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High Frequency Portfolio Analytics

User Manual

PortfolioEffectEstim R Package

High Frequency Price Estimators & Models

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1 Package Installation

PortfolioEffectEstim package for R relies on the rJava package, which assumes that Java runtime is installed and configured on your system. To install Java runtime and to configure your R engine to work with it, follow these steps:

1.1 Install Latest JDK/JRE Runtime

Download and install latest Java distribution (JDK or JRE) for your platform from Oracle's website

1.2 Configure Java Environment (Optional)

If you are using Windows, installation wizard from the previous step should have done everything for you. If you are on Linux or Mac and you used a tarball file, you will need to manually append the following lines to /etc/environment using your favorite text editor:

```
export JAVA_HOME=/path/to/java/folder
export PATH=$PATH:$JAVA_HOME/bin
```

Apply environment changes:

```
source /etc/environment
```

To complete with the set-up of Java environment inside R, run the following line:

```
sudo R CMD javareconf
```

1.3 Install Required Packages (Optional)

If you are manually installing PortfolioEffectEstim package (you don't want to use CRAN repositories for some reason), you would need to install all required package dependencies first. Start R from the command line or in your GUI editor and type

```
install.packages(c("PortfolioEffectHFT", "rJava"))
```

You are now ready to install the PortfolioEffectEstim package directly from www.portfolioeffect.com downloads section.

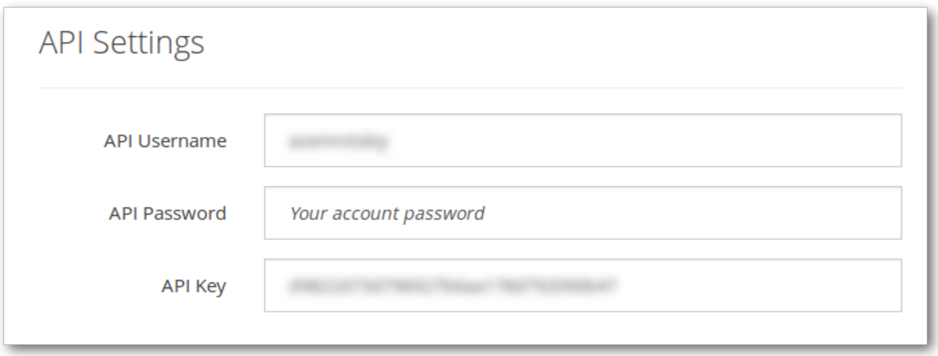
2 API Credentials

All portfolio computations are performed on PortfolioEffect cloud servers. To obtain a free non-professional account, you need to follow a quick sign-up process on our website: www.portfolioeffect.com/registration.

Please use a valid sign-up address - it will be used to email your account activation link.

2.1 Locate API Credentials

Log in to you account and locate your API credentials on the main page



2.2 Set API Credentials in R

Run the following commands to set your account API credentials for the PortfolioEffectEstim R Package installed. You will need to do it only once as your credentials are stored between sessions on your local machine to speed up future logons.

You would need to repeat this procedure if you change your account password or install PortfolioEffectEstim package on another computer.

```
require(PortfolioEffectEstim)
util_setCredentials("API Username", "API Password", "API Key")
```

You are now ready to call PortfolioEffectEstim methods.

3 Price Variance

3.1 Integrated Variance

Assume that the logarithmic eqilibrium price of a financial asset is given by the following diffusion process

$$X_t = \int_0^t \mu(s)ds + \int_0^t \sigma(s)dW(s) \tag{3.1}$$

where

- W_t is a standart Brownian Motion,
- the mean process μ is continuous and of finite variation,
- $\sigma(t) > 0$ denotes the cadlag instantaneous volatility.

The object of interest is the integrated variance (IV), i.e. the amount of variation at time point t accumulated over a past time interval Δ according to [1, Pigorsch et al.]:

$$IV_t = \int_{t-\Delta}^t \sigma^2(s)ds \tag{3.2}$$

3.1.1 Returns

Suppose there exist m intraday eqilibrium returns, the ith intraday return is then defined as:

$$r_i^{X(m)} = X_{i/m} - X_{(i-1)/m}, \quad i = 1, 2, \ldots, m. \tag{3.3}$$

3.2 Realized Variance

3.2.1 Assumptions

The equilibrium price process.

1. The logarithmic equilibrium price process p_t^X is a continuous stochastic volatility semimartingale. Specifically,
$$p_t^X = \alpha_t + m_t; \tag{3.4}$$
where α_t (with $\alpha_0 = 0$) is a predictable drift process of finite variation and m_t is a continuous local martingale defined as $\int_0^t \sigma_s dW_s$ with $W_t : t \geq 0$ denoting standard Brownian motion.
2. The spot volatility process σ_t is cadlag and bounded away from zero.
3. The process $\int_0^t \sigma_s^4 ds$ is bounded almost surely for all $t < \infty$.

3.2.2 Estimator

For discretely observed path X_{t_i}, $i = 0, \dots, n$ of the X, the realized quadratic variation could be estimated consistently using the realized variance measure, defined as [2, Zu and Boswijk, 2014]:

$$RV_t = \sum_{i=1}^{n} (X_{t_i} - X_{t_{(i-1)}})^2 \xrightarrow{p} IV \tag{3.5}$$

However the realized variance estimator for integrated volatility is not consistent when data is contaminated by market microstrcuture noise. In particular, when we only observe data with microstructure noise, the realized variance measure will diverge. When sampling frequency increases, realized variance actually estimates the sum of infinite many variances of noises.

3.2.3 Properties

- Convergence speed: $m^{1/2}$ (m - number of observation)
- Unbiased: no
- Consistent: no
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for for endogenous noise: no

3.3 Two Series Realized Variance

3.3.1 Assumptions

The microstructure noise.

1. The microstructure noise, $\epsilon_{t,i}$, has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.

3.3.2 Estimator

Two Scale Realized Variance (TSRV) estimates integrated volatility consistently. The idea is to use realized variance type estimators over two time scales to correct the effect of market microstructure noise. Define as:

$$[Y,Y]_t^{avg} = \frac{1}{K} \sum_{i=K}^{n} (Y_{t_i} - Y_{t_{(i-K)}})^2 \tag{3.6}$$

$$[Y,Y]_t^{all} = \sum_{i=1}^{n} (Y_{t_i} - Y_{t_{(i-1)}})^2 \tag{3.7}$$

$$\bar{n} = \frac{n - K + 1}{K}, \tag{3.8}$$

the TSRV estimator is defined as [2, Zu and Boswijk, 2008]:

$$TSRV_t = [Y,Y]_t^{(avg)} - \frac{\bar{n}}{n}[Y,Y]_t^{(all)} \xrightarrow{p} IV \tag{3.9}$$

A small sample refinement to the estimator give correction:

$$TSRV_t^{adjust} = \left(1 - \frac{\bar{n}}{n}\right)^{-1} TSRV_t \xrightarrow{p} IV. \tag{3.10}$$

If we have possibly dependent noise we should use an alternative estimator that is also based on the two time scales idea.

To pin down the optimal sampling frequency K [3, Zhang, Mykland, and Ait-Sahalia, 2005], one can minimize the expected asymptotic variance and to obtain

$$c^* = \left(\frac{16(E\epsilon^2)^2}{TE\eta^2}\right)^{1/3} \tag{3.11}$$

which can be consistently estimated from data in past time periods (before time $t_0 = 0$), using $\hat{E\epsilon^2}$ and an estimator of η^2. η^2 can be taken to be independent of K as long as one allocates sampling points to grids regularly. Hence one can choose c, and so also K, based on past data.

3.3.3 Properties

- Convergence speed: $m^{1/6}$ (m - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for endogenous noise: no

3.4 Multiple Series Realized Variance

3.4.1 Assumptions

Dependent Noise Structure [4, Podolskij and Vetter, 2009]:

1. The microstructure noise, $\epsilon_{t,i}$, has a zero mean, stationary, and strong mixing stochastic process, with the mixing coefficients decaying exponentially. In addition, $E[(\epsilon_{t,i})^{4+\kappa}]$, for some $\kappa > 0$.
2. The noise is independent of the price process.
3. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.

3.4.2 Estimator

Under most assumptions, this estimator violates the sufficiency principle, whence we define the average lag j realized volatility as

$$[Y,Y]_T^{(J)} = \frac{1}{J}\sum_{r=0}^{J-1}[Y,Y]_T^{(J,r)} = \frac{1}{J}\sum_{t=0}^{n-J}(Y_{t_{i+J}} - Y_{t_i})^2 \tag{3.12}$$

A generalization of TSRV can be defined for $1 \leq J < K \leq n$ as

$$MSRV_t = [Y,Y]_T^{(K)} - \frac{\bar{n}_K}{\bar{n}_J}[Y,Y]_T^{(J)} \xrightarrow{p} IV \tag{3.13}$$

thereby combining the two time scales J and K. Here $\bar{n}_K = (n-K+1)/K$ and similarly for $\bar{n}_J$.

3.4.3 Properties

- Convergence speed: $m^{1/4}$ (m - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: yes
- Allows for for endogenous noise: no

3.5 Modulated Realized Variance

3.5.1 Assumptions

The microstructure noise.

1. The microstructure noise, $\epsilon_{t,i}$, has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.

3.5.2 Estimator

Modulated Bipower Variation is written as [4, Podolskij and Vetter, 2009]:

$$MBV(Y,r,l)_n = n^{(r+l)/4-1/2}\sum_{m=1}^{M} \mid \bar{Y}_m^{(K)} \mid^r \mid \bar{Y}_{m+1}^{(K)} \mid, \quad r,l \geq 0; \tag{3.14}$$

$$Y_m^{(K)} = \frac{1}{n/M-K+1}\sum_{i=(m-1)n/M}^{mn/M-K}(Y_{(i+K)/n} - Y_{i/n}) \tag{3.15}$$

with

$$K = c_1 n^{1/2}, \quad M = \frac{n}{c_2 K} = \frac{n^{1/2}}{c_1 c_2} \tag{3.16}$$

We can choose the constants c_1 and c_2 from specific process:

$$c_1 = 0.25, \quad c_2 = 2. \tag{3.17}$$

Modulated Realized Variance is written as:

$$MRV(Y)_n = \frac{c_1 c_2 MBV(Y,2,0)_n - \nu_2 \hat{\omega}^2}{\nu_1} \xrightarrow{p} IV \tag{3.18}$$

where

$$\nu_1 = \frac{c_1(3c_2 - 4 + (2-c_2)^3 \bigvee 0)}{3(c_2-1)^2} \tag{3.19}$$

$$\nu_2 = \frac{2((c_2-1) \bigwedge 1)}{c_1(c_2-1)^2} \tag{3.20}$$

3.5.3 Properties

- Convergence speed: $m^{1/4}$ (m - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for for endogenous noise: no

3.6 Jump Robust Modulated Realized Variance

3.6.1 Assumptions

The microstructure noise.

1. The microstructure noise, $\epsilon_{t,i}$, has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.
4. The price is $Z = Y + J$ where Y is a noisy diffusion process and J denotes a finite activity jump process, that is, J exhibits finitely many jumps on compact intervals.

3.6.2 Estimator

We can construct consistent estimates for the integrated volatility, which are robust to noise and finite activity jumps [4, Podolskij and Vetter, 2009].

$$MBV(Y,r,l)_n = \frac{(c_1 c_2/\mu_1^2)(MBV(Z,1,1)_n - \nu_2 \hat{\omega}^2)}{\nu_1} \xrightarrow{p} IV \tag{3.21}$$

where

$$\nu_1 = \frac{c_1(3c_2 - 4 + (2 - c_2)^3 \bigvee 0)}{3(c_2 - 1)^2} \tag{3.22}$$

$$\nu_2 = \frac{2((c_2 - 1) \bigwedge 1)}{c_1(c_2 - 1)^2} \tag{3.23}$$

Mixed normal distribution with conditional variance [4, Podolskij and Vetter, 2009]:

$$\beta^2 = \frac{2c_1c_2}{\nu_1^2} \int_0^1 (\nu_1\sigma_u^2 + \nu_2\omega^2)^2 du \tag{3.24}$$

Consistent estimator of β is:

$$\beta_n^2 = \frac{2c_1^2c_2^2}{3\nu_1^2} MBV(Y, 4, 0)_n \tag{3.25}$$

We can choose the constants c_1 and c_2 that minimise the conditional variance. In order to compare our asymptotic variance with the corresponding results of other methods we assume that the volatility process σ is constant. In that case the conditional variance β^2 is minimised by

$$c_1 = \sqrt{\frac{18}{(c_2 - 1)(4 - c_2)} \cdot \frac{\omega}{\sigma}}, \quad c_2 = \frac{8}{5} \tag{3.26}$$

We can choose the constants c_1 and c_2 from specific process:

$$c_1 = 0.25, \quad c_2 = 2. \tag{3.27}$$

3.6.3 Properties

- Converges to integrated variance
- Convergence speed: $m^{1/6}$ (m - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: yes
- Allows for time dependent noise: no
- Allows for for endogenous noise: no

3.7 Kernel-based Realized Variance

3.7.1 Assumptions

The microstructure noise.

1. The microstructure noise, $\epsilon_{t,i}$, has zero mean and is distributed random variable.
2. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.

3.7.2 Kernel Types

Flat-Top Kernel	
Bartlett Kernel	$k(x) = 1 - x$
Epanichnikov Kernel	$k(x) = 1 - x^2$
Second order Kernel	$k(x) = 1 - 2x + x^2$
Non-Flat-Top Kernel	
Cubic Kernel	$k(x) = 1 - 3x^2 + 2x^3$
Parzen Kernel	$k(x) = \begin{cases} 2(1-x)^3 & x > 0.5 \\ 1 - 6x^2 + 6x^3 & x < 0.5, \end{cases}$
Tukey Hanning Kernel	$k(x) = \frac{(1 + \sin(\pi/2 - \pi x))}{2}$
Tukey Hanning Modified Kernel	$k(x) = \frac{(1 - \sin(\pi/2 - \pi(1-x)^2))}{2}$
Fifth Order Kernel	$k(x) = 1 - 10x^3 + 15x^4 - 6x^5$
Sixth Order Kernel	$k(x) = 1 - 15x^4 + 24x^5 - 10x^6$
Seventh Order Kernel	$k(x) = 1 - 21x^5 + 35x^6 - 15x^7$
Eighth Order Kernel	$k(x) = 1 - 28x^6 + 48x^7 - 21x^8$

3.7.3 Estimator

Kernel-based Realized Variance is [5, Barndorff-Nielsen et al., 2006]:

$$\tilde{K}(X_\delta) = \gamma_0(X_\delta) + \sum_{h=1}^{H} k\left(\frac{h-1}{H}\right)\tilde{\gamma}_h(X_\delta) \tag{3.28}$$

where $k()$ is kernel function and:

$$\tilde{\gamma}_h(X_\delta) = \gamma_h(X_\delta) + \gamma_{-h}(X_\delta) \tag{3.29}$$

$$\gamma_h(X_\delta) = \sum_{j=1}^{n}(X_{\delta j} - X_{\delta(j-1)})(X_{\delta(j-h)} - X_{\delta(j-h-1)}) \tag{3.30}$$

with $h = -H, \ldots, -1, 0, 1, \ldots, H$ and $n = [1/\delta]$.

Bandwidth for all kernrel is:

$$H = cn^{2/3} \tag{3.31}$$

In this case we have the asymptotic distribution given:

$$n^{1/6}\{\tilde{K}(X_\delta) - \int_0^t \sigma_u^2 du\} \xrightarrow{L_s} MN\left[0, 4ck_\bullet^{0,0} t \int_0^t \sigma_u^4 du + 4\omega^4 c^{-2}\{k'(0)^2 + k'(1)^2\}\right] \tag{3.32}$$

where $\omega = (1, 1, k(\frac{1}{H}), \ldots, k\left(\frac{H-1}{H}\right))^T$

$$k_\bullet^{0,0} = \int_0^1 k(x)^2 dx, \quad k_\bullet^{0,2} = \int_0^1 k(x)k''(x)dx \quad k_\bullet^{0,4} = \int_0^1 k(x)k''''(x)dx \tag{3.33}$$

The value of c which minimises the asymptotic variance is

$$c = d\frac{\omega^{4/3}}{(t\int_0^t \sigma_u^4 du)^{1/3}} \tag{3.34}$$

where

$$d = \left[\frac{2k^2(0)^2 + k'(1)^2}{k_\bullet^{0,0}}\right]^{1/2} \tag{3.35}$$

The lower bound for the asymptotic variance is $6dk_\bullet^{0,0}\omega^{4/3}(t\int_0^t \sigma_u^4 du)^{2/3}$

But for non-flat-top kernel we can rewiew special case.

Bandwidth for non-flat-top kernrel is:

$$H = cn^{1/2} \tag{3.36}$$

In this case the asymptotic distribution is given

$$n^{1/4}\{\tilde{K}(X_\delta) - \int_0^t \sigma_u^2 du\} \xrightarrow{L_s}$$
$$MN\left[0, 4ck_\bullet^{0,0} t \int_0^t \sigma_u^4 du - 8c^{-1}k_\bullet^{0,2}\omega^2\left(\int_0^t \sigma_u^2 du + \frac{\omega^2}{2}\right) + 4\omega^4 c^{-3}\{k'''(0) + k_\bullet^{0,4}\}\right] \tag{3.37}$$

For this class of kernels the value of $\hat{c}$ which minimises the asymptotic variance is

$$c \approx \sqrt{\frac{1}{k_\bullet^{0,0}}\left\{-k_\bullet^{0,2} + \sqrt{(k_\bullet^{0,2})^2 + 3k_\bullet^{0,0} f}\right\}} \tag{3.38}$$

3.7.4 Properties

Flat-Top Kernel

- Convergence speed: $m^{1/6}$ (m - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for for endogenous noise: no

Non-Flat-Top Kernel

- Convergence speed: $m^{1/4}$ (m - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: yes
- Allows for for endogenous noise: yes

3.8 IV Estimators Comparison

Type	Method	Unbiased	Consistent	Jump Robust	Time dependence noise	Endogenous noise
Plain	RV	✗	✗	✗	✗	✗
Subsampling	TSRV	✓	✓	✗	✗	✗
	MSRV	✓	✓	✗	✓	✗
Kernel	KRV_{FT}	✓	✓	✗	✗	✗
	KRV_{NFT}	✓	✓	✗	✓	✓
Range	MRV	✓	✓	✗	✗	✗
	MRV_{jump}	✓	✓	✓	✗	✗

Figure 3.1: Table Estimators

4 Price Noise Variance

4.1 RV Noise Variance

Assume that the observed (log) price is contaminated by market microstructure noise u (or measurement error), i.e.:

$$Y_{i/m} = X_{i/m} + u_{i/m}, \quad i = 1, 2, \ldots, m \tag{4.1}$$

where $X_{i/m}$ is the latent true, or so-called efficient, price that follows the semimartingale. In this case, the observed intraday return is given by:

$$r_i^{Y(m)} = r_i^{X(m)} + \epsilon_i^{(m)}, \quad i = 1, 2, \ldots, m, \tag{4.2}$$

i.e. by the efficient intraday return $r_i^{X(m)} = X_{i/m} - X_{(i-1)/m}$ and the intraday noise increment $\epsilon_i^{(m)} = u_{i/m} - u_{(i-1)/m}$.

The noise variance $E\epsilon^2$ can be estiated consistently by normalized realized variance over the whole interval $[0, 1]$ for noisy data [3, Zhang et al., 2005]:

$$\hat{\omega}^2 = \frac{1}{2n} \sum_{i=1}^{n} (Y_{t_i} - Y_{t_{i-1}})^2 \tag{4.3}$$

4.1.1 Properties

- Convergence speed: $m^{1/2}$ (m - number of observation)
- Unbiased: no
- Consistent: yes
- Jump Robust: yes
- Allows for time dependent noise: no

4.2 Autocovariance Noise Variance

The noise variance can be estimated as the negative of the first order autocovariance of observed returns [6, Oomen, 2005]:

$$\hat{\omega}^2 = \max\left\{0, -\frac{1}{n-1} \sum_{i=1}^{n} (Y_{t_i} - Y_{t_{i-1}})(Y_{t_{i-1}} - Y_{t_{i-2}})\right\} \tag{4.4}$$

4.2.1 Properties

- Convergence speed: $m^{1/2}$ (m - number of observation)

- Unbiased: no
- Consistent: yes
- Jump Robust: yes
- Allows for time dependent noise: no

5 Price Quarticity

5.1 Integrated Quarticity

The integrated quarticity is as described in [1, Pigorsch et al.]:

$$IQ = \int_{t-1}^{t} \sigma^4(s)ds. \tag{5.1}$$

5.2 Realized Quarticity

5.2.1 Assumptions

The microstructure noise.

1. The microstructure noise, $\epsilon_{t,i}$, has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.

5.2.2 Estimator

The realized fourth-power variation or realized quarticity, defined as [7, Corsi et al., 2005]:

$$RQ_t = \frac{M}{3} \sum_{j=1}^{M} r_{t,j}^4 \xrightarrow{p} IQ \tag{5.2}$$

where M is sampling frequency.

5.3 Realized Quadpower Quarticity

5.3.1 Assumptions

The microstructure noise.

1. The microstructure noise, $\epsilon_{t,i}$, has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.

5.3.2 Estimator

A more robust estimator then 5.2 on p. 17, especially in the presence of jumps, is the realized quad-power quarticity [7, Corsi et al., 2005]:

$$RQQ_t = M\frac{\pi^4}{4}\sum_{j=4}^{M} \mid r_{t,j} \mid\mid r_{t,j-1} \mid\mid r_{t,j-2} \mid\mid r_{t,j-3} \mid \xrightarrow{p} IQ \tag{5.3}$$

5.4 Modulated Realized Quarticity

5.4.1 Assumptions

The microstructure noise.

1. The microstructure noise, $\epsilon_{t,i}$, has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$ is $O(1)$.

5.4.2 Estimator

Modulated Realized Quarticity is written as [4, Podolskij and Vetter, 2009]:

$$MRQ(Y)_n = \frac{(c_1c_2/3)MBV(Y,4,0)_n - 2\nu_1\nu_2\hat{\omega}^2 MRV(Y)_n - \nu_2^2(\hat{\omega}^2)^2}{\nu_1^2} \xrightarrow{p} IQ \tag{5.4}$$

where MBV is written as 3.14 on p. 9,

$$\nu_1 = \frac{c_1(3c_2 - 4 + (2-c_2)^3 \bigvee 0)}{3(c_2-1)^2} \tag{5.5}$$

$$\nu_2 = \frac{2((c_2-1)\bigwedge 1)}{c_1(c_2-1)^2} \tag{5.6}$$

We can choose the constants c_1 and c_2 from specific process:

$$c_1 = 0.25, \quad c_2 = 2. \tag{5.7}$$

5.4.3 Properties

- Convergence speed: $m^{1/4}$ (m - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependence noise: no
- Allows for for endogenous noise: no

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