariance structures. *British Journal of Mathematical and Statistical Psychology*, 37, 62–83.

## Examples

```

## Generate non-normal data using monte1
set.seed(123)

## we will simulate data for 1000 subjects
N <- 1000

## R = the desired population correlation matrix among predictors
R <- matrix(c(1, .5, .5, 1), 2, 2)

## Consider a regression model with coefficient of determination (Rsq):
Rsq <- .50

## and vector of standardized regression coefficients
Beta <- sqrt(Rsq/t(sqrt(c(.5, .5)))) %*% R %*% sqrt(c(.5, .5)) * sqrt(c(.5, .5))

## generate non-normal data for the predictors (X)
## x1 has expected skew = 1 and kurtosis = 3
## x2 has expected skew = 2 and kurtosis = 5
X <- monte1(seed = 123, nvar = 2, nsub = N, cormat = R, skewvec = c(1, 2),
kurtvec = c(3, 5))$data

## generate criterion scores
y <- X %*% Beta + sqrt(1-Rsq)*rnorm(N)

## Create ADF Covariance Matrix of Covariances
adfCov(X, y)

#>      11      12      13      22      23      33
#> 11 3.438760 2.317159 2.269080 2.442003 1.962584 1.688631
#> 12 2.317159 3.171722 2.278212 3.349173 2.692097 2.028701
#> 13 2.269080 2.278212 2.303659 2.395033 2.149316 2.106310
#> 22 2.442003 3.349173 2.395033 6.275088 4.086652 2.687647
#> 23 1.962584 2.692097 2.149316 4.086652 3.287088 2.501094
#> 33 1.688631 2.028701 2.106310 2.687647 2.501094 2.818664

```

**bigen**

*Generate Correlated Binary Data*

## Description

Function for generating binary data with population thresholds.

## Usage

```
bigen(data, n, thresholds, seed = NULL)
```

## Arguments

<b>data</b>	Either a matrix of binary (0/1) indicators or a correlation matrix.
<b>n</b>	The desired sample size of the simulated data.
<b>thresholds</b>	If x is a correlation matrix, thresholds must be a vector of threshold cut points.
<b>seed</b>	Default = NULL. Optional seed for random number generator.

**Value**

data	Simulated binary data
r	Input or calculated (tetrachoric) correlation matrix

**Author(s)**

Niels G Waller

**Examples**

```
## Example: generating binary data to match
## an existing binary data matrix
##
## Generate correlated scores using factor
## analysis model
## X <- Z *L' + U*D
## Z is a vector of factor scores
## L is a factor loading matrix
## U is a matrix of unique factor scores
## D is a scaling matrix for U

N <- 5000

# Generate data from a single factor model
# factor pattern matrix
L <- matrix( rep(.707, 5), nrow = 5, ncol = 1)

# common factor scores
Z <- as.matrix(rnorm(N))

# unique factor scores
U <- matrix(rnorm(N *5), nrow = N, ncol = 5)
D <- diag(as.vector(sqrt(1 - L^2)))

# observed scores
X <- Z %*% t(L) + U %*% D

cat("\nCorrelation of continuous scores\n")
print(round(cor(X),3))

# desired difficulties (i.e., means) of
# the dichotomized scores
difficulties <- c(.2, .3, .4, .5, .6)

# cut the observed scores at these thresholds
# to approximate the above difficulties
thresholds <- qnorm(difficulties)

Binary <- matrix(0, N, ncol(X))
for(i in 1:ncol(X)){
  Binary[X[,i] <= thresholds[i],i] <- 1
}

cat("\nCorrelation of Binary scores\n")
print(round(cor(Binary), 3))
```

```

## Now use 'bigen' to generate binary data matrix with
## same correlations as in Binary

z <- bigen(data = Binary, n = N)

cat("\n\nnames in returned object\n")
print(names(z))

cat("\nCorrelation of Simulated binary scores\n")
print(round(cor(z$data), 3))

cat("Observed thresholds of simulated data:\n")
cat(apply(z$data, 2, mean))

```

**corSample***Sample Correlation Matrices from a Population Correlation Matrix***Description**

Sample correlation (covariance) matrices from a population correlation matrix (see Browne, 1968; Kshirsagar, 1959)

**Usage**

```
corSample(R, n)
```

**Arguments**

- |   |   |
|---|---|
| R | A population correlation matrix.  |
| n | Sample correlation (covariance) matrices will be generated assuming a sample size of n. |

**Value**

- |            |                            |
|------------|----------------------------|
| cor.sample | Sample correlation matrix. |
| cov.sample | Sample covariance matrix.  |

**Author(s)**

Niels Waller

**References**

- Browne, M. (1968). A comparison of factor analytic techniques. *Psychometrika*, 33(3), 267-334.  
 Kshirsagar, A. (1959). Bartlett decomposition and Wishart distribution. *The Annals of Mathematical Statistics*, 30(1), 239-241.

## Examples

```
R <- matrix(c(1, .5, .5, 1), 2, 2)
# generate a sample correlation from pop R with n = 25
out <- corSmooth(R, n = 25)
out$cor.sample
out$cov.sample
```

corSmooth

*Smooth a Non PD Correlation Matrix*

## Description

A function for smoothing a non-positive definite correlation matrix by the method of Knol and Berger (1991).

## Usage

```
corSmooth(R, eps = 1E8 * .Machine$double.eps)
```

## Arguments

R	A non-positive definite correlation matrix.
eps	Small positive number to control the size of the non-scaled smallest eigenvalue of the smoothed R matrix. Default = 1E8 * .Machine\$double.eps

## Value

Rsmoothed	A Smoothed (positive definite) correlation matrix.
-----------	--

## Author(s)

Niels Waller

## References

Knol, D. L., and Berger, M. P. F., (1991). Empirical comparison between factor analysis and multi-dimensional item response models. *Multivariate Behavioral Research*, 26, 457-477.

## Examples

```
## choose eigenvalues such that R is NPD
1 <- c(3.0749126, 0.9328397, 0.5523868, 0.4408609, -0.0010000)

## Generate NPD R
R <- genCorr(eigenval = 1, seed = 123)
print(eigen(R)$values)

#> [1] 3.0749126 0.9328397 0.5523868 0.4408609 -0.0010000

## Smooth R
Rsm<-corSmooth(R, eps = 1E8 * .Machine$double.eps)
print(eigen(Rsm)$values)

#> [1] 3.074184e+00 9.326669e-01 5.523345e-01 4.408146e-01 2.219607e-08
```

d2r

*Convert Degrees to Radians***Description**

A simple function to convert degrees to radians

**Usage**

```
d2r(deg)
```

**Arguments**

deg	Angle in degrees.
-----	-------------------

**Value**

Angle in radians.

**Examples**

```
d2r(90)
```

enhancement

*Find OLS Regression Coefficients that Exhibit Enhancement***Description**

Find OLS regression coefficients that exhibit a specified degree of enhancement.

**Usage**

```
enhancement(R, br, rr)
```

**Arguments**

R	Predictor correlation matrix.
br	Model R-squared = b' r. That is, br is the model coefficient of determination: $b'Rb = Rsq = br$
rr	Sum of squared predictor-criterion correlations (rxy). That is, rr = r'r = Sum(rxy^2)

**Value**

b	Vector of standardized regression coefficients.
r	Vector of predictor-criterion correlations.

**Author(s)**

Niels Waller

## References

Waller, N. G. (2011). The geometry of enhancement in multiple regression. *Psychometrika*, 76, 634–649.

## Examples

```
## Example: For a given predictor correlation matrix (R) generate
## regression coefficient vectors that produce enhancement (br - rr > 0)

## Predictor correlation matrix
R <- matrix(c( 1, .5, .25,
              .5, 1,   .30,
              .25, .30, 1), 3, 3)

## Model coefficient of determination
Rsq <- .60

output<-enhancement(R, br = Rsq, rr =.40)

r <- output$r
b <- output$b

##Standardized regression coefficients
print(t(b))

##Predictor-criterion correlations
print(t(r))

##Coefficient of determinations (b'r)
print(t(b) %*% r)

##Sum of squared correlations (r'r)
print(t(r) %*% r)
```

fungible

*Generate Fungible Regression Weights*

## Description

Generate fungible weights for OLS Regression Models.

## Usage

```
fungible(R.X, rxy, r.yhata.yhatb, sets, print = TRUE)
```

## Arguments

R.X	p x p Predictor correlation matrix.
rxy	p x 1 Vector of predictor-criterion correlations.
r.yhata.yhatb	Correlation between least squares (yhatb) and alternate-weight (yhata) composites.
sets	Number of returned sets of fungible weights.
print	Logical, if TRUE then print 5-point summaries of alternative weights.

**Value**

a	Number of sets x p matrix of fungible weights.
k	Number of sets x p matrix of k weights.
b	p x 1 vector of LS weights.
u	p x 1 vector of u weights.
r.yhata.yhatb	Correlation between yhata and yhatb.
r.y.yhatb	Correlation between y and yhatb.
cov.a	Expected covariance matrix for a.
cor.a	Expected correlation matrix for a.

**Author(s)**

Niels Waller

**References**

Waller, N. (2008). Fungible weights in multiple regression. *Psychometrika*, 73, 69–703.

**Examples**

```

## Predictor correlation matrix
R.X <- matrix(c(1.00,   .56,   .77,
               .56,   1.00,   .73,
               .77,   .73,   1.00), 3, 3)

## vector of predictor-criterion correlations
rxy <- c(.39, .34, .38)

## OLS standardized regression coefficients
b <- solve(R.X) %*% rxy

## Coefficient of determination (Rsq)
OLSRSQ <- t(b) %*% R.X %*% b

## theta controls the correlation between
## yhatb: predicted criterion scores using OLS coefficients
## yhata: predicted criterion scores using alternate weights
theta <- .01

## desired correlation between yhata and yhatb
r.yhata.yhatb <- sqrt(1 - (theta)/OLSRSQ)

## number of returned sets of fungible weight vectors
Nsets <- 50

output <- fungible(R.X, rxy, r.yhata.yhatb, sets = Nsets, print = TRUE)

```

---

<i>fungibleExtrema</i>	<i>Locate Extrema of Fungible Regression Weights</i>
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---

**Description**

Locate extrema of fungible regression weights.

**Usage**

```
fungibleExtrema(R.X, rxy, r.yhata.yhatb, Nstarts, MaxMin)
```

**Arguments**

<code>R.X</code>	<code>p x p</code> Predictor variable correlation matrix.
<code>rxy</code>	<code>p x 1</code> Vector of predictor-criterion correlations.
<code>r.yhata.yhatb</code>	Correlation between least squares ( <code>yhatb</code> ) and alternate-weight ( <code>yhata</code> ) composites.
<code>Nstarts</code>	Maximum number of (max) minimizations from random starting configurations.
<code>MaxMin</code>	Character: "Max" = maximize $\cos(a,b)$ ; "Min" = minimize $\cos(a,b)$ .

**Value**

<code>cos.ab</code>	cosine between OLS and alternate weights.
<code>a</code>	extrema of fungible weights.
<code>k</code>	<code>k</code> weights.
<code>z</code>	<code>z</code> weights: a normalized random vector.
<code>b</code>	OLS weights.
<code>u</code>	<code>p x 1</code> vector of <code>u</code> weights.
<code>r.yhata.yhatb</code>	Correlation between <code>yhata</code> and <code>yhatb</code> .
<code>r.y.yhatb</code>	Correlation between <code>y</code> and <code>yhatb</code> .
<code>gradient</code>	Gradient of converged solution.

**Author(s)**

Niels Waller and Jeff Jones

**References**

- Koopman, R. F. (1988). On the sensitivity of a composite to its weights. *Psychometrika*, 53(4), 547–552.
- Waller, N. & Jones, J. (2009). Locating the extrema of fungible regression weights in multiple regression. *Psychometrika*, 74, 589–602.

## Examples

```

## Example
## This is Koopman's Table 2 Example

R.X <- matrix(c(1.00, .69, .49, .39,
               .69, 1.00, .38, .19,
               .49, .38, 1.00, .27,
               .39, .19, .27, 1.00), 4, 4)

b <- c(.39, .22, .02, .43)
rxy <- R.X %*% b

OLSRSQ <- t(b) %*% R.X %*% b

## theta <- .02
## r.yhata.yhatb <- sqrt(1 - (theta)/OLSRSQ)

r.yhata.yhatb <- .90
set.seed(5)
output <- fungibleExtrema(R.X, rxy, r.yhata.yhatb, Nstarts = 500,
                           MaxMin = "Min")

## Scale to replicate Koopman
a <- output$a
a.old <- a
aRa <- t(a) %*% R.X %*% a

## Scale a such that a' R a = .68659
## vc = variance of composite
vc <- aRa
## sf = scale factor
sf <- .68659/vc
a <- as.numeric(sqrt(sf)) * a
cat("\nKoopman Scaling\n")
print(round(a, 2))

```

fungibleL

*Generate Fungible Logistic Regression Weights*

## Description

Generate fungible weights for Logistic Regression Models.

## Usage

```
fungibleL(X, y, Nsets = 1000, method = "LLM", RsqDelta = NULL,
           rLaLb = NULL, s = .3, Print = TRUE)
```

## Arguments

X	An n by nvar matrix of predictor scores without the leading column of ones.
---	---

y	An n by 1 vector of dichotomous criterion scores.
Nsets	The desired number of fungible coefficient vectors.
method	Character: "LLM" = Log-Likelihood method. "EM" = Ellipsoid Method. Default: method = "LLM".
RsqDelta	The desired decrement in the pseudo-R-squared - used when method = "LLM".
rLaLb	The desired correlation between the logits - used when method = "EM".
s	Scale factor for random deviates. s controls the range of random start values for the optimization routine. Recommended 0 <= s < 1. Default: s = 0.3.
Print	Boolean (TRUE/FALSE) for printing output summary.

## Details

fungibleL provides two methods for evaluating parameter sensitivity in logistic regression models by computing fungible logistic regression weights. For additional information on the underlying theory of these methods see Jones and Waller (in press).

## Value

model	A glm model object.
call	The function call to glm().
ftable	A data frame with the mle estimates and the minimum and maximum fungible coefficients.
lnLML	The maximum likelihood log likelihood value.
lnLf	The decremented, fungible log likelihood value.
pseudoRsq	The pseudo R-squared.
fungibleRsq	The fungible pseudo R-squared.
fungiblea	The Nsets by Nvar + 1 matrix of fungible (alternate) coefficients.
rLaLb	The correlation between the logits.
maxPosCoefChange	The maximum positive change in a single coefficient holding all other coefficients constant.
maxNegCoefChange	The maximum negative change in a single coefficient holding all other coefficients constant.

## Author(s)

Jeff Jones and Niels Waller

## References

Jones, J. A. & Waller, N. G. (in press). Fungible weights in logistic regression. *Psychological Methods*.

## Examples

```

# Example: Low Birth Weight Data from Hosmer Jr, D. W. & Lemeshow, S.(2000).
# low : low birth rate ( $0 \geq 2500$  grams,  $1 < 2500$  grams)
# race: 1 = white, 2 = black, 3 = other
# ftv : number of physician visits during the first trimester

library(MASS)
attach(birthwt)

race <- factor(race, labels = c("white", "black", "other"))
predictors <- cbind(lwt, model.matrix(~ race)[, -1])

# compute mle estimates
BWght.out <- glm(low ~ lwt + race, family = "binomial")

# compute fungible coefficients
fungible.LLM <- fungibleL(X = predictors, y = low, method = "LLM",
                             Nsets = 10, RsqDelta = .005, s = .3)

# Compare with Table 2.3 (page 38) Hosmer Jr, D. W. & Lemeshow, S.(2000).
# Applied logistic regression. New York, Wiley.

print(summary(BWght.out))
print(fungible.LLM$call)
print(fungible.LLM$ftable)
cat("\nMLE log likelihood      = ", fungible.LLM$lnML,
    "\nfungible log likelihood = ", fungible.LLM$lnLf)
cat("\nPseudo Rsq            = ", round(fungible.LLM$pseudoRsq, 3))
cat("\nungible Pseudo Rsq     = ", round(fungible.LLM$fungibleRsq, 3))

fungible.EM <- fungibleL(X = predictors, y = low, method = "EM" ,
                           Nsets = 10, rLaLb = 0.99)

print(fungible.EM$call)
print(fungible.EM$ftable)

cat("\nrLaLb = ", round(fungible.EM$rLaLb, 3))

```

## Description

Generate fungible correlation matrices. For a given vector of standardized regression coefficients, Beta, and a user-defined R-squared value, Rsq, find predictor correlation matrices, R, such that Beta' R Beta = Rsq. The size of the smallest eigenvalue (Lp) of R can be defined.

## Usage

```
fungibleR(R, Beta, Lp = .00, eps = 1e-08, Print.Warnings = TRUE)
```

**Arguments**

R	A p x p predictor correlation matrix.
Beta	A p x 1 vector of standardized regression coefficients.
Lp	Controls the size of the smallest eigenvalue of RstarLp.
eps	Convergence criterion.
Print.Warnings	Logical, default = TRUE. When TRUE, convergence failures are printed.

**Value**

R	Any input correlation matrix that satisfies Beta' R Beta = Rsq.
Beta	Input vector of std reg coefficients.
Rstar	A random fungible correlation matrix.
RstarLp	A fungible correlation matrix with a fixed minimum eigenvalue (RstarLp can be PD, PSD, or ID).
s	Scaling constant for Rstar.
sLp	Scaling constant for RstarLp.
Delta	Vector in the null space of vecp(Beta Beta').
Q	Left null space of Beta.
FrobNorm	Frobenius norm   R - Rstar  _F.
FrobNormLp	Frobenius norm   R - RstarLp  _F given random Delta.
converged	An integer code. 0 indicates successful completion.

**Author(s)**

Niels Waller

**References**

Waller, N. (2016). Fungible Correlation Matrices: A method for generating nonsingular, singular, and improper correlation matrices for Monte Carlo research.

**Examples**

```
library(fungible)

## ===== Example 1 =====
## Generate 5 random PD fungible R matrices
## that are consistent with a user-defined predictive
## structure: B' Rxx B = .30

set.seed(246)
## Create a 5 x 5 correlation matrix, R, with all r_ij = .25
R.ex1 <- matrix(.25, 5, 5)
diag(R.ex1) <- 1

## create a 5 x 1 vector of standardized regression coefficients,
## Beta.ex1
Beta.ex1 <- c(-.4, -.2, 0, .2, .4)
cat("\nModel Rsq = ", t(Beta.ex1) %*% R.ex1 %*% Beta.ex1)
```

```

## Generate fungible correlation matrices, Rstar, with smallest
## eigenvalues > 0.

Rstar.list <- list(rep(99,5))
i <- 0
while(i <= 5){
  out <- fungibleR(R = R.ex1, Beta = Beta.ex1, Lp = 1e-8, eps = 1e-8,
                     Print.Warnings = TRUE)
  if(out$converged==0){
    i <- i + 1
    Rstar.list[[i]] <- out$Rstar
  }
}

## Check Results
cat("\n *** Check Results ***")
for(i in 1:5){
  cat("\n\n" + "-----")
  cat("\nRstar", i, "\n")
  print(round(Rstar.list[[i]], 2),)
  cat("\neigenvalues of Rstar", i, "\n")
  print(eigen(Rstar.list[[i]])$values)
  cat("\nBeta' Rstar", i, "Beta = ",
      t(Beta.ex1) %*% Rstar.list[[i]] %*% Beta.ex1)
}

## ===== Example 2 =====
## Generate a PD fungible R matrix with a fixed smallest
## eigenvalue (Lp).

## Create a 5 x 5 correlation matrix, R, with all r_ij = .5
R <- matrix(.5, 5, 5)
diag(R) <- 1

## create a 5 x 1 vector of standardized regression coefficients, Beta,
## such that Beta_i = .1 for all i
Beta <- rep(.1, 5)

## Generate fungible correlation matrices (a) Rstar and (b) RstarLp.
## Set Lp = 0.12345678 so that the smallest eigenvalue (Lp) of RstarLp
## = 0.12345678
out <- fungibleR(R, Beta, Lp = 0.12345678, eps = 1e-10, Print.Warnings = TRUE)

## print R
cat("\nR: a user-specified seed matrix")
print(round(out$R,3))

## Rstar
cat("\nRstar: A random fungible correlation matrix for R")
print(round(out$Rstar,3))

cat("\nCoefficient of determination when using R\n")
print( t(Beta) %*% R %*% Beta )

```

```
cat("\nCoefficient of determination when using Rstar\n")
print( t(Beta) %*% out$Rstar %*% Beta)

## Eigenvalues of R
cat("\nEigenvalues of R\n")
print(round(eigen(out$R)$values, 9))

## Eigenvalues of Rstar
cat("\nEigenvalues of Rstar\n")
print(round(eigen(out$Rstar)$values, 9))

## What is the Frobenius norm (Euclidean distance) between
## R and Rstar
cat("\nFrobenious norm ||R - Rstar||\n")
print( out$FrobNorm)

## RstarLp is a random fungible correlation matrix with
## a fixed smallest eigenvalue of 0.12345678
cat("\nRstarLp: a random fungible correlation matrix with a user-defined
smallest eigenvalue\n")
print(round(out$RstarLp, 3))

## Eigenvalues of RstarLp
cat("\nEigenvalues of RstarLp")
print(eigen(out$RstarLp)$values, digits = 9)

cat("\nCoefficient of determination when using RstarLp\n")
print( t(Beta) %*% out$RstarLp %*% Beta)

## Check function convergence
if(out$converged) print("Failed to converge")

## ===== Example 3 =====
## This examples demonstrates how fungibleR can be used
## to generate improper correlation matrices (i.e., pseudo
## correlation matrices with negative eigenvalues).
library(fungible)

## We desire an improper correlation matrix that
## is close to a user-supplied seed matrix. Create an
## interesting seed matrix that reflects a Big Five
## factor structure.

set.seed(123)
minCrossLoading <- -.2
maxCrossLoading <- .2
F1 <- c(rep(.6,5),runif(20,minCrossLoading, maxCrossLoading))
F2 <- c(runif(5,minCrossLoading, maxCrossLoading), rep(.6,5),
       runif(15,minCrossLoading, maxCrossLoading))
F3 <- c(runif(10,minCrossLoading,maxCrossLoading), rep(.6,5),
       runif(10,minCrossLoading,maxCrossLoading) )
F4 <- c(runif(15,minCrossLoading,maxCrossLoading), rep(.6,5),
       runif(5,minCrossLoading,maxCrossLoading))
F5 <- c(runif(20,minCrossLoading,maxCrossLoading), rep(.6,5))
FacMat <- cbind(F1,F2,F3,F4,F5)
R.bfi <- FacMat %*% t(FacMat)
```

```

diag(R.bfi) <- 1

## Set Beta to a null vector to inform fungibleR that we are
## not interested in placing constraints on the predictive structure
## of the fungible R matrices.
Beta <- rep(0, 25)

## We seek a NPD fungible R matrix that is close to the bfi seed matrix.
## To find a suitable matrix we generate a large number (e.g., 50000)
## fungible R matrices. For illustration purposes I will set Nmatrices
## to a smaller number: 10.
Nmatrices<-10

## Initialize a list to contain the Nmatrices fungible R objects
RstarLp.list <- as.list( rep(0, Nmatrices) )
## Initialize a vector for the Nmatrices Frobeius norms ||R - RstarLp||
FrobLp.vec <- rep(0, Nmatrices)

## Constraint the smallest eigenvalue of RStarLp by setting
## Lp = -.1 (or any suitably chosen user-defined value).

## Generate Nmatrices fungibleR matrices and identify the NPD correlation
## matrix that is "closest" (has the smallest Frobenious norm) to the bfi
## seed matrix.
BestR.i <- 0
BestFrob <- 99
i <- 0

set.seed(1)
while(i < Nmatrices){
  out<-fungibleR(R = R.bfi, Beta, Lp = -.1, eps=1e-10)
  ## retain solution if algorithm converged
  if(out$converged == 0)
  {
    i<- i + 1
    ## print progress
    cat("\nGenerating matrix ", i, " Current minimum ||R - RstarLp|| = ",BestFrob)
    tmp <- FrobLp.vec[i] <- out$FrobNormLp #Frobenious Norm ||R - RstarLp||
    RstarLp.list[[i]]<-out$RstarLp
    if( tmp < BestFrob )
    {
      BestR.i <- i      # matrix with lowest ||R - RstarLp||
      BestFrob <- tmp  # value of lowest ||R - RstarLp||
    }
  }
}

# CloseR is an improper correlation matrix that is close to the seed matrix.
CloseR<-RstarLp.list[[BestR.i]]

plot(1:25, eigen(R.bfi)$values,
     type = "b",
     lwd = 2,

```

```

main = "Scree Plots for R and RstarLp",
cex.main = 1.5,
ylim = c(-.2,6),
ylab = "Eigenvalues",
xlab = "Dimensions")
points(1:25,eigen(CloseR)$values,
      type = "b",
      lty = 2,
      lwd = 2,
      col = "red")
abline(h = 0, col = "grey")
legend(legend=c(expression(paste(lambda[i]~" of R",sep = "")),
               expression(paste(lambda[i]~" of RstarLp",sep = ""))),
       lty=c(1,2),
       x = 17,y = 5.75,
       cex = 1.5,
       col=c("black","red"),
       text.width = 5.5,
       lwd = 2)

```

**genCorr***Generate Correlation Matrices with User-Defined Eigenvalues***Description**

Uses the Marsaglia and Olkin (1984) algorithm to generate correlation matrices with user-defined eigenvalues.

**Usage**

```
genCorr(eigenval, seed='rand')
```

**Arguments**

eigenval	A vector of eigenvalues that must sum to the order of the desired correlation matrix. For example: if you want a correlation matrix of order 4, then you need 4 eigenvalues that sum to 4. A warning message will display if sum(eigenval) != length(eigenval)
seed	Either a user supplied seed for the random number generator or 'rand' for a function generated seed. Default seed='rand'.

**Value**

Returns a correlation matrix with the eigen-structure specified by eigenval.

**Author(s)**

Jeff Jones

## References

- Jones, J. A. (2010). GenCorr: An R routine to generate correlation matrices from a user-defined eigenvalue structure. *Applied Psychological Measurement*, 34, 68-69.
- Marsaglia, G., & Olkin, I. (1984). Generating correlation matrices. *SIAM J. Sci. and Stat. Comput.*, 5, 470-475.

## Examples

```
## Example
## Generate a correlation matrix with user-specified eigenvalues
set.seed(123)
R <- genCorr(c(2.5, 1, 1, .3, .2))

print(round(R, 2))

#>      [,1]  [,2]  [,3]  [,4]  [,5]
#> [1,]  1.00  0.08 -0.07 -0.07  0.00
#> [2,]  0.08  1.00  0.00 -0.60  0.53
#> [3,] -0.07  0.00  1.00  0.51 -0.45
#> [4,] -0.07 -0.60  0.51  1.00 -0.75
#> [5,]  0.00  0.53 -0.45 -0.75  1.00

print(eigen(R)$values)

#[1] 2.5 1.0 1.0 0.3 0.2
```

## *kurt*

*Calculate Univariate Kurtosis for a Vector or Matrix*

## Description

Calculate univariate kurtosis for a vector or matrix (algorithm G2 in Joanes & Gill, 1998).

## Usage

`kurt(x)`

## Arguments

`x` Either a vector or matrix of numeric values.

## Value

Kurtosis for each column in `x`.

## Author(s)

Niels Waller

## References

Joanes, D. N. & Gill, C. A. (1998). Comparing measures of sample skewness and kurtosis. *The Statistician*, 47, 183-189.

## See Also

[skew](#)

## Examples

```
x <- matrix(rnorm(1000), 100, 10)
print(kurt(x))
```

monte

*Simulate Clustered Data with User-Defined Properties*

## Description

Function for simulating clustered data with user defined characteristics such as: within cluster indicator correlations, within cluster indicator skewness values, within cluster indicator kurtosis values, and cluster separations as indexed by each variable (indicator validities).

## Usage

```
monte(seed, nvar, nclus, clus.size, eta2, cor.list, random.cor,
      skew.list, kurt.list, secor, compactness, sortMeans)
```

## Arguments

seed	Required: An integer to be used as the random number seed.
nvar	Required: Number of variables to simulate.
nclus	Required: Number of clusters to simulate. <i>Note</i> that number of clusters must be equal to or greater than 2.
clus.size	Required: Number of objects in each cluster.
eta2	Required: A vector of indicator validities that range from 0 to 1. Higher numbers produce clusters with greater separation on that indicator.
cor.list	Optional: A list of correlation matrices. There should be one correlation matrix for each cluster. The first correlation matrix will represent the indicator correlations within cluster 1. The second correlation matrix will represent the indicator correlations for cluster 2. Etc.
random.cor	Optional: Set to TRUE to generate a common within cluster correlation matrix.
skew.list	Optional: A list of within cluster indicator skewness values.
kurt.list	Optional: A list of within cluster indicator kurtosis values.
secor	Optional: If 'random.cor = TRUE' then 'secor' determines the standard error of the simulated within group correlation matrices.
compactness	Optional: A vector of cluster compactness parameters. The meaning of this option is explained Waller et al. (1999). Basically, 'compactness' allows users some control over cluster overlap without changing indicator validities. See the example below for an illustration.
sortMeans	Optional: A logical that determines whether the latent means will be sorted by taxon. Default = TRUE

**Value**

data	The simulated data. The 1st column of 'data' denotes cluster membership.
lmn	The cluster indicator means.
f1	The factor loading matrix as described in Waller, et al. 1999.
fs	The unique values of the linearized factor scores.
call	The call.
nclus	Number of clusters.
nvar	Number of variables.
cor.list	The input within cluster correlation matrices.
skew.list	The input within cluster indicator skewness values.
kurt.list	The input within cluster indicator kurtosis values.
clus.size	The number of observations in each cluster.
eta2	Vector of indicator validities.
seed	The random number seed.

**Author(s)**

Niels Waller

**References**

- Fleishman, A. I (1978). A method for simulating non-normal distributions. *Psychometrika*, 43, 521-532.
- Vale, D. C., & Maurelli, V. A. (1983). Simulating multivariate nonnormal distributions. *Psychometrika*, 48, 465-471.
- Waller, N. G., Underhill, J. M., & Kaiser, H. A. (1999). A method for generating simulated plasmodes and artificial test clusters with user-defined shape, size, and orientation. *Multivariate Behavioral Research*, 34, 123-142.

**Examples**

```
## Example 1
## Simulating Fisher's Iris data
# The original data were reported in:
# Fisher, R. A. (1936) The use of multiple measurements in taxonomic
#     problems. Annals of Eugenics, 7, Part II, 179-188.
#
# This example includes 3 clusters. Each cluster represents
# an Iris species: Setosa, Versicolor, and Virginica.
# On each species, four variables were measured: Sepal Length,
# Sepal Width, Petal Length, and Petal Width.
#
# The within species (cluster) correlations of the flower
# indicators are as follows:
#
# Iris Type 1:
#      [,1] [,2] [,3] [,4]
# [1,] 1.000 0.743 0.267 0.178
# [2,] 0.743 1.000 0.278 0.233
# [3,] 0.267 0.278 1.000 0.332
```

```

# [4,] 0.178 0.233 0.332 1.000
#
# Iris Type 2
# [,1] [,2] [,3] [,4]
# [1,] 1.000 0.526 0.754 0.546
# [2,] 0.526 1.000 0.561 0.664
# [3,] 0.754 0.561 1.000 0.787
# [4,] 0.546 0.664 0.787 1.000
#
# Iris Type 3
# [,1] [,2] [,3] [,4]
# [1,] 1.000 0.457 0.864 0.281
# [2,] 0.457 1.000 0.401 0.538
# [3,] 0.864 0.401 1.000 0.322
# [4,] 0.281 0.538 0.322 1.000
#
# 'monte' expects a list of correlation matrices
#
#create a list of within species correlations
data(iris)
cormat <- cm <- lapply(split(iris[,1:4], iris[,5]), cor)

# create a list of within species indicator
# skewness and kurtosis
sk.lst <- list(c(0.120, 0.041, 0.106, 1.254),
                c(0.105, -0.363, -0.607, -0.031),
                c(0.118, 0.366, 0.549, -0.129) )

kt.lst <- list(c(-0.253, 0.955, 1.022, 1.719),
                c(-0.533,-0.366, 0.048, -0.410),
                c( 0.033, 0.706, -0.154, -0.602) )

#Generate a new sample of iris data
my.iris <- monte(seed=123, nvar = 4, nclus = 3, cor.list = cormat,
                  clus.size = c(50, 50, 50),
                  eta2=c(0.619, 0.401, 0.941, 0.929),
                  random.cor = FALSE,
                  skew.list = sk.lst,
                  kurt.list = kt.lst,
                  secor = .3, compactness=c(1, 1, 1),
                  sortMeans = TRUE)

summary(my.iris)
plot(my.iris)

# Now generate a new data set with the sample indicator validities
# as before but with different cluster compactness values.

my.iris2<-monte(seed = 123, nvar = 4, nclus = 3,
                  cor.list = cormat, clus.size = c(50, 50, 50),
                  eta2 = c(0.619, 0.401, 0.941, 0.929), random.cor = FALSE,
                  skew.list = sk.lst ,kurt.list = kt.lst,
                  secor = .3,

```

```

compactness=c(2, .5, .5),
sortMeans = TRUE)

summary(my.iris2)

# Notice that cluster 1 has been blow up whereas clusters 2 and 3 have been shrunk.
plot(my.iris2)

### Now compare your original results with the actual
## Fisher iris data
library(lattice)
data(iris)
super.sym <- trellis.par.get("superpose.symbol")
splom(~iris[1:4], groups = Species, data = iris,
      #panel = panel.superpose,
      key = list(title = "Three Varieties of Iris",
                 columns = 3,
                 points = list(pch = super.sym$pch[1:3],
                               col = super.sym$col[1:3]),
                 text = list(c("Setosa", "Versicolor", "Virginica"))))

#####
##### EXAMPLE 2 #####
#####

## Example 2
## Simulating data for Taxometric
## Monte Carlo Studies.
##
## In this four part example we will
## generate two group mixtures
## (Complement and Taxon groups)
## under four conditions.
##
## In all conditions
## base rate (BR) = .20
## 3 indicators
## indicator validities = .50
## (This means that 50 percent of the total
## variance is due to the mixture.)
##
##
## Condition 1:
## All variables have a slight degree
## of skewness (.10) and kurtosis (.10).
## Within group correlations = 0.00.
##
##
## Condition 2:
## In this condition we generate data in which the
## complement and taxon distributions differ in shape.
## In the complement group all indicators have
## skewness values of 1.75 and kurtosis values of 3.75.
## In the taxon group all indicators have skewness values
## of .50 and kurtosis values of 0.

```

```
## As in the previous condition, all within group
## correlations (nuisance covariance) are 0.00.
##
## Conditon 3:
## In this condition we retain all previous
## characteristics except that the within group
## indicator correlations now equal .80
## (they can differ between groups).
##
## Conditon 4:
## In this final condition we retain
## all previous data characteristics except that
## the variances of the indicators in the complement
## class are now 5 times the indicator variances
## in the taxon class (while maintaining indicator skewness,
## kurtosis, correlations, etc.).
```

##-----

```
library(lattice)

#####
##      Condition 1
#####
in.nvar <- 3  ##Number of variables
in.nclus <- 2 ##Number of taxa
in.seed <- 123
BR <- .20      ## Base rate of higher taxon

## Within taxon indicator skew and kurtosis
in.skew.list <- list(c(.1, .1, .1),c(.1, .1, .1))
in.kurt.list <- list(c(.1, .1, .1),c(.1, .1, .1))

## Indicator validities
in.eta2 <- c(.50, .50, .50)

## Groups sizes for Population
BigN <- 100000
in.clus.size <- c(BigN*(1-BR), BR * BigN)

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
                      nvar=in.nvar,
                      nclus = in.nclus,
                      clus.size = in.clus.size,
                      eta2 = in.eta2,
                      skew.list = in.skew.list,
                      kurt.list = in.kurt.list)

output <- summary(sample.data)
```

```

z <- data.frame(sample.data$data[sample(1:BigN, 600, replace=FALSE),])
z[,2:4] <- scale(z[,2:4])
names(z) <- c("id", "v1", "v2", "v3")

#trellis.device()
trellis.par.set( col.whitebg() )
print(
  cloud(v3 ~ v1 * v2,
    groups = as.factor(id), data=z,
    subpanel = panel.superpose,
    zlim=c(-4, 4),
    xlim=c(-4, 4),
    ylim=c(-4, 4),
    main="",
    screen = list(z = 20, x = -70)),
  position=c(.1, .5, .5, 1), more = TRUE)

#####
## Condition 2
#####

## Within taxon indicator skew and kurtosis
in.skew.list <- list(c(1.75, 1.75, 1.75),c(.50, .50, .50))
in.kurt.list <- list(c(3.75, 3.75, 3.75),c(0, 0, 0))

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
  nvar = in.nvar,
  nclus = in.nclus,
  clus.size = in.clus.size,
  eta2 = in.eta2,
  skew.list = in.skew.list,
  kurt.list = in.kurt.list)

output <- summary(sample.data)

z <- data.frame(sample.data$data[sample(1:BigN, 600, replace=FALSE),])
z[,2:4] <- scale(z[, 2:4])
names(z) <-c("id", "v1", "v2", "v3")

print(
  cloud(v3 ~ v1 * v2,
    groups = as.factor(id), data = z,
    subpanel = panel.superpose,
    zlim = c(-4, 4),
    xlim = c(-4, 4),
    ylim = c(-4, 4),
    main="",
    screen = list(z = 20, x = -70)),
  position = c(.5, .5, 1, 1), more = TRUE)

#####
## Condition 3
#####

```

```
#####
## Set within group correlations to .80
cormat <- matrix(.80, 3, 3)
diag(cormat) <- rep(1, 3)
in.cor.list <- list(cormat, cormat)

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
                      nvar = in.nvar,
                      nclus = in.nclus,
                      clus.size = in.clus.size,
                      eta2 = in.eta2,
                      skew.list = in.skew.list,
                      kurt.list = in.kurt.list,
                      cor.list = in.cor.list)

output <- summary(sample.data)

z <- data.frame(sample.data$data[sample(1:BigN, 600,
                                         replace = FALSE), ])
z[,2:4] <- scale(z[, 2:4])
names(z) <- c("id", "v1", "v2", "v3")

##trellis.device()
##trellis.par.set( col.whitebg() )
print(
  cloud(v3 ~ v1 * v2,
        groups = as.factor(id), data=z,
        subpanel = panel.superpose,
        zlim = c(-4, 4),
        xlim = c(-4, 4),
        ylim = c(-4, 4),
        main="",
        screen = list(z = 20, x = -70)),
  position = c(.1, .0, .5, .5), more = TRUE)

#####
##      Condition 4
#####

## Change compactness so that variance of
## complement indicators is 5 times
## greater than variance of taxon indicators

v <- ( 2 * sqrt(5))/(1 + sqrt(5))
in.compactness <- c(v, 2-v)

## Generate Population of scores with "monte"
sample.data <- monte(seed = in.seed,
                      nvar = in.nvar,
                      nclus = in.nclus,
                      clus.size = in.clus.size,
                      eta2 = in.eta2,
                      skew.list = in.skew.list,
                      kurt.list = in.kurt.list,
```

```

cor.list = in.cor.list,
compactness = in.compactness)

output <- summary(sample.data)

z <- data.frame(sample.data$data[sample(1:BigN, 600, replace = FALSE), ])
z[, 2:4] <- scale(z[, 2:4])
names(z) <- c("id", "v1", "v2", "v3")
print(
  cloud(v3 ~ v1 * v2,
    groups = as.factor(id), data=z,
    subpanel = panel.superpose,
    zlim = c(-4, 4),
    xlim = c(-4, 4),
    ylim = c(-4, 4),
    main="",
    screen = list(z = 20, x = -70)),
  position = c(.5, .0, 1, .5), more = TRUE)

```

**monte1**

*Simulate Multivariate Non-normal Data by Vale & Maurelli (1983)  
Method*

**Description**

Function for simulating multivariate nonnormal data by the methods described by Fleishman (1978) and Vale & Maurelli (1983).

**Usage**

```
monte1(seed, nvar, nsub, cormat, skewvec, kurtvec)
```

**Arguments**

seed	An integer to be used as the random number seed.
nvar	Number of variables to simulate.
nsub	Number of simulated subjects (response vectors).
cormat	The desired correlation matrix.
skewvec	A vector of indicator skewness values.
kurtvec	A vector of indicator kurtosis values.

**Value**

data	The simulated data.
call	The call.
nsub	Number of subjects.
nvar	Number of variables.
cormat	The desired correlation matrix.
skewvec	The desired indicator skewness values.
kurtvec	The desired indicator kurtosis values.
seed	The random number seed.

**Author(s)**

Niels Waller

**References**

- Fleishman, A. I (1978). A method for simulating non-normal distributions. *Psychometrika*, 43, 521-532.
- Vale, D. C., & Maurelli, V. A. (1983). Simulating multivariate nonnormal distributions. *Psychometrika*, 48, 465-471.

**See Also**

[monte](#), [summary.monte](#), [summary.monte1](#)

**Examples**

```
## Generate dimensional data for 4 variables.
## All correlations = .60; all variable
## skewness = 1.75;
## all variable kurtosis = 3.75

cormat <- matrix(.60,4,4)
diag(cormat) <- 1

nontaxon.dat <- monte1(seed = 123, nsub = 100000, nvar = 4, skewvec = rep(1.75, 4),
                           kurtvec = rep(3.75, 4), cormat = cormat)

print(cor(nontaxon.dat$data), digits = 3)
print(apply(nontaxon.dat$data, 2, skew), digits = 3)
print(apply(nontaxon.dat$data, 2, kurt), digits = 3)
```

normalCor

*Compute Normal-Theory Covariances for Correlations*

**Description**

Compute normal-theory covariances for correlations

**Usage**

`normalCor(R, Nobs)`

**Arguments**

- |      |                                 |
|------|---------------------------------|
| R    | a p x p matrix of correlations. |
| Nobs | Number of observations.         |

**Value**

A normal-theory covariance matrix of correlations.

**Author(s)**

Jeff Jones and Niels Waller

**References**

Nel, D.G. (1985). A matrix derivation of the asymptotic covariance matrix of sample correlation coefficients. *Linear algebra and its applications*, 67, 137–145.

**See Also**

[adfCor](#)

**Examples**

```
data(Harman23.cor)
normalCor(Harman23.cor$cov, Nobs = 305)
```

**plot.monte**

*Plot Method for Class Monte*

**Description**

plot method for class "monte"

**Usage**

```
## S3 method for class 'monte'
plot(x, ...)
```

**Arguments**

- x An object of class 'monte', usually, a result of a call to `monte`.
- ... Optional arguments passed to plotting function.

**Value**

The function `plot.monte` creates a scatter plot of matrices plot (a splom plot). Cluster membership is denoted by different colors in the plot.

**Examples**

```
#plot(monte.object)
```

**r2d***Convert Radians to Degrees***Description**

Convert radian measure to degrees.

**Usage**

```
r2d(radian)
```

**Arguments**

radian	Radian measure of an angle
--------	----------------------------

**Value**

Degree measure of an angle

**Examples**

```
r2d(.5*pi)
```

**rarc***Rotate Points on the Surface on an N-Dimensional Ellipsoid***Description**

Rotate between two points on the surface on an n-dimensional ellipsoid. The hyper-ellipsoid is composed of all points, B, such that  $B' Rxx B = Rsq$ . Vector B contains standardized regression coefficients.

**Usage**

```
rarc(Rxx, Rsq, b1, b2, Npoints)
```

**Arguments**

Rxx	Predictor correlation matrix.
Rsq	Model coefficient of determination.
b1	First point on ellipsoid. If b1 and b2 are scalars then choose scaled eigenvectors v[b1] and v[b2] as the start and end vectors.
b2	Second point on ellipsoid. If b1 and b2 are scalars then choose scaled eigenvectors v[b1] and v[b2] as the start and end vectors.
Npoints	Generate “Npoints” +1 OLS coefficient vectors between b1 and b2.

**Value**

b	N+1 sets of OLS coefficient vectors between b1 and b2.
---	--

## Author(s)

Niels Waller and Jeff Jones.

## References

Waller, N. G. & Jones, J. A. (2011). Investigating the performance of alternate regression weights by studying all possible criteria in regression models with a fixed set of predictors. *Psychometrika*, 76, 410-439.

## Examples

```

## Example
## GRE/GPA Data
##-----
R <- Rxx <- matrix(c(1.00, .56, .77,
                     .56, 1.00, .73,
                     .77, .73, 1.00), 3, 3)

## GPA validity correlations
rxy <- c(.39, .34, .38)
b <- solve(Rxx) %*% rxy

Rsq <- t(b) %*% Rxx %*% b
N <- 200

b <- rarc(Rxx = R, Rsq, b1 = 1, b2 = 3, Npoints = N)

## compute validity vectors
r <- Rxx %*% b
N <- N + 1
Rsq.r <- Rsq.unit <- rep(0, N)

for(i in 1:N){
  ## eval performance of unit weights
  Rsq.unit[i] <- (t(sign(r[,i])) %*% r[,i])^2 /
    (t(sign(r[,i])) %*% R %*% sign(r[,i]))

  ## eval performance of correlation weights
  Rsq.r[i] <- (t(r[,i]) %*% r[,i])^2 / (t(r[,i]) %*% R %*% r[,i])
}

cat("\nAverage relative performance of unit weights across elliptical arc:",
    round(mean(Rsq.unit)/Rsq,3) )
cat("\n\nAverage relative performance of r weights across elliptical arc:",
    round(mean(Rsq.r)/Rsq,3) )

plot(seq(0, 90, length = N), Rsq.r, typ = "l",
      ylim = c(0, .20),
      xlim = c(0, 95),
      lwd = 3,
      ylab = expression(R^2),
      xlab = expression(paste("Degrees from ",b[1]," in the direction of ",b[2])),
      cex.lab = 1.25, lab = c(10, 5, 5))
points(seq(0, 90, length = N), Rsq.unit,

```

```

type = "l",
lty = 2, lwd = 3)
legend(x = 0,y = .12,
       legend = c("r weights", "unit weights"),
       lty = c(1, 2),
       lwd = c(4, 3),
       cex = 1.5)

```

**rcone***Generate a Cone of Regression Coefficient Vectors***Description**

Compute a cone of regression vectors with a constant R-squared around a target vector.

**Usage**

```
rcone(R,Rsq,b,axis1,axis2,deg,Npoints=360)
```

**Arguments**

R	Predictor correlation matrix.
Rsq	Coefficient of determination.
b	Target vector of OLS regression coefficients.
axis1	1st axis of rotation plane.
axis2	2nd axis of rotation plane.
deg	All vectors b.i will be ‘deg’ degrees from b.
Npoints	Number of rotation vectors, default = 360.

**Value**

b.i	Npoints values of b.i
-----	-----------------------

**Author(s)**

Niels Waller and Jeff Jones

**References**

Waller, N. G. & Jones, J. A. (2011). Investigating the performance of alternate regression weights by studying all possible criteria in regression models with a fixed set of predictors. *Psychometrika*, 76, 410-439.

## Examples

```
R <- matrix(.5, 4, 4)
diag(R) <- 1

Npoints <- 1000
Rsq <- .40
NumDeg <- 20
V <- eigen(R)$vectors

## create b parallel to v[,3]
## rotate in the 2 - 4 plane
b <- V[,3]
bsq <- t(b) %*% R %*% b
b <- b * sqrt(Rsq/bsq)
b.i <- rcone(R, Rsq,b, V[,2], V[,4], deg = NumDeg, Npoints)

t(b.i[,1]) %*% R %*% b.i[,1]
t(b.i[,25]) %*% R %*% b.i[,25]
```

**rellipsoid**

*Generate Uniformly Spaced OLS Regression Coefficients that Yield a User-Supplied R-Squared Value*

## Description

Given predictor matrix R, generate OLS regression coefficients that yield a user-supplied R-Squared value. These regression coefficient vectors will be uniformly spaced on the surface of a (hyper) ellipsoid.

## Usage

```
rellipsoid(R, Rsq, Npoints)
```

## Arguments

R	A p x p predictor correlation matrix.
Rsq	A user-supplied R-squared value.
Npoints	Desired number of generated regression vectors.

## Value

b	A p x Npoints matrix of regression coefficients
---	---

## Author(s)

Niels Waller and Jeff Jones.

## References

Waller, N. G. and Jones, J. A. (2011). Investigating the performance of alternate regression weights by studying all possible criteria in regression models with a fixed set of predictors. *Psychometrika*, 76, 410-439.

## Examples

```

## generate uniformly distributed regression vectors
## on the surface of a 14-dimensional ellipsoid
N <- 10000
Rsq <- .21

# Correlations from page 224 WAIS-III manual
# The Psychological Corporation (1997).
wais3 <- matrix(
  c(1, .76, .58, .43, .75, .75, .42, .54, .41, .57, .64, .54, .50, .53,
    .76, 1, .57, .36, .69, .71, .45, .52, .36, .63, .68, .51, .47, .54,
    .58, .57, 1, .45, .65, .60, .47, .48, .43, .59, .60, .49, .56, .47,
    .43, .36, .45, 1, .37, .40, .60, .30, .32, .34, .35, .28, .35, .29,
    .75, .69, .65, .37, 1, .70, .44, .54, .34, .59, .62, .54, .45, .50,
    .75, .71, .60, .40, .70, 1, .42, .51, .44, .53, .60, .50, .52, .44,
    .42, .45, .47, .60, .44, .42, 1, .46, .49, .47, .43, .27, .50, .42,
    .54, .52, .48, .30, .54, .51, .46, 1, .45, .50, .58, .55, .53, .56,
    .41, .36, .43, .32, .34, .44, .49, .45, 1, .47, .49, .41, .70, .38,
    .57, .63, .59, .34, .59, .53, .47, .50, .47, 1, .63, .62, .58, .66,
    .64, .68, .60, .35, .62, .60, .43, .58, .49, .63, 1, .59, .50, .59,
    .54, .51, .49, .28, .54, .50, .27, .55, .41, .62, .59, 1, .48, .53,
    .50, .47, .56, .35, .45, .52, .50, .53, .70, .58, .50, .48, 1, .51,
    .53, .54, .47, .29, .50, .44, .42, .56, .38, .66, .59, .53, .51, 1),
  nrow = 14, ncol = 14)

R <- wais3[1:6,1:6]
b <- rellipsoid(R, Rsq, Npoints = N)
b <- b$b
#
plot(b[1,],b[2,])

```

## Description

Computes Normal Theory and ADF Standard Errors and CIs for Standardized Regression Coefficients

## Usage

```
seBeta(X, y, cov.x = NULL, cov.xy = NULL, var.y = NULL, Nobs = NULL,
alpha = 0.05, estimator = "ADF", digits = 3)
```

## Arguments

X	Matrix of predictor scores.
y	Vector of criterion scores.
cov.x	Covariance or correlation matrix of predictors.
cov.xy	Vector of covariances or correlations between predictors and criterion.
var.y	Criterion variance.

Nobs	Number of observations.
alpha	Desired Type I error rate; default = .05.
estimator	'ADF' or 'Normal' confidence intervals - requires raw X and raw y; default = 'ADF'.
digits	Number of significant digits to print; default = 3.

**Value**

cov.Beta	Normal theory or ADF covariance matrix of standardized regression coefficients.
se.Beta	standard errors for standardized regression coefficients.
alpha	desired Type-I error rate.
CI.Beta	Normal theory or ADF (1-alpha)% confidence intervals for standardized regression coefficients.
estimator	estimator = "ADF" or "Normal".

**Author(s)**

Jeff Jones and Niels Waller

**References**

Jones, J. A., and Waller, N. G. (2015). The Normal-Theory and Asymptotic Distribution-Free (ADF) covariance matrix of standardized regression coefficients: Theoretical extensions and finite sample behavior. *Psychometrika*, 80, 365-378.

**Examples**

```
library(MASS)

set.seed(123)

R <- matrix(.5, 3, 3)
diag(R) <- 1
X <- mvrnorm(n = 200, mu = rep(0, 3), Sigma = R, empirical = TRUE)
Beta <- c(.2, .3, .4)
y <- X%*% Beta + .64 * scale(rnorm(200))
seBeta(X, y, Nobs = 200, alpha = .05, estimator = 'ADF')

# 95% CIs for Standardized Regression Coefficients:
#
#          lbound estimate ubound
# beta_1  0.104    0.223  0.341
# beta_2  0.245    0.359  0.473
# beta_3  0.245    0.360  0.476
```

---

seBetaCor	<i>Standard Errors and CIs for Standardized Regression Coefficients from Correlations</i>
-----------	---

---

**Description**

Computes Normal Theory and ADF Standard Errors and CIs for Standardized Regression Coefficients from Correlations

**Usage**

```
seBetaCor(R, rxy, Nobs, alpha=.05, digits=3, covmat = 'normal')
```

**Arguments**

R	A p x p predictor correlation matrix.
rxy	A p x 1 vector of predictor-criterion correlations
Nobs	Number of observations.
alpha	Desired Type I error rate; default = .05.
digits	Number of significant digits to print; default = 3.
covmat	String = 'normal' (the default) or a (p+1)p/2 x (p+1)p/2 covariance matrix of correlations. The default option computes an asymptotic covariance matrix under the assumption of multivariate normal data. Users can supply a covariance matrix under asymptotic distribution free (ADF) or elliptical distributions when available.

**Value**

cov.Beta	Covariance matrix of standardized regression coefficients.
se.Beta	Vector of standard errors for the standardized regression coefficients.
alpha	Type-I error rate.
CI.Beta	(1-alpha)% confidence intervals for standardized regression coefficients.

**Author(s)**

Jeff Jones and Niels Waller

**References**

- Jones, J. A, and Waller, N. G. (2013). The Normal-Theory and asymptotic distribution-free (ADF) covariance matrix of standardized regression coefficients: Theoretical extensions and finite sample behavior. Technical Report (052913)[TR052913]
- Nel, D.A.G. (1985). A matrix derivation of the asymptotic covariance matrix of sample correlation coefficients. *Linear Algebra and its Applications*, 67, 137-145.
- Yuan, K. and Chan, W. (2011). Biases and standard errors of standardized regression coefficients. *Psychometrika*, 76(4), 670–690.

## Examples

```
R <- matrix(c(1.0000, 0.3511, 0.3661,
             0.3511, 1.0000, 0.4359,
             0.3661, 0.4359, 1.0000), 3, 3)

rxy <- c(0.5820, 0.6997, 0.7621)
Nobs <- 46
out <- seBetaCor(R = R, rxy = rxy, Nobs = Nobs)

# 95% CIs for Standardized Regression Coefficients:
#
#      lbound estimate ubound
# beta_1  0.107    0.263  0.419
# beta_2  0.231    0.391  0.552
# beta_3  0.337    0.495  0.653
```

**seBetaFixed**

*Covariance Matrix and Standard Errors for Standardized Regression Coefficients for Fixed Predictors*

## Description

Computes Normal Theory Covariance Matrix and Standard Errors for Standardized Regression Coefficients for Fixed Predictors

## Usage

```
seBetaFixed(X, y, cov.x = NULL, cov.xy = NULL, var.y = NULL, var.error = NULL,
            Nobs = NULL)
```

## Arguments

X	Matrix of predictor scores.
y	Vector of criterion scores.
cov.x	Covariance or correlation matrix of predictors.
cov.xy	Vector of covariances or correlations between predictors and criterion.
var.y	Criterion variance.
var.error	Optional argument to supply the error variance: var(y - yhat).
Nobs	Number of observations.

## Value

cov.Beta	Normal theory covariance matrix of standardized regression coefficients for fixed predictors.
se.Beta	Standard errors for standardized regression coefficients for fixed predictors.

## Author(s)

Jeff Jones and Niels Waller

## References

Yuan, K. & Chan, W. (2011). Biases and standard errors of standardized regression coefficients. *Psychometrika*, 76(4), 670-690.

## See Also

[seBeta](#)

## Examples

```
## We will generate some data and pretend that the Predictors are being held fixed

library(MASS)
R <- matrix(.5, 3, 3); diag(R) <- 1
Beta <- c(.2, .3, .4)

rm(list = ".Random.seed", envir = globalenv()); set.seed(123)
X <- mvrnorm(n = 200, mu = rep(0, 3), Sigma = R, empirical = TRUE)
y <- X %*% Beta + .64*scale(rnorm(200))

seBetaFixed(X, y)

# $covBeta
#          b1          b2          b3
# b1  0.003275127 -0.001235665 -0.001274303
# b2 -0.001235665  0.003037100 -0.001491736
# b3 -0.001274303 -0.001491736  0.002830157
#
# $seBeta
#          b1          b2          b3
# 0.05722872 0.05510989 0.05319922

## you can also supply covariances instead of raw data

seBetaFixed(cov.x = cov(X), cov.xy = cov(X, y), var.y = var(y), Nobs = 200)

# $covBeta
#          b1          b2          b3
# b1  0.003275127 -0.001235665 -0.001274303
# b2 -0.001235665  0.003037100 -0.001491736
# b3 -0.001274303 -0.001491736  0.002830157
#
# $seBeta
#          b1          b2          b3
# 0.05722872 0.05510989 0.05319922
```

## Description

Calculate univariate skewness for vector or matrix (algorithm G1 in Joanes & Gill, 1998).

**Usage**

```
skew(x)
```

**Arguments**

**x** Either a vector or matrix of numeric values.

**Value**

Skewness for each column in **x**.

**Author(s)**

Niels Waller

**References**

Joanes, D. N. & Gill, C. A. (1998). Comparing measures of sample skewness and kurtosis. *The Statistician*, 47, 183-189.

**See Also**

[kurt](#)

**Examples**

```
x <- matrix(rnorm(1000), 100, 10)
skew(x)
```

**summary.monte**

*Summary Method for an Object of Class Monte*

**Description**

summary method for class "monte"

**Usage**

```
## S3 method for class 'monte'
summary(object, digits = 3, compute.validities = FALSE, Total.stats=TRUE, ...)
```

**Arguments**

<b>object</b>	An object of class <b>monte</b> , usually, a result of a call to <b>monte</b> .
<b>digits</b>	Number of digits to print. Default = 3.
<b>compute.validities</b>	Logical: If TRUE then the program will calculate the indicator validities ( $\eta^2$ ) for the generated data.
<b>Total.stats</b>	Logical: If TRUE then the program will return the following statistics for the total sample: (1) indicator correlation matrix, (2) indicator skewness, (3) indicator kurtosis.
<b>...</b>	Optional arguments.

**Value**

Various descriptive statistics will be computed within groups including"

1. clus.size Number of objects within each group.
2. centroids Group centroids.
3. var.matrix Within group variances.
4. Ratio of within group variances (currently printed but not saved).
5. cor.list Expected within group correlations.
6. obs.cor Observed within group correlations.
7. skew.list Expected within group indicator skewness values.
8. obs.skew Observed within group indicator skewness values.
9. kurt.list Expected within group indicator kurtosis values.
10. obs.kurt Observed within group indicator kurtosis values.
11. validities Observed indicator validities.
12. Total.cor Total sample correlation matrix.
13. Total.skew Total sample indicator skewness.
14. Total.kurt Total sample indicator kurtosis.

**Examples**

```
## set up a 'monte' run for the Fisher iris data

sk.lst <- list(c(0.120,  0.041,  0.106,  1.254),
                 c(0.105, -0.363, -0.607, -0.031),
                 c(0.118,  0.366,  0.549, -0.129) ) #
```

  

```
kt.lst <- list(c(-0.253, 0.955,  1.022,  1.719),
                  c(-0.533,-0.366,  0.048, -0.410),
                  c( 0.033, 0.706, -0.154, -0.602))
```

  

```
cormat <- lapply(split(iris[,1:4],iris[,5]), cor)
```

  

```
my.iris <- monte(seed = 123, nvar = 4, nclus = 3, cor.list = cormat,
                  clus.size = c(50, 50, 50),
                  eta2 = c(0.619, 0.401, 0.941, 0.929),
                  random.cor = FALSE,
                  skew.list = sk.lst, kurt.list = kt.lst,
                  secor = .3,
                  compactness = c(1, 1, 1),
                  sortMeans = TRUE)
summary(my.iris)
```

**summary.monte1***Summary Method for an Object of Class Monte1***Description**

summary method for class "monte1"

**Usage**

```
## S3 method for class 'monte1'
summary(object, digits=3, ...)
```

**Arguments**

object	An object of class <code>monte1</code> , usually, a result of a call to <code>monte1</code> .
digits	Number of significant digits to print in final results.
...	Additional argument affecting the summary produced.

**Value**

Various descriptive statistics will be computed including"

1. Expected correlation matrix.
2. Observed correlation matrix.
3. Expected indicator skewness values.
4. Observed indicator skewness values.
5. Expected indicator kurtosis values.
6. Observed indicator kurtosis values.

**Examples**

```
## Generate dimensional data for 4 variables.
## All correlations = .60; all variable
## skewness = 1.75;
## all variable kurtosis = 3.75

cormat <- matrix(.60, 4, 4)
diag(cormat) <- 1

nontaxon.dat <- monte1(seed = 123, nsub = 100000, nvar = 4, skewvec = rep(1.75, 4),
                        kurtvec = rep(3.75, 4), cormat = cormat)

summary(nontaxon.dat)
```

---

<code>tetcor</code>	<i>Compute ML Tetrachoric Correlations</i>
---------------------	--

---

## Description

Compute ML tetrachoric correlations with optional bias correction and smoothing.

## Usage

```
tetcor(X, y = NULL, BiasCorrect, stderror, Smooth = TRUE, max.iter)
```

## Arguments

<code>X</code>	Either a matrix or vector of (0/1) binary data.
<code>y</code>	An optional(if X is a matrix) vector of (0/1) binary data.
<code>BiasCorrect</code>	A logical that determines whether bias correction (Brown & Benedetti, 1977) is performed. Default = TRUE.
<code>stderror</code>	A logical that determines whether standard errors are calculated. Default = FALSE.
<code>Smooth</code>	A logical which determines whether the tetrachoric correlation matrix should be smoothed. A smoothed matrix is always positive definite.
<code>max.iter</code>	Maximum number of iterations. Default = 50.

## Value

If `stderror` = FALSE, `tetcor` returns a matrix of tetrachoric correlations. If `stderror` = TRUE then `tetcor` returns a list the first component of which is a matrix of tetrachoric correlations and the second component is a matrix of standard errors (see Hamdan, 1970).

## Author(s)

Niels Waller

## References

- Brown, M. B. & Benedetti, J. K. (1977). On the mean and variance of the tetrachoric correlation coefficient. *Psychometrika*, 42, 347–355.
- Divgi, D. R. (1979) Calculation of the tetrachoric correlation coefficient. *Psychometrika*, 44, 169-172.
- Hamdan, M. A. (1970). The equivalence of tetrachoric and maximum likelihood estimates of rho in 2 by 2 tables. *Biometrika*, 57, 212-215.

## Examples

```
## generate bivariate normal data
library(MASS)
set.seed(123)
rho <- .85
xy <- mvrnorm(100000, mu = c(0,0), Sigma = matrix(c(1, rho, rho, 1), ncol = 2))

# dichotomize at difficulty values
```

```

p1 <- .7
p2 <- .1
xy[,1] <- xy[,1] < qnorm(p1)
xy[,2] <- xy[,2] < qnorm(p2)

print( apply(xy,2,mean), digits = 2)
#[1] 0.700 0.099

tetcor(X = xy, BiasCorrect = TRUE,
       stderror = TRUE, Smooth = TRUE, max.iter = 5000)

# $r
# [,1]      [,2]
# [1,] 1.0000000 0.8552535
# [2,] 0.8552535 1.0000000
#
# $se
# [,1]      [,2]
# [1,] 1.00000000 0.01458171
# [2,] 0.01458171 1.00000000
#
# $Warnings
# list()

```

**tetcorQuasi**

*Correlation between a Naturally and an Artificially Dichotomized Variable*

**Description**

A function to compute Ulrich and Wirtz's correlation of a naturally and an artificially dichotomized variable.

**Usage**

```
tetcorQuasi(x, y = NULL)
```

**Arguments**

- x An N x 2 matrix or an N x 1 vector of binary responses coded 0/1.
- y An optional (if x is a vector) vector of 0/1 responses.

**Value**

A quasi tetrachoric correlation

...

**Author(s)**

Niels Waller

## References

Ulrich, R. & Wirtz, M. (2004). On the correlation of a naturally and an artificially dichotomized variable. *British Journal of Mathematical and Statistical Psychology*, 57, 235-252.

## Examples

```
set.seed(321)
Nsubj <- 5000

## Generate mvn data with rxy = .5
R <- matrix(c(1, .5, .5, 1), 2, 2)
X <- MASS::mvrnorm(n = Nsubj, mu = c(0, 0), Sigma = R, empirical = TRUE)

## dichotomize data
thresholds <- qnorm(c(.2, .3))
binaryData <- matrix(0, Nsubj, 2)

for(i in 1:2){
  binaryData[X[,i] <= thresholds[i],i] <- 1
}

## calculate Pearson correlation
cat("\nPearson r: ", round(cor(X)[1,2], 2))

## calculate Pearson Phi correlation
cat("\nPhi r: ", round(cor(binaryData)[1,2], 2))

## calculate tetrachoric correlation
cat("\nTetrachoric r: ", round(tetcor(binaryData)$r[1,2], 2))

## calculate Quasi-tetrachoric correlation
cat("\nQuasi-tetrachoric r: ", round(tetcorQuasi(binaryData), 2))
```

vcos

*Compute the Cosine Between Two Vectors*

## Description

Compute the cosine between two vectors.

## Usage

```
vcos(x, y)
```

## Arguments

- |   |                 |
|---|-----------------|
| x | A p x 1 vector. |
| y | A p x 1 vector. |

## Value

Cosine between x and y

**Examples**

```
x <- rnorm(5)
y <- rnorm(5)
vcos(x, y)
```

---

vnorm

*Norm a Vector to Unit Length*

---

**Description**

Norm a vector to unit length.

**Usage**

`vnorm(x)`

**Arguments**

`x` An n by 1 vector.

**Value**

`x` scaled to unit length.

**Author(s)**

Niels Waller

**Examples**

```
x <- rnorm(5)
v <- vnorm(x)
```

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