Analysis of Stable Matchings in R: Package matchingMarkets

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Abstract

R package **matchingMarkets** implements structural estimators to correct for the sample selection bias from observed outcomes in matching markets. It also contains R code for matching algorithms such as the deferred-acceptance algorithm for college admissions, the top-trading-cycles algorithm for house allocation and a partitioning linear program for the roommates problem.

Keywords: market design, stable matching, endogeneity, selection models, Bayesian methods, econometrics, R.

1. Introduction

The `matchingMarkets` package contains R and C++ code for the estimation of structural models that correct for the sample selection bias of observed outcomes in matching markets. Matching is concerned with who transacts with whom, and how. For example, who works at which job, which students go to which school, who forms a workgroup with whom, and so on. The empirical analysis of matching markets is naturally subject to sample selection problems. If agents match assortatively on characteristics unobserved to the analyst but correlated with both the exogenous variable and the outcome of interest, regression estimates will generally be biased.

The `matchingMarkets` package contains

1. *Bayes estimators.* The estimators implemented in function `stabit` and `stabit2` correct for the selection bias from endogenous matching. The current package version provides solutions for two commonly observed matching processes: (i) the *group formation problem* with fixed group sizes, (ii) the *roommates problem* with transferable utility, and (iii) the college admissions problem. These processes determine which matches are observed – and which are not – and this is a sample selection problem.

2. *Post-estimation tools.* Function `mfx` computes marginal effects from coefficients in binary outcome and selection equations and `khb` implements the Karlson-Holm-Breen test for confounding due to sample selection.

3. *Design matrix generation.* The estimators are based on independent variables for all feasible, i.e., observed and counterfactual, matches in the market. Generating the char-

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acteristics of all feasible matches from individual-level data is a combinatorial problem. The `stabit` function has an argument `method="model.frame"` that returns a design matrix based on pre-specified transformations to generate counterfactual matches.

4. *Algorithms.* The package also contains several matching algorithms such as the deferred acceptance algorithm (`daa`) for stable marriages and college admissions, the top-trading-cycles algorithm (`ttc`) for house allocation and a partioning linear program (`plp`) for the stable roommates problem. These can be used to obtain stable matchings from simulated or real preference data.

5. *Data.* In addition to the `baac00` dataset from borrowing groups in Thailands largest agricultural lending program, the package provides functions to simulate one's own data from matching markets. `stabsim` generates individual-level data and the `stabit` function has an argument `simulation` which generates group-level data and determines which groups are observed in equilibrium based on an underlying linear stochastic model.

Frequently Asked Questions

- *Why can I not use the classic Heckman correction?*

  Estimators such as the Heckman (1979) correction (in package sampleSelection) or double selection models are inappropriate for this class of selection problems. To see this, note that a simple first stage discrete choice model assumes that an observed match reveals match partners' preferences over each other. In a matching market, however, agents can only choose from the set of partners who would be willing to form a match with them and we do not observe the players' relevant choice sets.

- *Do I need an instrumental variable to estimate the model?*

  Short answer: No. Long answer: The characteristics of other agents in the market serve as the source of exogenous variation necessary to identify the model. The identifying exclusion restriction is that characteristics of all agents in the market affect the matching, i.e., who matches with whom, but it is only the characteristics of the match partners that affect the outcome of a particular match once it is formed. No additional instruments are required for identification (Sørensen 2007a).

- *What are the main assumptions underlying the estimator?*

  The approach has certain limitations rooted in its restrictive economic assumptions.

  1. The matching models are *complete information* models. That is, agents are assumed to have a complete knowledge of the qualities of other market participants.
  2. The models are *static equilibrium* models. This implies that (i) the observed matching must be an equilibrium, i.e., no two agents would prefer to leave their current partners in order to form a new match (definition of pairwise stability), and (ii) the equilibrium must be unique for the likelihood function of the model to be well defined (Bresnahan and Reiss 1991).
  3. Uniqueness results can be obtained in two ways. First, as is common in the industrial organization literature, by imposing suitable *preference restrictions*. A suitable restriction on agents' preferences that guarantees a unique equilibrium is

alignment (Pycia 2012). In a group formation model, (pairwise) preference alignment states that any two agents who belong to the same groups must prefer the same group over the other. Second, by choosing a *market assigment* based on matching algorithms that produce a unique stable matching, such as the well-studied Gale and Shapley (1962) deferred acceptance algorithm.

4. Finally, the models assume *bivariate normality* of the errors in selection and outcome equation. If that assumption fails, the estimator is generally inconsistent and can provide misleading inference in small samples.

The remainder of the paper is structured as follows. Section 2 clearly motivates the importance of correcting for sorting bias that arises from endogenous matching in group formation. Section 3 outlines the multi-index sample selection problem, develops the structural model and discusses the identification strategy. Section 4 presents Monte-Carlo evidence of the robustness of the estimator in small samples. Section 5 provides replication code and data for an application of the method in microfinance group formation (see Klein 2015a).

2. Example of sorting bias: omitted variables

This section clearly motivates the importance of sorting bias in this setup. The basic idea starts with the result in Klein (2015a) that project correlation and risk type positively co-move across groups due to endogenous matching. If we just look at the effect of group correlatedness on repayment, we get an upward-biased coefficient, since high within-group correlatedness is also proxying for safer risk levels.

The sorting bias that arises by systematic matching into groups in one-sided matching market is best illustrated with an example. The focus here is on the bias that results when variables influencing peer selection are not observed in the data. Appendix A continues this example by illustrating the bias arising when variables influencing peer selection are measured with error.

Consider a market for group loans with four borrowers A, B, C and D, who have no pledgeable collateral, and one lender, who offers a pooling contract. Assume that the borrowers prefer to take loans in groups of two over the outside option of remaining unmatched. There are three feasible group constellations or matchings. One possibility is that borrower A matches with borrower B and borrower C matches with borrower D. Denote this matching $\mu_1 = \{AB, CD\}$. The other two possible matchings are $\mu_2 = \{AC, BD\}$ and $\mu_3 = \{AD, BC\}$. I will refer to $M = \{\mu_1, \mu_2, \mu_3\}$ as the set of feasible matchings.

Which of these matchings is observed depends on all four borrowers' preferences over feasible matches. Each of the six potential matches between any two borrowers i and j has an associated match valuation, V_{ij}. Using the equilibrium characterisation under non-transferable utility in Klein (2015a), the equilibrium condition for μ_1 can be written in the form of the inequality

$$max\{V_{AB}, V_{CD}\} \quad > \quad max\{V_{AC}, V_{BD}, V_{AD}, V_{BC}\}. \tag{1}$$

The inequality states that the equilibrium matching contains the match with the largest of the six match valuations. The intuition is that those two borrowers who form the match with the highest valuation have no incentive to deviate. In this simple example, the second equilibrium group is formed by the two residual borrowers. Put differently, the valuation of every non-equilibrium group must be smaller than the opportunity costs of its members to leave their equilibrium groups AB and CD and form a new group.

The equilibrium condition is based on the six match valuation equations. These equations are taken from the model in Ghatak (1999), with two modifications. First, I denote the probability of default as $d_i := 1 - p_i$ and assume that $d_i d_j$ is close to zero and therefore negligible. Second, I assume all borrowers are of the same exposure type but differ in exposure intensity γ. This allows for a clearer exposition of the match valuations as below:

$$V_{ij} = u_{i,j} + u_{j,i} = -q(p_i + p_j) + 2qp_ip_j + 2q\gamma_i\gamma_j \quad (2)$$

$$\overset{d_id_j=0}{=} -q(d_i + d_j) + 2q\epsilon_{ij} \quad (3)$$

$$= \alpha_1\epsilon_{ij} + \eta_{ij}. \quad (4)$$

Here, d_i and d_j give the risk type (probability of default) of borrower i and j. The parameter α_1 in Eqn 4 equals $2q$. When borrowers' risk types are unobservable, the term $-q(d_i + d_j)$ is captured in the match-specific error term η_{ij}.

For this example, let the characteristics of the four borrowers be as given in Table 1. Furthermore, let the interest payment be $r = 2$ and set the joint liability payment to $q = 1$. The six valuations, V_{ij}, are then given in Table 2.

Table 1: Borrower-level characteristics.
- p_i: success prob.
- d_i: failure prob.
- γ_i: risk exposure

	p_i	d_i	γ_i
A	0.8	0.2	0.3
B	0.7	0.3	0.4
C	0.7	0.3	0.2
D	0.6	0.4	0.3

Table 2: Group-level variable values of
- $(d_i + d_j)$: failure prob.
- ϵ_{ij}: risk exposure
- V_{ij}: group valuation
- $Y_{ij}^{\beta_1=0}$: group outcome

	$(d_i + d_j)$	$\epsilon_{ij} = \gamma_i\gamma_j$	V_{ij}	$Y_{ij}^{\beta_1=0}$
AB	0.5	0.12	-0.26	3.5
AC	0.5	0.06	-0.38	3.5
AD	0.6	0.09	-0.42	3.4
BC	0.6	0.08	-0.44	3.4
BD	0.7	0.12	-0.46	3.3
CD	0.7	0.06	-0.58	3.3

Which groups are observed in equilibrium is determined by the condition in Eqn 1. In this example, the values in Table 1 were chosen such that the observed component γ_i and the unobserved component p_i are uncorrelated, i.e. $cor(\gamma_i, p_i) = 0$. Table 2 illustrates how, for observed equilibrium groups, the independent variable ϵ_{ij} will be correlated with the error term η_{ij} when matching is on both these variables, i.e. when $\alpha_1 \neq 0$. Simple algebra confirms that for the set of feasible groups in Table 2, the correlation between ϵ_{ij} and η_{ij} is zero. For the equilibrium groups in $\mu = \{AB, CD\}$, however, we find $cov(\epsilon_{ij}, \eta_{ij}) = +0.06$.

Let the group outcome, Y^*_{ij}, be given by the expected repayment:

$$Y^*_{ij} = (r+q)(p_i+p_j) - 2qp_ip_j - 2q\epsilon_{ij} \quad (5)$$

$$\overset{d_id_j=0}{=} 2r - (r-q)(d_i+d_j) - 2q\epsilon_{ij} \quad (6)$$

$$= \beta_0 + \beta_1\epsilon_{ij} + \varepsilon_{ij}. \quad (7)$$

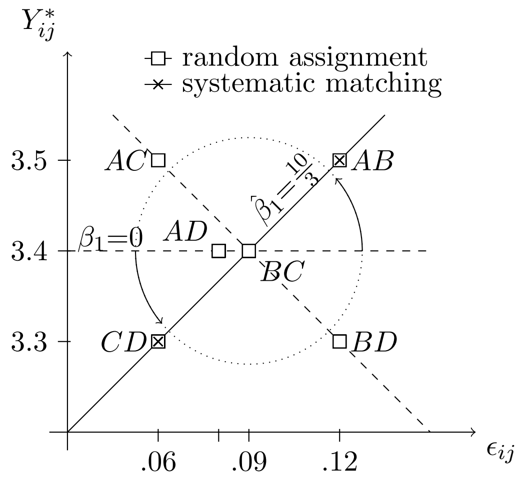
Note that $\frac{\partial V_{ij}}{\partial p_i} = -q+2qp_j > 0$ for $p_j > 0.5$ and $\frac{\partial Y^*_{ij}}{\partial p_i} = r+q-2qp_j > 0$ for $r > q$. That is, both match valuation and match outcome are increasing in risk type. In fact, the unobservable component in the outcome equation is $\varepsilon_{ij} = \delta\eta_{ij}$ with $\delta = (r-q)/q = +1$ and the outcome equation can be rewritten as

$$Y^*_{ij} = \beta_0 + \beta_1\epsilon_{ij} + \delta\eta_{ij}. \quad (8)$$

Now consider estimating the parameter β_1. Assume, for simplicity, that the true coefficient is $\beta_1 = 0$. That is, the group outcome Y^*_{ij} only depends on the unobservable risk type. A simple OLS based on the observed data points yields an upwards-biased coefficient of $\hat{\beta}_1 = +10/3$ (see Figure 1). It is clear that the source of the bias is the correlation between the independent variables and the error term. For the expected value of $\hat{\beta}_1$, we have $\mathbb{E}[\hat{\beta}_1] = \beta_1 + \frac{cov(\epsilon_{ij},\varepsilon_{ij})}{var(\epsilon_{ij})} = 0 + \frac{0.06}{0.018} = 10/3$.

Figure 1 also illustrates how the bias resolves when groups are assigned randomly. Then, the expected marginal effect $\hat{\beta}_1$ can be seen as the equally weighted average of the OLS estimates for the three equiprobable, feasible group constellations, i.e. $\frac{1}{3}(+\frac{10}{3} - \frac{10}{3} + 0) = 0$. A comparison of the coefficient estimate for the endogenously formed groups ($\hat{\beta}_1$=10/3) and the random assignment ($\hat{\beta}^*_1$=0) separates the bias from sorting.

Figure 1: Bias on coefficient pertaining to risk exposure term ϵ_{ij} from endogenous sorting when risk type is unobserved. Bias resolves (i) under random assignment, or (ii) when matching is independent of risk exposure, i.e. $\alpha_1 = 0$.



The matching model in the econometric analysis below tries to control for the bias by estimating both the matching and outcome equations simultaneously. The variation in borrower types across markets will serve the role of an instrumental variable and help to identify the coefficients in the outcome equation.

3. Multi-index sample selection

This section develops a structural empirical model to estimate the direct (or causal) repayment effect of project correlation, net of sorting bias. Technically, the equilibrium groups constitute a self-selected sample.[1] The selection problem at hand differs substantially from that in Heckman (1979), who proposes a two-stage correction. The latter entails the estimation of the selection and outcome equations simultaneously and explicitly models the dependence structure of the error terms.

3.1. Problem statement

In the four-borrower example from Section 2, the first-stage selection mechanism that determines which borrower groups are observed (and which are not) is a one-sided matching game and not a simple discrete choice as in the Heckman model. A discrete choice model assumes that an observed match reveals group partners' preferences over each other. However, the observed matching is the outcome of complex interactions and conflicts of interest between the players in the market.

To make this point clearer, consider the example in Section 2 where preferences are aligned and any borrower strictly prefers matching with partner A or B over C or D. Assume we observe the match of agents C and D in a market of four borrowers A, B, C and D. With a discrete choice model, we would infer that C's choice of partner D suggests that $u_{C,D} > u_{C,A}$. This restriction on the latent match valuations can then be used to derive the likelihood. However, such a conclusion has potential flaws in matching markets. In such markets, players B, C and D compete over a match with borrower A. If borrower A prefers to match with B instead of C, then we observe the match CD from the example although it may well hold that $u_{C,A} > u_{C,D}$. In particular, borrowers can only choose from the set of partners who would be willing to form a match with them. However, we do not observe the players' relevant choice sets. This makes direct inference based on a discrete choice model impossible, even if it accounts for social interactions such as the models in Brock and Durlauf (2007) and Ciliberto and Tamer (2009).

In response to this problem, Sørensen (2007b) generalises the single-index Heckman sample selection model to multi-index sample selection models that allow for selection based on game theoretical models by relaxing the index property. The index property requires that two matches, such as AB and CD, that have the same probability of being observed also have the same conditioning of unobserved characteristics. This requirement fails in matching markets. Here, for safe types A and B the unobserved characteristics are truncated from *below* since they would be unable to match with a safe type if their unobserved characteristics were low. Following the same logic, for risky types C and D the unobserved characteristics are truncated from *above*.

In matching markets, therefore, the index property is violated and a multi-index selection model is called for. This model is a system of two equations. The first equation determines when the outcome is observed, while the second equation determines the outcome.

[1] Wooldridge (2002, Chapter 17) provides a comprehensive textbook treatment of sample selection models.

3.2. Structural empirical model

The first part of the structural model is the selection equation. The selection process can be written as the following system of match equations

$$V_G = W_G\alpha + \eta_G. \tag{9}$$

There are $|\Omega|$ equations, where Ω is the set of feasible groups in the market. $V \in \mathbb{R}^{|\Omega|}$ is a vector of latents and $W \in \mathbb{R}^{|\Omega| \times k}$ a matrix of k characteristics for all feasible groups. $\alpha \in \mathbb{R}^k$ is a parameter vector and $\eta \in \mathbb{R}^{|\Omega|}$ a vector of random errors. Whether a group, and therefore its outcome Y_G, is observed in equilibrium is indicated by $D_G = 1\left[V_G \in \Gamma_\mu\right]$. This is an indicator function with $D_G = 1$ if Y_G is observed, and 0 otherwise. Y_G is observed iff a group is part of the equilibrium matching μ in the market. That is, its group valuation is in the set of valuations Γ_μ that satisfy the equilibrium condition.[2] This set of valuations is the link between the structural empirical model and the equilibrium characterisations derived in Klein (2015a) (for non-transferable utility) and Proposition B.1 (for transferable utility) in Appendix B. With $V \in \mathbb{R}^{|\Omega|}$, the vector of all valuations in the market, the equilibrium condition can be written as a collection of inequalities that give upper and lower bounds on the match valuations as follows

$$V \in \Gamma_\mu \Leftrightarrow \left[V_G < \overline{V_G}\ \forall G \notin \mu\right] \Leftrightarrow \left[V_G > \underline{V_G}\ \forall G \in \mu\right]. \tag{10}$$

Substitution of the match valuations in Eqn 9 into the equilibrium condition above, allows us to state the condition on the error terms

$$\mu \text{ is stable} \Leftrightarrow \eta \in \Gamma_\mu - W\alpha. \tag{11}$$

The likelihood of the matching model is then

$$L(\mu;\alpha) = \mathbb{P}\left(\eta \in \Gamma_\mu - W\alpha\right) = \int 1\left[\eta \in \Gamma_\mu - W\alpha\right] dF(\eta), \tag{12}$$

where $1[\cdot]$ is the indicator function and estimates for α could, in principle, be obtained by maximising this function. When several independent matching markets are observed, the likelihood is the product over these markets. To normalise parameter level, the constant term is excluded from W.

The second part of the model is the outcome equation. The binary outcome is given as $Y_G = 1[Y_G^* > 0]$, where the latent group outcome variable Y_G^* is

$$Y_G^* = X_G\beta + \varepsilon_G, \tag{13}$$

with $\varepsilon_G := \delta\eta_G + \zeta_G$, where ζ_G is a random error. This specification allows for a linear relationship between the error terms in the selection and outcome equation with covariance δ. The design matrices $X \in \mathbb{R}^{|\mu|}$ and $W \in \mathbb{R}^{|\Omega|}$ do not necessarily contain distinct explanatory variables.

3.3. Distribution of error terms

Figure 2 summarises the structural model. If there are unobservables, captured in the error term, that determine both match valuation (the decision who matches with whom in the market) and the outcome, then η and ε are correlated and we have an endogeneity problem.

[2] The classical Heckman (1979) model is a special case where $D_G = 1[V_G \geq 0]$ and the set of feasible valuations is simply $\Gamma = [0, +\infty)$.

Figure 2: The structural empirical model.
- $X_{G\notin\mu}$ characteristics of non-equilibrium groups
- $X_{G\in\mu}$ characteristics of equilibrium groups
- $D_{G\in\mu}$ equ. indicator
- $Y_{G\in\mu}$ equ. outcome
- η, ε correlated latents

$$
\begin{tikzcd}
 & & \eta_{G\in\mu} \arrow[dl] \arrow[rr, dash, dashed, bend left] & & \varepsilon_{G\in\mu} \arrow[dl] \\
X_{G\notin\mu}, \eta_{G\notin\mu} \arrow[r] & D_{G\in\mu} & & Y_{G\in\mu} & \\
 & & X_{G\in\mu} \arrow[ul] \arrow[ur] & &
\end{tikzcd}
$$

The joint distribution of ε_G and η_G is assumed bivariate normal with mean zero, and constant covariance δ.

$$\begin{pmatrix} \varepsilon_G \\ \eta_G \end{pmatrix} \sim N\left(0, \begin{bmatrix} \sigma_\xi^2 + \delta^2 & \delta \\ \delta & 1 \end{bmatrix}\right) \tag{14}$$

Here, the variance of the error term of the outcome equation σ_ε^2 is $var(\delta\eta + \xi) = \delta^2 + \sigma_\xi^2$. To normalise parameter scale, the variance of η and ζ is set to 1, which simplifies σ_ε^2 to $1 + \delta^2$ in the estimation. If the covariance δ were zero, the marginal distributions of ε_G and η_G would be independent and the selection problem would vanish. That is, the observed outcomes would be a random sample from the population of interest.

3.4. Identification

The structural model allows for correlation between ε and η, and imposes necessary equilibrium conditions on the valuations of both observed and unobserved groups. The interaction in the market makes estimation computationally involved but overcomes the identification problem.

Identification requires exogenous variation. In this model, it is the characteristics of the other agents in the market that provide the exogenous variation. To illustrate, recall the example in Section 2 with valuation Eqn 4 and outcome Eqn 8. The characteristics in the outcome equation of group AB are simply $X = (X_{AB})$. The characteristics in the selection equation are $W = (X_{AB}, X_{CD}, X_{AC}, X_{AD}, X_{BC}, X_{BD})$, and the independent elements of W are then $W' = (X_{CD}, X_{AC}, X_{AD}, X_{BC}, X_{BD})$. The identifying assumption is thus that the characteristics of agents outside the match (those comprised in W') are exogenous, i.e., uncorrelated with the error terms. Put differently, the exclusion restriction is that D (which groups are observed in equilibrium) depends the characteristics of all agents in the market, while the outcome of the equilibrium groups only depends on the characteristics of the members of those groups.[3]

[3]This identification assumption also holds in the presence of aggregate shocks because shocks which are common across potential groups in a village do not affect group formation. In the TU model, for example, the equilibrium condition is $V_L + V_M > \max\{V_{L'} + V_{M'}\}$, where the LHS is the match valuation of equilibrium groups and the RHS is the maximum of all counterfactual groups in the market. Observe that a common shock term, added on both sides, would simply cancel out. A similar argument can be made in the NTU model because aggregate shocks add a constant to all match valuations.

In particular, other agents' characteristics are not used as instruments in a traditional sense. Rather than entering the selection equation directly, they pose restrictions on the match valuations by determining the bounds in the estimation.

3.5. Estimation

In the estimation, I follow Sørensen (2007a), who uses Bayesian inference with a Gibbs sampling algorithm that performs Markov Chain Monte Carlo (MCMC) simulations from truncated normal distributions. The latent outcome and valuation variables, Y^* and V, are treated as nuisance parameters and sampled from truncated Normal distributions that enforce sufficient conditions for the draws to come from the equilibrium of the group formation game. For the posterior distributions, see Klein (2015a). For an illustration of the simulation of the posteriors, see Appendix C.

The conjugate prior distributions of parameters α, β and δ are Normal and denoted by $N(\bar{\alpha}, \Sigma_\alpha)$, $N(\bar{\beta}, \Sigma_\beta)$ and $N(\bar{\delta}, \sigma^2_\delta)$. In the estimation, the prior distributions of α and β have mean zero and variance-covariance matrix $\Sigma_\beta = (\frac{1}{|\mu|}X'X)^{-1}$ and $\Sigma_\alpha = (\frac{1}{|\Omega|}W'W)^{-1}$. This is the widely studied and used g-prior (Zellner 1986). For δ, the prior distribution has mean zero and variance 10. For this parameter, the prior variance is at least 40 times larger than the posterior variance in all estimated models. This confirms that the prior is fairly uninformative.

Under the assumption of transferable utility, estimation is computationally complex due to the valuation of the equilibrium bounds. Here, estimation does not simply involve a maximisation of a given set of valuations as in Klein (2015a). Instead, the bounds in Eqns 23 and 24 derived in Appendix B require a maximisation over the set of all feasible *matchings* in the market. This involves solving a partitioning linear program.

In the context of this paper, with two groups per market, obtaining the bounds is relatively straightforward. The Bank for Agriculture and Agricultural Cooperatives (BAAC) data comprise 30 two-group villages and nine one-group villages, each with a maximum of 5 borrowers per group. For each two-group village we thus have up to $\binom{10}{5} = 252$ feasible groups and roughly $252 \times 30 = 7,560$ bounds per iteration to calculate. On a laptop with a quad core 2.90 GHz processor running the 64 bit version of R 3.1.1 under Ubuntu Linux, the estimator produces 800,000 iterations in under 20 minutes.[4]

[4]There are clear limitations to the feasibility of the TU estimation using larger data sets, however. For example, let the Bayesian estimator presented in this paper take more than a million iterations to converge on a given data set. Nauss (2003) presents an optimising branch-and-bound algorithm for the related generalised assignment problem (GAP). Dependent on the problem, this algorithm can find an optimal assignment in less than one second. A linear programming algorithm can, in principle, be implemented based on a partitioning linear program (see Quint 1991) to evaluate the upper bounds in Eqn 23 and lower bounds in Eqn 24 that satisfy the equilibrium conditions Assuming the linear programming algorithm can evaluate a bound in just 0.7 seconds, it would take $1,000,000 \times 7,560 \times 0.7$ seconds ≈ 167 years to estimate the model. Even if one assumes a weaker stability concept – *individual stability* – where only individuals (not coalitions) can block a matching, we would still have 27 feasible groups per village (the original 2 groups plus $5 \times 5 = 25$ one-for-one swaps) and thus $27 \times 30 = 810$ bounds to evaluate per iteration. If one further assumes that the number of necessary iterations decreases linearly with the number of bounds that need to be evaluated (taking the TU model estimated in this paper as a reference point), we would still require approximately $100,000 \times 810 \times 0.7$ seconds ≈ 1.8 years.

4. Monte Carlo experiments

This section presents the R[5] software package **matchingMarkets** (Klein 2015b) that implements the estimator developed in Section 3 and presents results of Monte Carlo experiments.

4.1. A simple example

I first provide a brief overview of the basic functionality of the **matchingMarkets** package and introduce the model specification used in the Monte Carlo experiments. The required R packages are installed from the CRAN archive. In a first step, `stabsim` simulates individual-level, independent variables. The code below generates data for 1,000 markets with two groups of size five each.

```
R> ## Simulate individual-level, independent variables
R> library(matchingMarkets)
R> idata <- stabsim(m=1000, ind=5, seed=123, gpm=2)
R> head(idata)
  m.id g.id        pi wst  R
1    1    1 0.6437888   0 NA
2    1    1 0.8941526   0 NA
3    1    1 0.7044885   1 NA
4    1    1 0.9415087   0 NA
5    1    1 0.9702336   0 NA
6    1    2 0.5227782   0 NA
```

The resulting data frame contains market and (tentative) group identifiers, two independent variables *pi* and *wst* and a vector of NAs for the dependent variable. Next we apply the function `stabit` that serves three purposes. First, it specifies the list of variables to be included in selection and outcome equations. Second, it generates group-level variables based on group members' individual characteristics. For example, the operation `add="pi"` generates the group average for variable `pi`. The operation `ieq="wst"` produces the probability that two randomly drawn group members have the same value of `wst`. Third, it draws group-level unobservables that enter selection and outcome equation.

```
R> ## Simulate group-level variables (takes a minute to complete...)
R> mdata <- stabit(x=idata, selection=list(ieq="wst"),
+ outcome=list(ieq="wst"), simulation="NTU", method="model.frame")
```

```
R> mdata <- mdata$model.frame
R> head(mdata$OUT, 4)
  m.id g.id intercept wst.ieq          R         xi    epsilon
1    1    1         1     0.4 -0.7822129 -1.6679419 -0.3822129
2    1    2         1     0.4  0.1709600  0.2145388  0.5709600
3    2    1         1     0.4 -0.3740809 -1.3165104  0.0259191
4    2    2         1     0.4  0.1172254  1.4414618  0.5172254
```

[5]The R project for statistical computing (R Core Team 2014) at http://www.r-project.org/.

```
R> head(mdata$SEL, 4)
  m.id g.id wst.ieq D         V        eta
1    1    1     0.4 1 2.9714581  2.5714581
2    1    2     0.4 1 1.1128423  0.7128423
3    1    3     0.6 0 0.2560828 -0.3439172
4    1    4     0.4 0 1.9985088  1.5985088
```

It is seen that the data frame for the selection equation `mdata$SEL` stacks the two equilibrium groups from the outcome equation `mdata$OUT` at the top (with `D=1`) and all counterfactual groups (with `D=0`) below, for each market. In the `stabit` function, the argument `simulation="NTU"` indicates that the function draws random errors `eta` and `xi` to determine the dependent variables V and R in selection and outcome equation, respectively. Here, the equilibrium selection is based on the conditions for the group formation game with non-transferable utility in Klein (2015a). The selection equation determines which groups are observed $D = 1$ and which are not $D = 0$.

$$D = 1[V \text{ satisfies equilibrium condition}] \tag{15}$$

$$V = \alpha \cdot \text{wst.ieq} + \eta, \text{ with } \eta \sim N(0,1) \tag{16}$$

The outcome equation determines the group outcome R.

$$R = \beta \cdot \text{wst.ieq} + \varepsilon, \text{ with } \varepsilon = \delta\eta + \xi \text{ and } \xi \sim N(0,1) \tag{17}$$

In the `matchingMarkets` package, the true parameters are hardcoded as $\alpha = 1$; $\beta = -1$; $\delta = 0.5$. Now, estimating the outcome equation of this model with OLS yields upward biased estimates of the slope coefficient β.

```
R> ## Naive OLS estimation
R> lm(R ~ wst.ieq, data=mdata$OUT)$coefficients
(Intercept)     wst.ieq
  0.3059781  -0.2766853
```

The source of this bias is the positive correlation between ε and the exogenous variable wst.ieq.

```
R> ## epsilon is correlated with independent variables
R> with(mdata$OUT, cor(epsilon, wst.ieq))
[1] 0.1030035
```

The intuition behind this bias is given in the example in Section 2. From Eqn 17, we know that $\varepsilon = \delta\eta + \xi$. Thus, conditional on η, the unobservables in the outcome equation are independent of the exogenous variables (because ξ does not enter the selection equation).

```
R> ## xi is uncorrelated with independent variables
R> with(mdata$OUT, cor(xi, wst.ieq))
[1] 0.009238994
```

The selection problem is resolved when the residual from the selection equation, $\hat{\eta}$, is controlled for in the outcome equation.

```
R> ## 1st stage: obtain fitted value for eta
R> lm.sel <- lm(V ~ -1 + wst.ieq, data=mdata$SEL); lm.sel$coefficients
 wst.ieq
1.004587
R> eta <- lm.sel$resid[mdata$SEL$D==1]
R> ## 2nd stage: control for eta
R> lm(R ~ wst.ieq + eta, data=mdata$OUT)$coefficients
(Intercept)     wst.ieq         eta
-0.03005002 -0.95630149  0.50448486
```

In most real-world applications, however, the match valuations in `V` are not observed. The solution is to estimate the selection equation by imposing equilibrium bounds (as derived in Proposition B.1) on the latent match valuations and this is the procedure I follow in the Monte Carlo experiments below. .

4.2. Simulation results

The Monte Carlo experiments are designed to test for the validity of the estimator. I continue to use the variable `wst.ieq` from the original model. The true parameters are defined as seen in the first row of Table 3. The table is composed of three blocks, each representing a different market setting and sampling strategy. The first block gives the results of a benchmark experiment that aims to see whether the structural model can reduce the bias of standard OLS estimates. Experiment 1 tests the robustness of the estimator when applied to a random sample of the groups' members. Experiment 2 works with the full population of group members but uses random samples from the counterfactual groups to reduce the computational burden arising from the combinatorics of large groups. I discuss motivation, set-up, implementation and results of each experiment in turn.

Benchmark study

The following steps replicate the results of the benchmark experiment in Table 3. The R code for replication is available in the documentation of function `mce`.

Implementation:

1. Following the nature of the data in the BAAC 2000 survey, I simulate individual-level, independent variables for 40 two-group markets with groups of size five.

```
R> idata <- stabsim(m=40, ind=5, seed=123, gpm=2)
```

2. Repeat the following steps for `i=1` to `1000`.

   a) Draw group-level unobservables ξ and η that determine both (i) which groups are observed in equilibrium and (ii) the equilibrium group outcomes.

   ```
   R> mdata <- stabit(x=idata, selection=list(ieq="wst"), outcome=
   + list(ieq="wst"), simulation="NTU", method="model.frame", seed=i)
   ```

b) Obtain parameter estimates using (i) OLS and (ii) the structural model.

```
R> ols <- stabit(x=mdata, method="outcome", niter=400000)
R> fit <- stabit(x=mdata, method="NTU", niter=400000)
```

Interpretation: The results for the benchmark study in Table 3 confirm the upward bias in the OLS estimates of the slope coefficient β. It is seen that the structural model successfully reduces the bias resulting from endogenous matching into groups. Note that the modes of the simulated posterior distributions in the first row of Figure 3 correspond to the true values in the first row of Table 3.

The benchmark study works with the full population of borrowers. The two experiments below investigate the robustness of the estimator for the practically more relevant case of working with random samples from the population of interest.

Table 3: Monte Carlo results for 40 two-group markets (based on 512 draws)

	Parameters	α	(Intercept)	β	δ	σ^2_ξ
	True values	1	0	-1	0.5	1
Benchmark: All group members (5/5); all counterfactual groups (250/250)						
5/5; 250/250	OLS	–	0.304	-0.282	–	1.501
	Structural	0.958	-0.112	-0.802	0.469	1.072
Experiment 1: 5 randomly sampled group members; all counterfactual groups						
5/6; 250/250	OLS	–	0.377	-0.242	–	1.676
	Structural	0.748	-0.108	-0.642	0.570	0.973
Experiment 2: All group members; 250 randomly sampled counterfactual groups						
6/6; 250/922	OLS	–	0.365	-0.121	–	1.695
	Structural	1.052	0.048	-0.707	0.567	0.978

Experiment 1: randomly sampled group members

While group sizes at Grameen Bank, for example, have evolved to five members, self-help group and village lending schemes operate with up to 30 members. Surveys, such as the BAAC 2000 survey (Townsend 2000) studied here, are therefore often restricted to a random sample of the groups' members.

Set-up: I continue to work with a sample of five borrowers per group but take original group sizes to be six borrowers. This means that one group member is dropped at random.

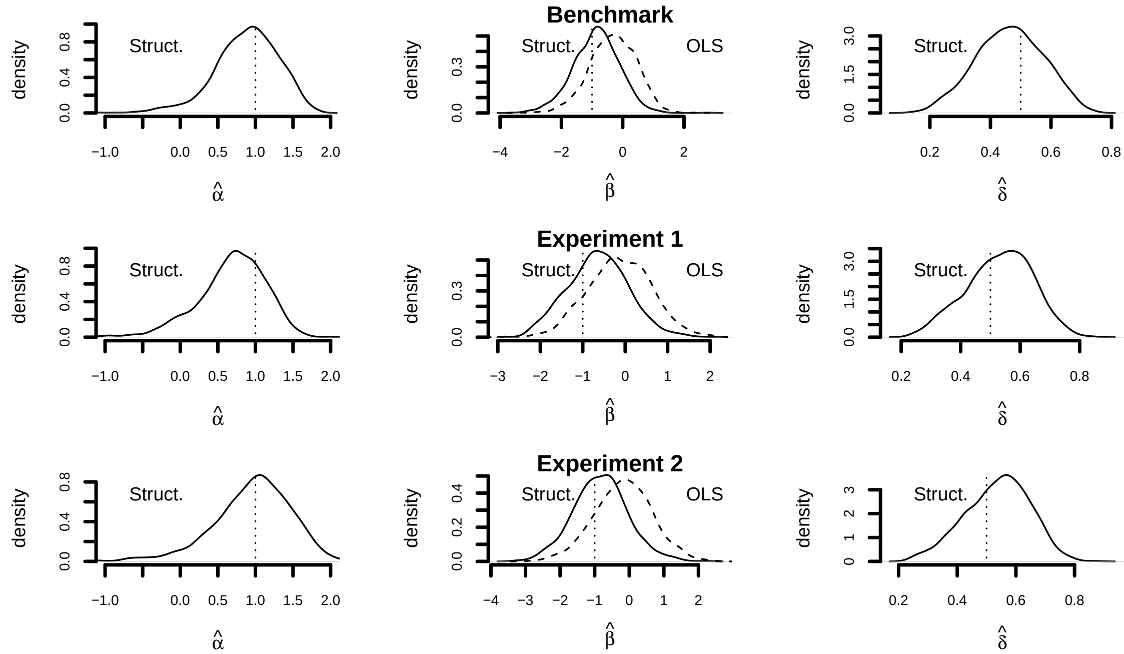
Implementation:

1. Simulate group-level, independent variables for all $\binom{2n}{n}$ feasible groups of size $n = 6$ in two-group markets.

2. Repeat the following steps 1,000 times.

   a) Draw group-level unobservables ξ and η that determine both (i) which groups are observed in equilibrium and (ii) the equilibrium group outcomes.

Figure 3: Posterior distributions of parameters for benchmark simulations. True value are given by vertical, dotted lines. Structural model and OLS estimates for 1,000 draws are given by straight and dashed lines, respectively.



b) Randomly drop one member per equilibrium group.

c) Generate new group-level, independent variables from the reduced sample of group members (leaving the equilibrium group indicator, D, and group outcomes, R, unchanged).

d) Obtain parameter estimates using (i) OLS and (ii) the structural model.

Interpretation: The results for Experiment 1 in Table 3 display clear evidence of attenuation bias (see Wooldridge 2002, Chapter 4.4.2) in both the OLS and structural estimates. The random sampling of group members induces measurement error in the group-level, independent variables that biases the slope estimates towards zero.

Experiment 2: randomly sampled counterfactual groups

While data on the full population of group members solves the attenuation problem encountered in Experiment 1, it creates another problem for statistical analysis. The BAAC 1997 survey (Townsend 1997), for example, comprises data from two-group markets with up to 20 members resulting in $\binom{40}{20} \approx 137.85$ billion feasible groups per market which renders the analysis computationally intractable.

Set-up: As in Experiment 1, the original group size is taken to be six members. In two-group markets, this results in $\binom{12}{6} - 2 = 922$ counterfactual groups, from which 250 groups are sampled at random for the analysis.

Implementation:

1. Simulate group-level, independent variables for all $\binom{2n}{n}$ feasible groups of size $n = 6$ in two-group markets.
2. Repeat the following steps 1,000 times.
   a) Draw group-level unobservables ξ and η that determine both (i) which groups are observed in equilibrium and (ii) the equilibrium group outcomes.
   b) Randomly draw 250 groups from the set of counterfactual groups.
   c) Obtain paramter estimates using (i) OLS and (ii) the structural model.

Interpretation: The results for Experiment 2 in Table 3 suggest that working with a random sample of counterfactual groups does not affect the mode of the posterior distribution of the coefficients. However, the standard deviation of the posterior distribution of β increases from $\hat{\sigma}_{\hat{\beta}} = 0.75$ to 0.79 (not reported in Table 3). A possible explanation is that the random sampling relaxes the equilibrium bounds which results in increased uncertainty in the parameter estimates.

5. Application in microfinance

This section contains R code to replicate the results of the structural model in Table III of Klein (2015a). To begin with, load the individual-level data contained in the `matchingMarkets` package (Klein 2015b) and standardise the variables. The 292 borrowers, are nested within 68 groups and 39 markets.

```
R> ## 1. Load individual-level data
R> library("matchingMarkets")
R> data(baac00)
R> baac00$pi            <- baac00$pi + (1-baac00$pi)*0.5
R> baac00$loan_size     <- baac00$loan_size/sd(baac00$loan_size)
R> baac00$loan_size2    <- baac00$loan_size^2
R> baac00$lngroup_agei <- baac00$lngroup_agei/sd(baac00$lngroup_agei)
```

In the next step, specify variables and variable transformations for selection and outcome equation. The function `stabit` generates the group-level design matrix and runs the Gibbs sampler with 800,000 iterations to obtain the results of the structural model.

```
R> ## 2-a. Run Gibbs sampler
R> klein15a <- stabit(x=baac00, method="NTU",
+     selection = list(inv="pi",ieq="wst"),
+     outcome   = list(add="pi",inv="pi",ieq="wst",
+                      add=c("loan_size","loan_size2","lngroup_agei")),
+     offsetOut=1, binary=TRUE, gPrior=TRUE, marketFE=TRUE, niter=800000
+   )
```

Alternatively, the results can be loaded directly from the package.

```
R> ## 2-b. Load data and get marginal effects
R> data(klein15a)
R> mfx(m=klein15a)$mfx.selection[1:2,]
            mx  s.e. t.stat p.val stars
pi.inv  -0.778 0.992 -0.785 0.216
wst.ieq  0.356 0.119  2.984 0.001    **
R> mfx(m=klein15a)$mfx.outcome[c(1:5,20),]
                      mx  s.e. t.stat p.val stars
pi.inv             1.571 1.813  0.867 0.195
wst.ieq           -0.586 0.243 -2.413 0.010    **
loan_size.add      0.970 0.362  2.680 0.005    **
loan_size2.add    -0.187 0.080 -2.338 0.012     *
lngroup_agei.add -0.395 0.109 -3.609 0.000   ***
delta              0.512 0.127  4.028 0.000   ***
```

The results on the attenuation bias in Experiment 1, Section 4, suggests that the difference in the parameter estimates of the Probit and the structural model underestimate the true confounding effect of endogenous matching. Specifically, the selection problem arising from endogenous group formation – while already strongly significant – is still likely to be underrated because attenuation results in an *underestimation* of the positive Probit coefficient and, at the same time, an *overestimation* of the negative coefficient from the structural model.

The function `khb` implements the statistical test for confounding in Probit models proposed in Karlson, Holm, and Breen (2012). The function takes as arguments the data frame of independent variables `X=X`, the dependent variable `y=R`, and the name of the confounding variable `z="eta"`.

```
R> ## 3. Test for confounding from sample selection
R> ## model matrix
R> M   <- klein15a$model.list
R> X   <- do.call(rbind.data.frame, M$X)
R> eta <- c(klein15a$coefs$eta, rep(0, length(M$X)-length(M$W)))
R> X   <- cbind(X,eta)
R> ## outcome
R> y <- unlist(M$R)
R> ## KHB test
R> head(khb(X=X, y=y, z="eta"),5)

Karlson-Holm-Breen method
Null hypothesis: Change in coefficient is not attributable to confounding by z.
                 p.value
pi.inv            0.5581
wst.ieq           0.0480
loan_size.add     0.9034
loan_size2.add    0.0442
lngroup_agei.add  0.0412
```

The procedure tests for differences in probit and structural model coefficients. It confirms that the probit estimates for variables `wst.ieq`, `loan_size2.add` and `lngroup_agei` are significantly confounded as a result of endogenous matching.

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A. Example of sorting bias: measurement error

The following example illustrates the bias arising when variables influencing peer selection are measured with error. The analysis below is based on the Ghatak (1999) model of group formation with match valuations given by

$$V_{ij} \;=\; \alpha_1(d_i + d_j) + \eta_{ij}. \tag{18}$$

Here, d_i and d_j give the risk type (probability of default) of borrower i and j and α_1 is a coefficient. The unobserved match valuation is captured by the match-specific error term η_{ij}. Now let A and B be safe types, denoted $d_A = d_B = 5\%$, and C and D risky types ($d_C = d_D = 15\%$). Further, let the interest payment be $r = 120$ and the joint liability payment $q = 20$. The six valuations, V_{ij}, are then given in Table 4.

Table 4: Match valuations* of all feasible groups

	A	B	C	D
A (d_A=5%)		$-2+\eta_{AB}$	$-4+\eta_{AC}$	$-4+\eta_{AD}$
B (d_B=5%)			$-4+\eta_{BC}$	$-4+\eta_{BD}$
C (d_C=15%)				$-6+\eta_{CD}$
D (d_D=15%)				

*parameters: $d_i = 1 - p_i$; $r = 120$; $q = 20$; $\alpha_1 = -q = -20$

The outcome Y^*_{ij}, is a latent variable that gives the expected loan repayment for a group comprising borrower i and j as

$$Y^*_{ij} \;=\; 2r + (q - r)(d_i + d_j) + \varepsilon_{ij} \tag{19}$$

$$\;=\; \beta_0 + \beta_1(d_i + d_j) + \varepsilon_{ij}, \tag{20}$$

where Y^*_{ij} determines the binary variable Y_{ij} that indicates successful repayment of group ij by the following threshold rule $Y_{ij} = 1[Y^*_{ij} > 0]$. Which outcomes are observed is determined by the equilibrium matching. Now consider estimating the parameter β_1. The selection problem arises when the equilibrium is not independent of the outcome, i.e., when the distribution of ε is not independent of the distribution of η. To investigate the nature of the selection bias that arises in this example, note the true value of $\beta_1 = (q - r) = -100$, and $\beta_0 = 2r = 240$. The observed match outcomes are given in Table 5 according to Eqn 20.

Table 5: Match outcomes with systematic matching (partially unobservable characteristics).*

	A	B ($\bar{d}$=10%)	C	D ($\bar{d}$=10%)
A (d_A=5%)		230		
B				
C (d_C=15%)				210
D				

*true parameters: $\beta_0 = 240$; $\beta_1 = -100$; $r = 120$; $q = 20$

Assume first that the researcher observes the characteristics of one borrower for every matched group with an error.[6] Thus part of the borrowers outcome-relevant quality is unobserved and

[6] For example, interviews were conducted with one randomly selected group member who was also interviewed on some of the characteristics of her fellow group member. Such a sampling strategy is used by Carpenter and Sadoulet (2000), Lensink and Mehrteab (2003), and Ahlin and Townsend (2007). Assume, for example, that the characteristics of the second group member are reported as the sample average, $\bar{d} = 10\%$.

therefore captured by the error term. Let the reported characteristics of borrowers B and D be the sample mean of $\bar{d} = 10\%$ and the characteristics of borrowers A and C be $d_A = 5\%$ and $d_C = 15\%$ as above.

The outcome for group AB is 230, and group CD has an outcome of 210, and a natural estimate of β_1 is -200 ($= [230 - 210]/[0.15 - 0.25]$). However, given the nature of the matching in this example, the estimate is severely downward biased. To see this, recall the omitted characteristics of borrowers B and D that lead to measurement error in our explanatory variable. The true model is $Y^*_{ij} = \beta_0 + (d_i + d_j)\beta_1 + \varepsilon_{ij}$. However, we estimate $Y^*_{ij} = \beta_0 + (d_i + \bar{d})\beta_1 + \varepsilon'_{ij}$ with $\varepsilon'_{ij} = (d_j - \bar{d})\beta_1 + \varepsilon_{ij}$. Now, if d_j is correlated with d_i, then d_i is correlated with ε'_{ij} and the estimate of β_1 is biased. Specifically, because of assortative matching in the market we have $cov(\varepsilon'_{ij}, d_i) < 0$ and the estimate is downward biased.[7]

If we observed the omitted borrower characteristics $d_B = 5\%$ and $d_D = 15\%$ the measurement error resolves. The natural and unbiased estimate of β_1 is -100 ($= [230 - 210]/[0.1 - 0.3]$). The bias is a consequence of the systematic selection of the observed sample of outcomes. Table 6 shows that we can obtain unbiased estimates – even if the quality of B and D is unobserved – when we observe the outcomes at random.

Table 6: Match outcomes with random assignment (partially unobservable characteristics)*

	A	B ($\bar{d}$=10%)	C	D ($\bar{d}$=10%)
A (d_A=5%)		230	220	220
B			220	220
C (d_C=15%)				210
D				

*true parameters: $\beta_0 = 240$; $\beta_1 = -100$

The above is essentially the outcome of the experiment outlined in the introductory section. A comparison of the coefficient estimate for the endogenously formed groups ($\hat{\beta}_1$=-200) and the random assignment ($\hat{\beta}^*_1$=-100) separates the direct effect of risk type from selection bias (Figure 4).

Figure 4 illustrates the decomposition of ex-ante (sorting) and ex-post effects on lending outcomes. The latent outcome variable, Y^*_G, gives group G's outcome. Following Ghatak (1999), the risk type of group G is given by the sum of its observed borrower risk types $X_G = \sum_{i \in G} d_i$. The dashed line gives the estimated relationship between group risk and lending outcome for a random assignment of borrowers into groups. This estimate coincides with the true underlying relationship, and an increase in group risk by 10% lowers the outcome by 10 units. The solid line gives the estimated relationship for the observed equilibrium $\mu_1 = \{AB, CD\}$. Here, the equilibrium matching is the result of an assortative matching of borrowers based on their risk type. This systematic matching leads to a downward bias in the linear probability model. If there was random assignment of borrowers to groups, the two lines would overlap completely.

To see how this experimental result can be obtained from non-experimental data, I adapt an example from Sørensen (2007a). Consider observing a second market with similar borrowers but with two additional borrowers A' and D' of risk-types $d_{A'} = 0\%$ and $d_{D'} = 20\%$ (Table

[7] Assortative matching on risk-type implies that $cov[d_j, d_i] > 0$ in the last argument of the straightforward algebra: $cov(\varepsilon'_{ij},\, d_i) = cov[(d_j - \bar{d})\beta_1 + \varepsilon_{ij},\, d_i] = cov[d_j\beta_1 - \bar{d}\beta_1 + \varepsilon_{ij},\, d_i] = \beta_1 \cdot cov[d_j,\, d_i] = -100 \cdot cov[d_j,\, d_i] <$ 0. If we observed the group constellations at random, i.e., a random sample with $cov[d_j, d_i] = 0$, the bias resolves.

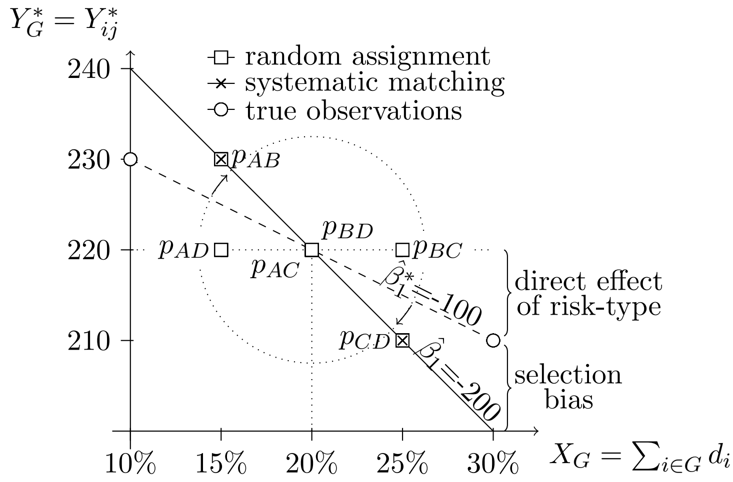


Figure 4: Decomposition of direct effect of risk-type on lending outcomes and selection effect.

7). The presence of A' and D' changes the relative rankings in the market.[8] Again, we only observe the risk-type of one borrower per match and again the estimate of β is biased downwards, -192.9 in this case. However, a direct comparison of the two markets shows that the expected group repayment of borrower B and D increases by 5 and 10 units when their match partners' default risk reduces by 5% and 10% respectively. A natural estimate of β is -100.

Table 7: Match outcomes with exogenous variation (partially unobservable characteristics)

	A'	A	B	C	D	D'
A' ($d_{A'} = 0\%$)			**235**			
A ($d_A = 5\%$)			230		**220**	
B						
C ($d_C = 15\%$)					210	**205**
D						
D'						

[8] Suppose B breaks up her match with A to match with the safer A'. This in turn may lead D to break up her current match with C to match with the safer single A. C then matches with the remaining high risk D'.

B. Equilibrium characterisation

Under transferable utility, agents can write binding contracts that specify how to share the total pay-off generated by the collective of all players in the market. As a result, the coordinated efforts of borrowers lead to a matching that maximises the total market pay-off. This pay-off is then shared between the players according to the binding contracts.

Stability and uniqueness

The group formation game under transferable utility is a special case of the Kaneko and Wooders (1982) partitioning game. A partitioning game consists of a finite set of N players $\mathcal{N} = \{1, ..., N\}$ and a characteristic function V that assigns a value to each group of players, $G \subset \mathcal{N}$. In the partitioning game, only certain coalitions – so-called *basic coalitions* – can create value. These coalitions are subsets of $\mathcal{N}$. In the group formation game, the basic coalitions are all groups of size n. The collection of these basic coalitions Ω is the set of feasible matches, i.e. borrowing groups. It can be written as $\Omega = \{G \subset \mathcal{N} : |G| = n\}$.[9]

For the partitioning game, Quint (1991) shows that the equilibrium[10] coincides with the set of optimal solutions to the dual of a linear programming problem.

Equilibrium characterisation

Let M denote the set of feasible matchings (or group constellations μ) in the sense that each agent is matched exactly once. This set comprises both the observed group constellation in village t and all unobserved group constellations.[11] The latter contain equally sized, alternative groups in the same village composed of borrowers from the observed groups. Using this definition, the objective function of the PLP can be rewritten as below. An *optimal partitioning* is a matching μ that maximises the total valuation in the market. This matching is such that it solves the following maximisation problem.

$$\max_{\mu \in M} \sum_{G \in \mu} V_G \tag{21}$$

The equilibrium condition for a coalition-wise (or core) stable matching is given by the following inequality, where $M \backslash \tilde{\mu}$ gives the set of feasible deviations from the equilibrium matching $\tilde{\mu}$.

$$\sum_{G' \in \tilde{\mu}} V_{G'} \quad > \quad \max_{\mu \in M \backslash \tilde{\mu}} \sum_{G \in \mu} V_G \tag{22}$$

The condition can be restated in two simple inequalities that impose upper bounds for non-equilibrium matchings and lower bounds for the equilibrium matchings. Proposition B.1

[9] In the empirical context of this paper, the set of feasible groups is obtained by generating all k-for-k borrower swaps across two groups in the same village. The total number of swaps in a village with two groups of five borrowers is given by $5 \times 5 = 25$ 1-for-1 swaps, $10 \times 10 = 100$ 2-for-2 swaps, $10 \times 10 = 100$ 3-for-3 swaps and $5 \times 5 = 25$ 4-for-4 swaps ($= 250$ in total).

[10] Specifically, the set of feasible allocations that cannot be improved upon. This is also referred to as the 'core' in the matching literature.

[11] In a context with two groups per market, the number of feasible matchings $|M|$ is half the number of feasible matches $|\Omega|$.

summarises the conditions for pairwise stability based on the bounds $\overline{V_G^*}$ and $\underline{V_G^*}$ derived below.

Proposition B.1 *The matching μ is stable iff $V_G < \overline{V_G^*}$ $\forall G \notin \mu$. Equivalently, the matching μ is stable iff $V_G > \underline{V_G^*}$ $\forall G \in \mu$.*

Proof B.1 *A matching is stable if deviation is unattractive. Alternative matchings are therefore bound to have lower valuations than observed ones. This naturally leads to upper bounds $V_G < \overline{V_G^*}$ for the valuation of matches $G \in \mu$, not contained in the equilibrium matching $\tilde{\mu}$.*

$$V_G \quad < \quad \sum_{G' \in \tilde{\mu}} V_{G'} - \max_{\mu \in M \setminus \tilde{\mu}} \sum_{G'' \in \mu \setminus G} V_{G''} =: \overline{V_G^*} \tag{23}$$

The upper bounds $\overline{V_G^*}$ are increasing in the valuation of the equilibrium matching $\tilde{\mu}$ (first term on RHS of the inequality in Eqn 23), and decreasing in the valuation of the optimal group constellation of all remaining borrowers not contained in G (second term). It is possible to invert the inequalities to obtain a lower bound $V_G > \underline{V_G^*}$ for the valuation of the equilibrium match $G \in \tilde{\mu}$.

$$V_G \quad > \quad \max_{\mu \in M \setminus \tilde{\mu}} \sum_{G' \in \mu} V_{G'} - \sum_{G'' \in \tilde{\mu} \setminus G} V_{G''} =: \underline{V_G^*} \tag{24}$$

The lower bound $\underline{V_G^*}$ is increasing in the valuation of the most attractive non-equilibrium matching (first term on RHS of the inequality in Eqn 24) and decreasing in the valuations of the other equilibrium matches without G (second term).

These conditions are equivalent, but both are important for estimation as they impose different bounds on the latent valuation variables. These inequalities will be used in the econometric model to truncate the valuations of feasible groupings.

C. Simulation of posterior distribution

The Bayesian estimator uses the data augmentation approach (proposed by Albert and Chib 1993) that treats the latent outcome and valuation variables as nuissance parameters. The following four steps illustrate the first iteration of the estimator for the first-stage matching model.

C.1. Match valuations for unobserved groups

The algorithm starts by simulating the latent match valuations for unobserved groups conditional on the data and parameters. In the first iteration illustrated here, the slope parameter alpha (blue asterisk) and the match valuations of the equilibrium groups (red asterisks) are initially set to zero and match valuations for unobserved groups (black circles) are drawn from a normal distribution with mean zero. For the observed groups to be in equilibrium, the match valuation of the unobserved groups must be lower than the maximum equilibrium group valuation. The draws from the normal are therefore censored from above (gray shades).

C.2. Match valuations for first observed group

In the next step, the match valuation is drawn from a normal distribution with mean zero (conditional mean given by the dashed line). The equilibrium condition holds because the valuation of the second equilibrium group is larger than that of any non-equilibrium group (indicated by the yellow shades). Thus the valuation can be drawn from an uncensored normal.

C.3. Match valuations for second observed group

Same procedure as in step 2.

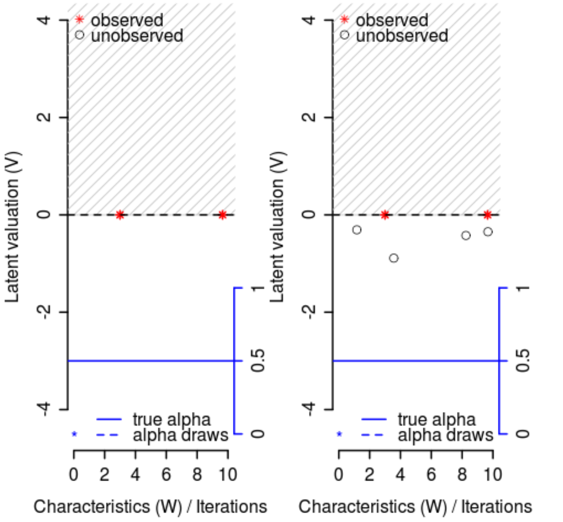
C.4. Alpha slope parameter

Fit a regression based on the given valuations and data (solid line) and draw alpha (dashed blue line) from a normal distribution with mean and standard deviation of the estimated slope parameter. Use the new alpha draw in the next iteration to simulate the latent match valuations, etc etc.

Table 8: Simulation of posterior distribution: Conditional Draws for match valuations and model parameters

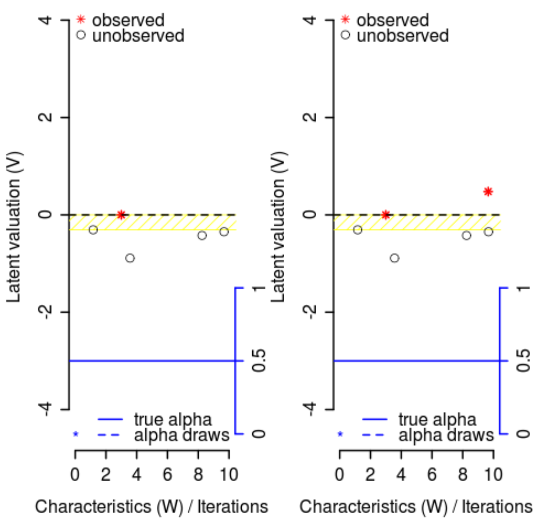
1. Match valuations for unobserved groups

before. after.



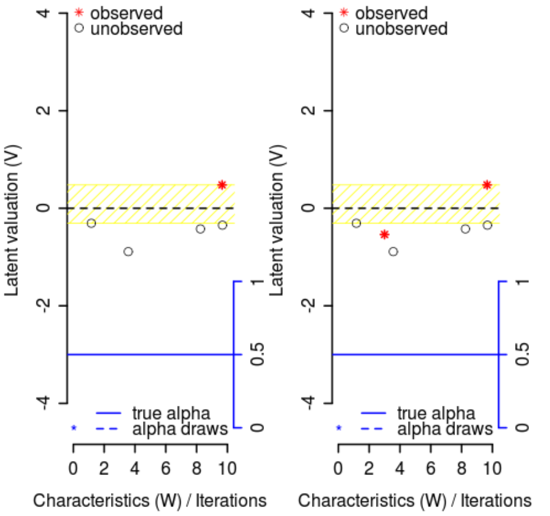
2. Match valuation for 1st observed group

before. after.



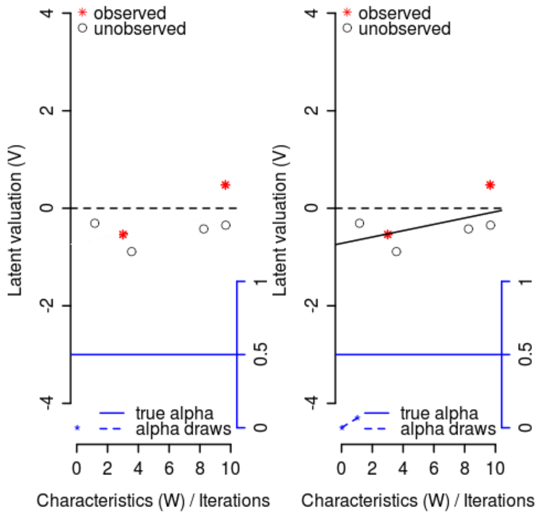
3. Match valuation for 2nd observed group

before. after.



4. Alpha slope parameter

before. after.



D. Measurement error in group-level analysis

The 1997 baseline survey used in Ahlin and Townsend (2007) restricts their measures of project covariation to the village level rather than the group level variable studied in this paper. Their *worst_year* measure therefore gives the probability that two randomly chosen agents from the same *village* are of the same exposure type. This measure is subject to a measurement error that results in an upward bias of the coefficient pertaining to project covariation, which could explain the rather surprising finding that covariation improves repayment in Ahlin and Townsend (2007). The endogeneity problem of village-level project covariation measure is stated and derived formally in Proposition D.1.

Proposition D.1 *The village-level measure in Ahlin and Townsend (2007) leads to an upward bias in the regression coefficient unless the matching into groups is random w.r.t. exposure type.*

Proof D.1 *I illustrate the bias using two exposure types A and B with proportion θ_A and $\theta_B := 1-\theta_A$ respectively, (see Ahlin 2009). The probability of drawing two agents of the same type $s = A, B$ is θ_s^2. The measure used in Ahlin and Townsend (2007) for village or market t is then simply given by*

$$\tilde{X}_t \quad = \quad \sum_{s\in\{A,B\}} \theta_{st}^2. \tag{25}$$

If agents do match on exposure type, i.e. they anti-diversify, the true village-level average of this measure will always be higher than the measure used by the authors. For example, under assortative matching on exposure type, one group will be homogeneous in the leading exposure type, say A. The probability of drawing two group members of the same type in this group is 1. The residual group has proportion $2\theta_B$ of B-types and proportion $2(\theta_A - \frac{1}{2})$ of A-types, and the average village-level project covariation is

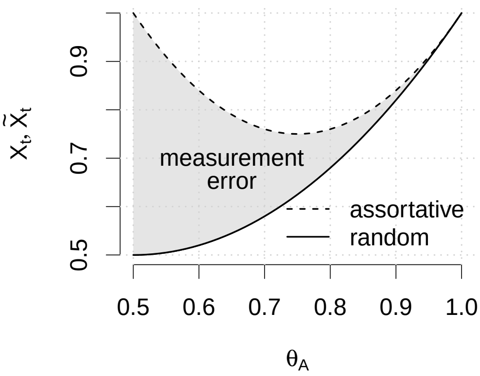
$$X_t \quad = \quad \frac{1}{2}\left[1 + \left(2\left[\theta_{At} - \frac{1}{2}\right]\right)^2 + (2\theta_{Bt})^2\right]. \tag{26}$$

The values for X_t (assortative matching on exposure type) and $\tilde{X}_t$ (random matching) are plotted for different levels of θ_A in Figure 5. It is clear from the figure and fairly intuitive that anti-diversification results in a higher village-level project covariation; that is, the dotted line is always above the solid line. What is interesting about this figure is that the measurement error (shaded area in Figure 5a) from using $\tilde{X}_t$, when matching is assortative and the true measure should be X_t, is negatively related to the erroneous measure $\tilde{X}_t$ (see Figure 5b). Following a standard argument[12], this results in an upward bias of the coefficient pertaining to project covariation.

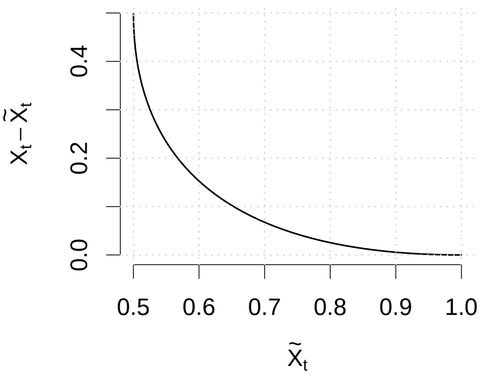
[12] Let the true specification be $Y_{Gt} = \beta_0 + \beta_1 X_t + \varepsilon_{Gt}$. For this specification it holds that $\mathbb{E}[\varepsilon_{Gt}|X_t] = 0$. Furthermore, let X_t be the true value of X and $\tilde{X}_t$, the covariation measure that falsely assumes random matching on risk exposure type. We estimate $Y_{Gt} = \beta_0' + \beta_1'\tilde{X}_t + \varepsilon_{Gt}'$ with $\varepsilon_{Gt}' = \beta_1(X_t - \tilde{X}_t) + \varepsilon_{Gt}$. If $(X_t - \tilde{X}_t)$ is negatively correlated with $\tilde{X}_t$ (see Figure 5), then $\tilde{X}_t$ is negatively correlated with the error term ε_{Gt}' and $\hat{\beta}_1$ is biased and inconsistent. Because $\beta_1 < 0$, the bias is upwards.

Figure 5: Measurement error in group-level variables

(a) Erroneous measure $\tilde{X}_t$ and correct measure X_t against share of leading exposure type A



(b) Measurement error $\tilde{X}_t - X_t$ against erroneous measure $\tilde{X}_t$



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