Anti-Social Conductance? Estimating Social Distance In The Presence of Antagonistic Relationships

Andrew C. Thomas

June 27, 2009

In chapter ??, I put forth a method for considering social distances in relational data using electrical conductors as a model for friendships, then using circuit theory to estimate the strength of connection between two individuals as being proportional to the current produced when a fixed voltage difference is applied between points.

While this approach is able to consider ties of any positive value, the method cannot be immediately extended to antagonistic relationships while perfectly maintaining the metaphor. By considering the flow of current as a means of propagating a message from a source to a target, I model a "negative tie" as a connection where the message is transformed rather than reversed in direction. I then demonstrate how this method allows for effective distances between two individuals to vary above or below the underlying dyadic distance, and propose a family of stochastic processes that can determine the time progression of a social network.

1 Enemy Metaphors: Messages In Bottles, or Social Chromodynamics

The concept of social conductance uses two metaphors to describe social interaction: circuit pathways, which replace a social tie with an electrical conductance; and signal propagation, such that the flow of electric current represents the transmission of information, and that larger conductances represent stronger signals being transmitted.

Consider the notion that current represents the flow of information. If this is in the form of a message, the only variation would be in the strength of the signal rather than the content. If electric current is substituted with another type of flow, one with similar physical properties but otherwise transmutable, the current analogy can be augmented with a quantity describing signal content.

One interpretation is that current flow is represented by the transmission of a binary signal within a physical medium. Among the allegories that fit this model, we have:

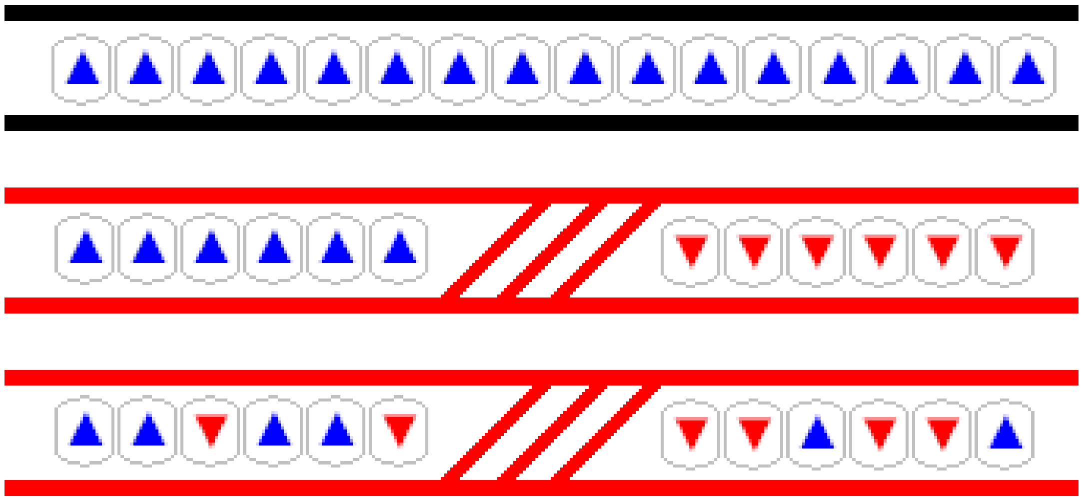

Figure 1: Signal transmission through "friend" and "enemy" connections. In the friend case (uppermost), signals are transmitted with perfect fidelity. Enemy connections reverse all signals, as shown in the bottom two connections.

- The outcome of a fair coin flip, heads or tails.
- A declaration of feeling: "She loves me, she loves me not."

A friendly tie is described as one that transmits each signal it receives with perfect clarity; in an "enemy" tie, the signal transmitted is the opposite of the one received; see Figure 1 for a demonstration. In aggregate, a node will receive a collection of signals from nodes with higher electric potential, and transmit to nodes with lower electric potential in a degree proportional to both the potential difference and the conductance between the source node and its respective targets; the signal transmitted will either be as received, to friendly connections, or flipped around to enemies. [1]

For a relational data system consisting of "friend" and "enemy" ties of varying magnitudes, the total current flow will be identical to one where the system contains identical ties magnitudes, but only friendly ties. The only modification will be in the nature of the signal received in the end.

[1]Clearly, this analogy is not perfectly suited to true social communication; over time, messages from enemies would be ignored once the relationship became clear to both parties. However, its properties are compelling enough that I am willing to proceed with it for the purposes of prediction.

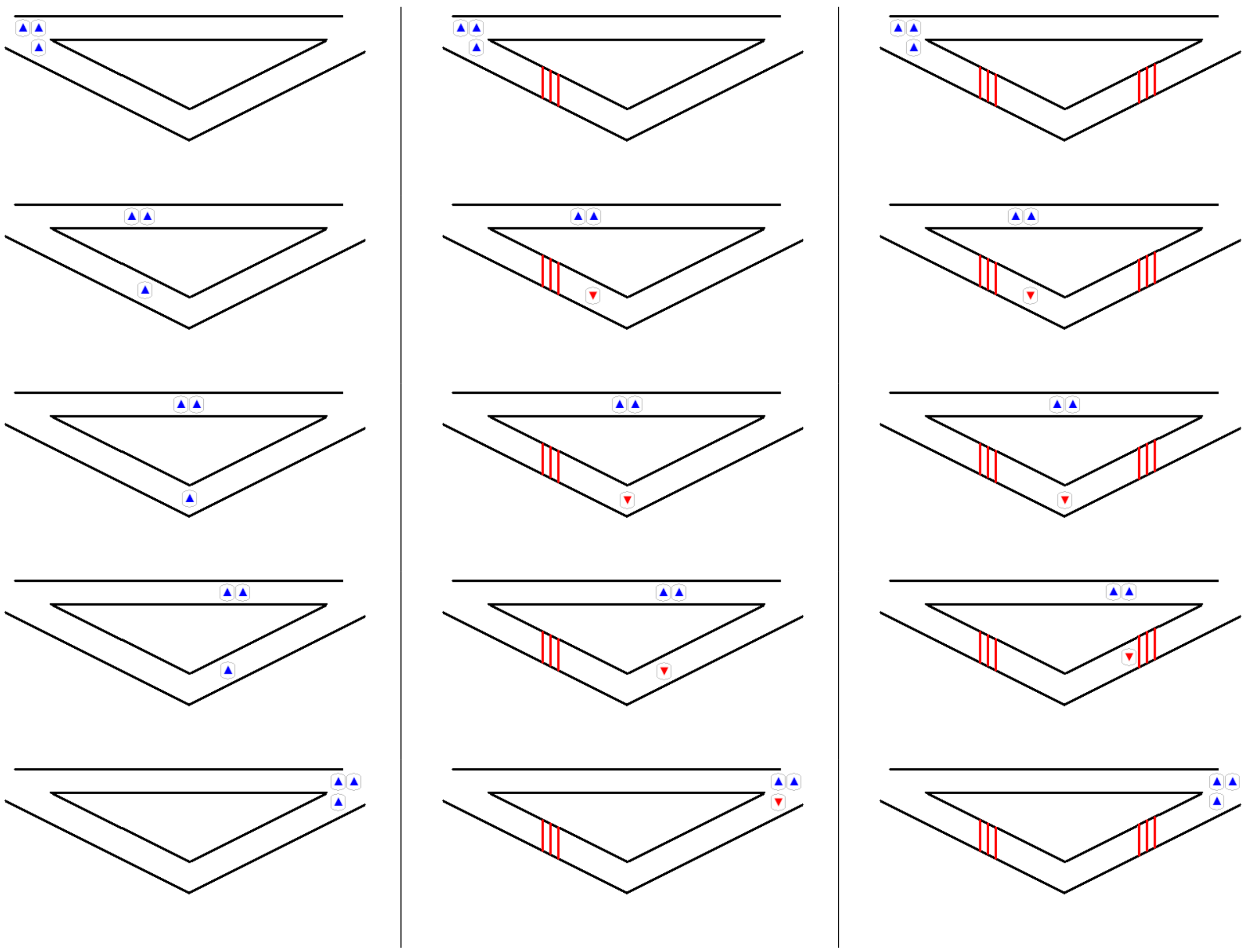

Figure 2: Social conductance with friend-of-friend, friend-of-enemy and enemy-of-enemy triads. Friend-of-friend connections maintain an increased signal strength and perfect fidelity, while one enemy connection on the indirect path causes the signal fidelity to decrease. With an enemy-of-enemy connection, the signal flip is undone, restoring an identical signal fidelty as in the friend of friend case.

1.1 Definitions: Social Conductance in Partly Antagonistic Networks

The method of social conductance (included in this R package) comes about by applying a fixed potential difference across two nodes and calculating the resulting current flow; for a unit value of potential difference, the equivalent conductance is equal in magnitude to the current produced. In this case, there is an additional element: what fraction of the current received by the target is opposite that which was sent initially.

Given nodes i and j, let (B_{ij}, R_{ij}) be the current received at node j of each type[2]. Define:

- Strength of Signal $I_{ij} = B_{ij} + R_{ij}$, or the total current flowing from node i
- Fidelity of Signal, $f_{ij} = \frac{B_{ij}-R_{ij}}{B_{ij}+R_{ij}}$. Equal current of each type results in a message consisting entirely of noise, since the signals offset each other completely.

From this definition, social distance measures can be constructed; one possibility is that the equivalent social conductance $\bar{C}_{ij}$ is defined as $I_{ij}f_{ij}$, and social distance as defined as the inverse of that measure.

2 Examples of Triads

As is shown in Figure friend-friend, there are three forms of triad that deserve immediate description: how an existing friendship is affected by a third party, and this party's respective relationships in forming a triad.

In the case where the third person is on good terms with both partners, the effective strength of the tie increases. If the underlying friendships all have social conductance G, it is as if each relationship is composed of a direct tie G plus an indirect tie of strength $G/2$, for a total strength $3G/2$.

Supposing that the third party has one enemy and one friend in the triad, the signal strength is maintained at $3G/2$, but the effective fidelity of the connection is $1/2$, reducing the effective social conductance to $3G/4$ and lowering the strength of the original friendship. It is also worth noting that the direct enemy connection is itself mitigated, decreasing in magnitude from an original conductance of $-G$ to a lesser $-3G/4$.

In the case where the original two parties share a common enemy, any negations along pathways are cancelled, leaving an equivalent social conductance of $3G/2$ as before.

[2]For lack of better symbology, let B and R represent "blue" and "red" respectively for each type of signal, as coded in the diagrams.

3 Algorithm for a Complete Social Network

Since the total current flow between two nodes is identical in the cases where ties are friendly or antagonistic in nature, the effective strength and fidelity of a tie can be determined through a two-step process. First, the effective tie strength is calculated as if all ties in the system are friendly. Then, fidelity is calculated using the following method:

- Determine the electric potential levels at each node, as calculated from the tie strength algorithm, and note their order from highest (the source) to lowest (the sink).
- Calculate the total current flowing out of the first node, in terms of both upright and inverted currents.
- For each outgoing current from the node, replace the outgoing current with a sum of upright and inverted current equal in magnitude to the original signal and in proportion to the balance in the total current.
- For each outgoing current where the tie is antagonistic, switch the proportions of upright and inverted currents.
- Repeat these steps for each node in order of descending electric potential.

Only one pass is necessary to solve for the current balance in each edge, and therefore for each source-sink pair in the system.

References

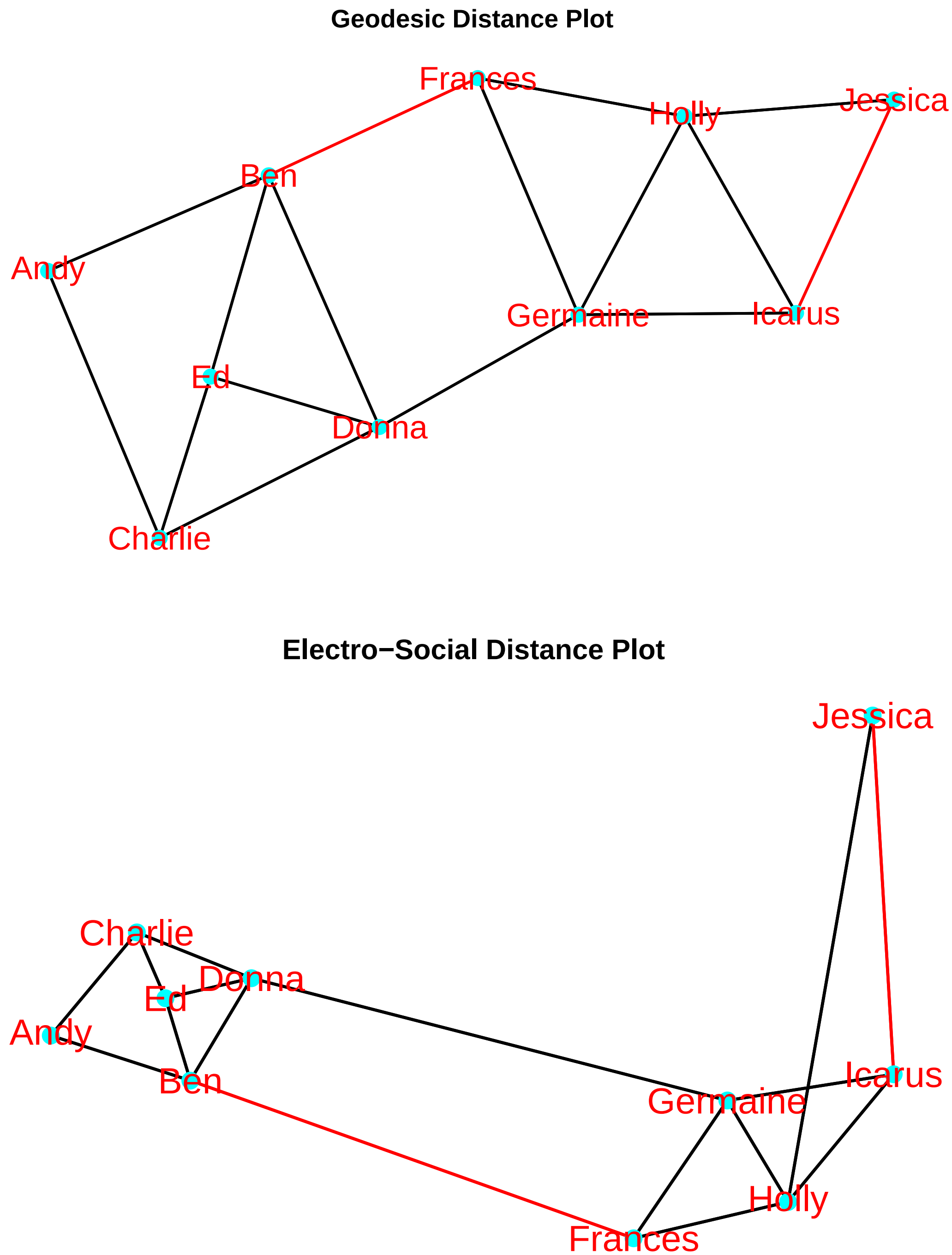

Figure 3: Two plots of a hypothetical ten-node social network with two antagonistic connections. Above, the distances between nodes represent geodesic path lengths; below, distances are calculated as the inverse of effective social conductance, divided by the absolute value of the signal fidelity.