Comparison of Estimators for SeriesB Series

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Abstract

We use the BIC criterion to select the best long-memory ARMA(p, q) model with $p, q = 0, 1, 2, 3$ and long-memory model specifications FD, FGN and PLA. Also the best ARMA(p, q), $p, q = 0, 1, 2, 3$ with no long-memory component is determined using the BIC.

Keywords: BIC, long-memory models .

1. Introduction

Series B from Box, Jenkins, and Reinsel (2008) consists of the closing price of IBM common stock, daily, May 17 1961 to November 2 1962. There are $n = 369$ consecutive values. Denoting the close price on successive trading days by $z_t, t = 1, \ldots, n$, the returns are defined by $r_t = \log(z_t) - \log(z_{t-1}), t = 2, \ldots, n$. From the autocorrelation (ACF) for the returns, shown in Figure 1, we see there is no suggestion of signficant autocorrelation and we conclude the returns are white noise. However the absolute returns, $|r_t|$, exhibit a slowly decaying autocorrelation.

Exact maximum likelihood was used to fit the autoregressive moving-average model of order (p, q) by itself as well as convolved with three types of long-memory: fractionally differenced white noise, fractional Gaussian noise, and power-law-decay-autocorrelation. The BIC criterion was used to select the best model among the four types taking $p, q = 0, \ldots, 3$. In all there are 64 models. In the short-memory case the best model was with $p = q = 1$ while in the long-memory case, $p = q = 2$ was best. The best fitting models for each of the four types are compared in Table using their relative plausibility. For likelihoods, the relative plausibility is defined by $R = L/L^*$, where L is the likelihood and L^* is the largest likelihood out of the 64 models fitted. Similarly for the BIC, $R_{\text{BIC}} = \exp\{-0.5 * (\text{BIC} - \text{BIC}^*)\}$. So in terms of the BIC, we see that

The models are compared in Table in terms of their relative likelihoods,

```
R> require("FGN")
R> require("xtable")
R> layout(matrix(c(1,2), ncol=1, nrow=2))
R> r <- diff(log(SeriesB))
R> acf(r, main="Returns, Series B")
R> z <- abs(r)
R> acf(z, main="Absolute Returns")
```

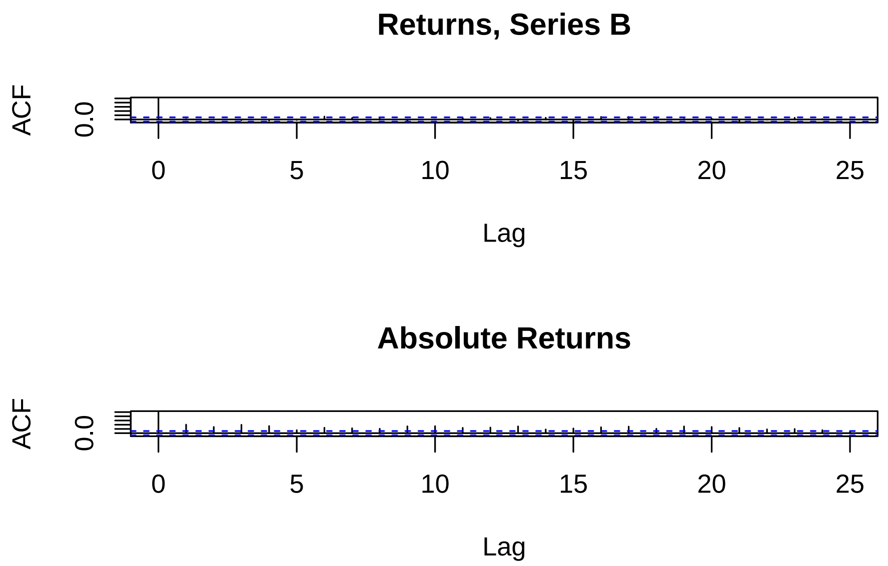

Figure 1: IBM Stock Prices Returns,

Exact MLE and Whittle MLE are implemented in McLeod and Veenstra (2013). For further results see Veenstra (2013).

2. Find the Best Model

```
R> z <- z - mean(z)
R> n <- length(z)
R> P <- Q <- 3
R> #
R> TotalTimes <- numeric(4)
R> names(TotalTimes) <- c("FD", "FGN", "PLA", "NONE")
R> #FD
R> numMod <- (P+1)*(Q+1)
R> outMod <- vector("list", numMod)
R> ii <- 0
R> #takes about 42 seconds on unit
R> startTime <- proc.time()
R> for (p in 0:P)
+     for (q in 0:Q) {
+     ii <- ii+1
+     k <- p+q+2
+     order <- c(p,0,q)
+     ans <- earfima(z, order=order, lmodel="FD")
+     Le <- ans$LL
```

```
+     bice <- -2*Le+k*log(n)
+     out <- c(p,q,Le,bice)
+     names(out) <- c("p","q","Le","bice")
+     outMod[[ii]] <- out
+     }
R> endTime <- proc.time()
R> totalTime <- endTime-startTime
R> TotalTimes[1] <- totalTime[1]
R> m<-matrix(unlist(outMod),byrow=TRUE,ncol=4)
R> dimnames(m)[[2]]<- c("p","q","Le","bice")
R> ind1 <- which.min(m[,"bice"])
R> mc<-rep(" ", 16)
R> mc[ind1]<-"*"
R> dimnames(m)[[1]]<-mc
R> mFD<-m
R> #
R> #FGN
R> numMod <- (P+1)*(Q+1)
R> outMod <- vector("list", numMod)
R> ii <- 0
R> #takes about 42 seconds on unit
R> startTime <- proc.time()
R> for (p in 0:P)
+     for (q in 0:Q) {
+     ii <- ii+1
+     k <- p+q+2
+     order <- c(p,0,q)
+     ans <- earfima(z, order=order, lmodel="FGN")
+     Le <- ans$LL
+     bice <- -2*Le+k*log(n)
+     out <- c(p,q,Le,bice)
+     names(out) <- c("p","q","Le","bice")
+     outMod[[ii]] <- out
+     }
R> endTime <- proc.time()
R> totalTime <- endTime-startTime
R> TotalTimes[2] <- totalTime[1]
R> m<-matrix(unlist(outMod),byrow=TRUE,ncol=4)
R> dimnames(m)[[2]]<- c("p","q","Le","bice")
R> ind1 <- which.min(m[,"bice"])
R> mc<-rep(" ", 16)
R> mc[ind1]<-"*"
R> dimnames(m)[[1]]<-mc
R> mFGN<-m
R> #
R> #PLA
R> numMod <- (P+1)*(Q+1)
```

```
R> outMod <- vector("list", numMod)
R> ii <- 0
R> #takes about 42 seconds on unit
R> startTime <- proc.time()
R> for (p in 0:P)
+     for (q in 0:Q) {
+     ii <- ii+1
+     k <- p+q+2
+     order <- c(p,0,q)
+     ans <- earfima(z, order=order, lmodel="PLA")
+     Le <- ans$LL
+     bice <- -2*Le+k*log(n)
+     out <- c(p,q,Le,bice)
+     names(out) <- c("p","q","Le","bice")
+     outMod[[ii]] <- out
+     }
R> endTime <- proc.time()
R> totalTime <- endTime-startTime
R> TotalTimes[3] <- totalTime[1]
R> m<-matrix(unlist(outMod),byrow=TRUE,ncol=4)
R> dimnames(m)[[2]]<- c("p","q","Le","bice")
R> ind1 <- which.min(m[,"bice"])
R> mc<-rep(" ", 16)
R> mc[ind1]<-"*"
R> dimnames(m)[[1]]<-mc
R> mPLA<-m
R> #
R> #
R> #NONE
R> numMod <- (P+1)*(Q+1)
R> outMod <- vector("list", numMod)
R> ii <- 0
R> #takes about 42 seconds on unit
R> startTime <- proc.time()
R> for (p in 0:P)
+     for (q in 0:Q) {
+     ii <- ii+1
+     k <- p+q+2
+     order <- c(p,0,q)
+     ans <- earfima(z, order=order, lmodel="NONE")
+     Le <- ans$LL
+     bice <- -2*Le+k*log(n)
+     out <- c(p,q,Le,bice)
+     names(out) <- c("p","q","Le","bice")
+     outMod[[ii]] <- out
+     }
R> endTime <- proc.time()
```

```
R> totalTime <- endTime-startTime
R> TotalTimes[4] <- totalTime[1]
R> m<-matrix(unlist(outMod),byrow=TRUE,ncol=4)
R> dimnames(m)[[2]]<- c("p","q","Le","bice")
R> ind1 <- which.min(m[,"bice"])
R> mc<-rep(" ", 16)
R> mc[ind1]<-"*"
R> dimnames(m)[[1]]<-mc
R> mNONE<-m
R> #
R> LLs <- c(mFD["*",3],mFGN["*",3],mPLA["*",3],mNONE["*",3])
R> LLmax <- max(LLs)
R> RLs <- exp(LLs-LLmax)
R> names(RLs) <- c("FD","FGN","PLA","NONE")
R> #
R> bics <- c(mFD["*",4],mFGN["*",4],mPLA["*",4],mNONE["*",4])
R> bicmin <- min(bics)
R> RELs <- exp(-0.5*(bics-bicmin))
R> names(RELs) <- c("FD","FGN","PLA","NONE")
R> tb <- matrix(c(RLs,RELs)*100, byrow=TRUE, nrow=2)
R> dimnames(tb) <- list(c("RL","REL"), names(RELs))
R> tbSeriesB <- tb
R> #dump("tbSeriesB", file="d:/R/CRAN/FGN/vig/tbSeriesB.R")
R> #
R> TotalTimes

  FD  FGN  PLA NONE
4.48 4.95 5.07 1.43

R> sum(TotalTimes)

[1] 15.93

R> #
R> mFD

  p q       Le      bice
  0 0 1646.106 -3280.396
  0 1 1646.943 -3276.162
  0 2 1649.078 -3274.524
  0 3 1651.269 -3272.997
  1 0 1646.574 -3275.423
  1 1 1647.678 -3271.724
  1 2 1649.616 -3269.691
  1 3 1651.638 -3267.827
  2 0 1651.212 -3278.792
```

```
  2 1 1651.565 -3273.590
* 2 2 1658.799 -3282.149
  2 3 1655.121 -3268.886
  3 0 1652.336 -3275.132
  3 1 1652.799 -3270.149
  3 2 1659.282 -3277.208
  3 3 1660.313 -3273.361
```

```
R> mFGN
```

```
  p q       Le      bice
  0 0 1644.637 -3277.458
  0 1 1647.900 -3278.075
  0 2 1648.713 -3273.794
  0 3 1651.637 -3273.734
  1 0 1646.244 -3274.765
  1 1 1648.309 -3272.987
  1 2 1649.335 -3269.129
  1 3 1651.848 -3268.248
  2 0 1651.581 -3279.530
  2 1 1651.654 -3273.768
* 2 2 1658.860 -3282.272
  2 3 1655.143 -3268.930
  3 0 1651.860 -3274.180
  3 1 1652.463 -3269.477
  3 2 1655.161 -3268.964
  3 3 1660.606 -3273.947
```

```
R> mPLA
```

```
  p q       Le      bice
  0 0 1645.946 -3280.076
  0 1 1646.996 -3276.268
  0 2 1648.996 -3274.361
  0 3 1651.308 -3273.076
  1 0 1646.506 -3275.289
  1 1 1647.753 -3271.874
  1 2 1649.571 -3269.602
  1 3 1651.653 -3267.857
  2 0 1651.330 -3279.027
  2 1 1651.618 -3273.695
* 2 2 1658.697 -3281.946
  2 3 1655.116 -3268.876
  3 0 1652.280 -3275.019
  3 1 1652.772 -3270.095
  3 2 1659.370 -3277.383
  3 3 1660.354 -3273.442
```

```
R> mNONE

  p q       Le      bice
  0 0 1594.776 -3177.736
  0 1 1620.334 -3222.944
  0 2 1622.806 -3221.979
  0 3 1633.459 -3237.378
  1 0 1627.736 -3237.747
* 1 1 1647.824 -3272.015
  1 2 1650.292 -3271.045
  1 3 1646.392 -3257.336
  2 0 1632.228 -3240.824
  2 1 1649.929 -3270.318
  2 2 1651.021 -3266.593
  2 3 1651.118 -3260.880
  3 0 1647.982 -3266.423
  3 1 1649.551 -3263.654
  3 2 1650.468 -3259.579
  3 3 1658.360 -3269.455

R> #
R> tbSeriesB

          FD FGN      PLA        NONE
RL  94.04804 100 84.96072 0.001610195
REL 94.04804 100 84.96072 0.592551897
```

3. Summary

	FD	FGN	PLA	NONE
RL	94.0	100.0	85.0	0.0
REL	94.0	100.0	85.0	0.6

Table 1: Relative Plausibility for Fitted Models

References

Box GEP, Jenkins GM, Reinsel GC (2008). *Time Series Analysis: Forecasting and Control*. Wiley, 4th edition.

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