Stochastic Differential Equations with Solutions

Stochastic Differential Equations

We consider the model as the parametric Itô stochastic differential equation :

$$dX_t = \mu(\theta, t, X_t)dt + \sigma(\vartheta, t, X_t)dW_t, \qquad t \geq 0, X_0 = \zeta \tag{1}$$

where $\{W_t, t \geq 0\}$ is a standard Wiener process, $\mu : \Theta \times [0,T] \times \mathbb{R} \to \mathbb{R}$, called the drift coefficient, and $\sigma : \Xi \times [0,T] \times \mathbb{R} \to \mathbb{R}^+$, called the diffusion coefficient, are known functions except the unknown parameters θ and ϑ, $\Theta \subset \mathbb{R}$, $\Xi \subset \mathbb{R}$ and $\mathbb{E}(\zeta^2) < \infty$.

Itô Lemma

$$df(t,x_t) = \left(\frac{\partial f(t,x_t)}{\partial t} + \mu(\theta,t,x_t)\frac{\partial f(t,x_t)}{\partial x} + \frac{1}{2}\sigma^2(\vartheta,t,x_t)\frac{\partial^2 f(t,x_t)}{\partial x^2} \right) dt + \sigma(\vartheta,t,x_t)\frac{\partial f(t,x_t)}{\partial x} dW_t \tag{2}$$

Solution of SDE [1]

SDE	***Solution***
$dX_t = \alpha X_t dt + \beta X_t dW_t$	$X_t = X_0 \exp\left(\left(\alpha - \frac{1}{2}\beta\right)t + \beta W_t\right), \quad X_0 > 0.$
$dX_t = (\alpha X_t + \beta)dt + \lambda dW_t$	$X_t = e^{\alpha t}\left(X_0 + \frac{\beta}{\alpha}\left(1 - e^{-\alpha t}\right)\right) + \lambda \int_0^t e^{-\alpha s} dW_s.$
$dX_t = \left(\frac{2}{1+t}X_t + \beta(1+t^2)\right)dt + \beta(1+t^2)dW_t$	$X_t = \left(\frac{1+t}{1+t_0}\right)^2 X_0 + \beta(1+t^2)(W_t - W_{t_0} + t - t_0).$
$dX_t = \alpha X_t dt + \beta dW_t$	$X_t = e^{-\alpha t}\left(X_0 + \beta \int_0^t e^{\alpha s} dW_s\right).$
$dX_t = \frac{1}{2}\alpha(\alpha-1)X_t^{1-2/\alpha}dt + \alpha X_t^{1-1/\alpha}dW_t$	$X_t = \left(W_t + X_0^{1/\alpha}\right)^{\alpha}, \quad X_0 > 0.$
$dX_t = \frac{1}{2}\alpha^2 X_t dt + \alpha X_t dW_t$	$X_t = X_0 \exp(\alpha W_t).$
$dX_t = \frac{1}{2}(\ln\alpha)^2 X_t dt + (\ln\alpha)X_t dW_t$	$X_t = X_0 \exp(W_t \ln\alpha), \quad \alpha > 0.$
$dX_t = -\frac{1}{2}\alpha^2 X_t dt + \alpha\sqrt{1 - X_t^2}dW_t$	$X_t = \sin(\alpha W_t + \arcsin X_0), \quad \lvert X_0 \rvert \leq 1$
$dX_t = -\frac{1}{2}\alpha^2 X_t dt - \alpha\sqrt{1 - X_t^2}dW_t$	$X_t = \cos(\alpha W_t + \arccos X_0), \quad \lvert X_0 \rvert \leq 1$
$dX_t = -X_t(2\ln X_t + 1)dt - 2X_t\sqrt{-\ln X_t}dW_t$	$X_t = \exp\left(-\left(W_t + \sqrt{-\ln X_t}\right)^2\right), \quad X_0 \leq 1.$
$dX_t = \frac{1}{2}\alpha^2 m X_t^{2m-1}dt + \alpha X_t^m dW_t$	$X_t = \left(X_0^{1-m} - \alpha(m-1)W_t\right)^{1/(1-m)}, m \neq 1, X_0 > 0.$
$dX_t = -\beta^2 X_t(1 - X_t^2)dt + \beta(1 - X_t^2)dW_t$	$X_t = \frac{(1+X_0)\exp(2\beta W_t) + X_0 - 1}{(1+X_0)\exp(\beta W_t) - X_0 + 1}.$
$dX_t = \frac{1}{3}X_t^{1/3}dt + X_t^{2/3}dW_t$	$X_t = \left(X_0^{1/3} + \frac{1}{3}W_t\right)^3.$
$dX_t = -(\alpha + \beta^2 X_t)(1 - X_t^2)dt + \beta(1 - X_t^2)dW_t$	$X_t = \frac{(1+X_0)\exp(-2\alpha(t-t_0) + 2\beta W_t) + X_0 - 1}{(1+X_0)\exp(-2\alpha(t-t_0) + \beta W_t) - X_0 + 1}.$

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