The markovchain Package: A Package for Easily Handling Discrete Markov Chains in R

Giorgio Alfredo Spedicato, Tae Seung Kang, Sai Bhargav Yalamanchi and Deepak Yadav

Abstract

The **markovchain** package aims to fill a gap within the R framework providing S4 classes and methods for easily handling discrete time Markov chains, homogeneous and simple inhomogeneous ones as well as continuous time Markov chains. The S4 classes for handling and analysing discrete and continuous time Markov chains are presented, as well as functions and method for performing probabilistic and statistical analysis. Finally, some examples in which the package's functions are applied to Economics, Finance and Natural Sciences topics are shown.

Keywords: discrete time Markov chains, continuous time Markov chains, transition matrices, communicating classes, periodicity, first passage time, stationary distributions..

1. Introduction

Markov chains represent a class of stochastic processes of great interest for the wide spectrum of practical applications. In particular, discrete time Markov chains (DTMC) permit to model the transition probabilities between discrete states by the aid of matrices. Various R packages deal with models that are based on Markov chains:

- **msm** (Jackson 2011) handles Multi-State Models for panel data;
- **mcmcR** (Geyer and Johnson 2013) implements Monte Carlo Markov Chain approach;
- **hmm** (Himmelmann and www.linhi.com 2010) fits hidden Markov models with covariates;
- **mstate** fits Multi-State Models based on Markov chains for survival analysis (de Wreede, Fiocco, and Putter 2011).

Nevertheless, the R statistical environment (R Core Team 2013) seems to lack a simple package that coherently defines S4 classes for discrete Markov chains and allows to perform probabilistic analysis, statistical inference and applications. For the sake of completeness, **markovchain** is the second package specifically dedicated to DTMC analysis, being **DTMCPack** (Nicholson 2013) the first one. Notwithstanding, **markovchain** package (Spedicato 2016) aims to offer more flexibility in handling DTMC than other existing solutions, providing S4 classes for both homogeneous and non-homogeneous Markov chains as well as methods suited to perform statistical and probabilistic analysis.
The **markovchain** package depends on the following R packages: **expm** (Goulet, Dutang,

Maechler, Firth, Shapira, Stadelmann, and expm-developers@lists.R-forge.R-project.org 2013) to perform efficient matrices powers; **igraph** (Csardi and Nepusz 2006) to perform pretty plotting of `markovchain` objects and **matlab** (Roebuck 2011), that contains functions for matrix management and calculations that emulate those within MATLAB environment. Moreover, other scientific softwares provide functions specifically designed to analyze DTMC, as Mathematica 9 (Wolfram Research 2013b).

The paper is structured as follows: Section 2 briefly reviews mathematics and definitions regarding DTMC, Section 3 discusses how to handle and manage Markov chain objects within the package, Section 4 and Section 5 show how to perform probabilistic and statistical modelling, while Section 6 presents some applied examples from various fields analyzed by means of the **markovchain** package.

2. Review of core mathematical concepts

2.1. General Definitions

A DTMC is a sequence of random variables X_1, X_2 , . . . , X_n, . . . characterized by the Markov property (also known as memoryless property, see Equation 1). The Markov property states that the distribution of the forthcoming state X_{n+1} depends only on the current state X_n and doesn't depend on the previous ones X_{n-1}, $X_{n-2}, \ldots,$ X_1.

$$Pr\left(X_{n+1}=x_{n+1}\left|X_1=x_1,X_2=x_{2,\ldots},X_n=x_n\right.\right)=Pr\left(X_{n+1}=x_{n+1}\left|X_n=x_n\right.\right). \tag{1}$$

The set of possible states $S=\{s_1,s_2,...,s_r\}$ of X_n can be finite or countable and it is named the state space of the chain.

The chain moves from one state to another (this change is named either 'transition' or 'step') and the probability p_{ij} to move from state s_i to state s_j in one step is named transition probability:

$$p_{ij}=Pr\left(X_1=s_j\left|X_0=s_i\right.\right). \tag{2}$$

The probability of moving from state i to j in n steps is denoted by $p_{ij}^{(n)}=Pr\left(X_n=s_j\left|X_0=s_i\right.\right)$.

A DTMC is called time-homogeneous if the property shown in Equation 3 holds. Time homogeneity implies no change in the underlying transition probabilities as time goes on.

$$Pr\left(X_{n+1}=s_j\left|X_n=s_i\right.\right)=Pr\left(X_n=s_j\left|X_{n-1}=s_i\right.\right). \tag{3}$$

If the Markov chain is time-homogeneous, then $p_{ij}=Pr\left(X_{k+1}=s_j\left|X_k=s_i\right.\right)$ and $p_{ij}^{(n)}=Pr\left(X_{n+k}=s_j\left|X_k=s_i\right.\right)$, where $k>0$.

The probability distribution of transitions from one state to another can be represented into a transition matrix $P=(p_{ij})_{i,j}$, where each element of position (i,j) represents the transition probability p_{ij}. E.g., if $r=3$ the transition matrix P is shown in Equation 4

$$P=\left[\begin{array}{ccc} p_{11} & p_{12} & p_{13}\\ p_{21} & p_{22} & p_{23}\\ p_{31} & p_{32} & p_{33} \end{array}\right]. \tag{4}$$

The distribution over the states can be written in the form of a stochastic row vector x (the term stochastic means that $\sum_i x_i = 1, x_i \geq 0$): e.g., if the current state of x is s_2, $x = (0\,1\,0)$. As a consequence, the relation between $x^{(1)}$ and $x^{(0)}$ is $x^{(1)} = x^{(0)}P$ and, recursively, we get $x^{(2)} = x^{(0)}P^2$ and $x^{(n)} = x^{(0)}P^n$, $n > 0$.

DTMC are explained in most theory books on stochastic processes, see Brémaud (1999) and Dobrow (2016) for example. Valuable references online available are: Konstantopoulos (2009), Snell (1999) and Bard (2000).

2.2. Properties and classification of states

A state s_j is said accessible from state s_i (written $s_i \rightarrow s_j$) if a system started in state s_i has a positive probability to reach the state s_j at a certain point, i.e., $\exists n > 0 : p_{ij}^n > 0$. If both $s_i \rightarrow s_j$ and $s_j \rightarrow s_i$, then s_i and s_j are said to communicate.

A communicating class is defined to be a set of states that communicate. A DTMC can be composed by one or more communicating classes. If the DTMC is composed by only one communicating class (i.e., if all states in the chain communicate), then it is said irreducible. A communicating class is said to be closed if no states outside of the class can be reached from any state inside it.

If $p_{ii} = 1$, s_i is defined as absorbing state: an absorbing state corresponds to a closed communicating class composed by one state only.

The canonic form of a DTMC transition matrix is a matrix having a block form, where the closed communicating classes are shown at the beginning of the diagonal matrix.

A state s_i has period k_i if any return to state s_i must occur in multiplies of k_i steps, that is $k_i = gcd\left\{n : Pr\left(X_n = s_i \left| X_0 = s_i\right.\right) > 0\right\}$, where gcd is the greatest common divisor. If $k_i = 1$ the state s_i is said to be aperiodic, else if $k_i > 1$ the state s_i is periodic with period k_i. Loosely speaking, s_i is periodic if it can only return to itself after a fixed number of transitions $k_i > 1$ (or multiple of k_i), else it is aperiodic.

If states s_i and s_j belong to the same communicating class, then they have the same period k_i. As a consequence, each of the states of an irreducible DTMC share the same periodicity. This periodicity is also considered the DTMC periodicity. It is possible to classify states according to their periodicity. Let $T^{x \rightarrow x}$ is the number of periods to go back to state x knowing that the chain starts in x.

- A state x is recurrent if $P(T^{x \rightarrow x} < +\infty) = 1$ (equivalently $P(T^{x \rightarrow x} = +\infty) = 0$). In addition:
    1. A state x is null recurrent if in addition $E(T^{x \rightarrow x}) = +\infty$.
    2. A state x is positive recurrent if in addition $E(T^{x \rightarrow x}) < +\infty$.
    3. A state x is absorbing if in addition $P(T^{x \rightarrow x} = 1) = 1$.
- A state x is transient if $P(T^{x \rightarrow x} < +\infty) < 1$ (equivalently $P(T^{x \rightarrow x} = +\infty) > 0$).

It is possible to analyze the timing to reach a certain state. The first passage time from state s_i to state s_j is the number T_{ij} of steps taken by the chain until it arrives for the first time to state s_j, given that $X_0 = s_i$. The probability distribution of T_{ij} is defined by Equation 5

$$h_{ij}{}^{(n)} = Pr\left(T_{ij} = n\right) = Pr\left(X_n = s_j, X_{n-1} \neq s_j, \ldots, X_1 \neq s_j | X_0 = s_i\right) \quad (5)$$

and can be found recursively using Equation 6, given that ${h_{ij}}^{(n)} = p_{ij}$.

$$ {h_{ij}}^{(n)} = \sum_{k \in S - \{s_j\}} p_{ik} {h_{kj}}^{(n-1)}. \tag{6} $$

If in the definition of the first passage time we let $s_i = s_j$, we obtain the first return time $T_i = inf\{n \geq 1 : X_n = s_i | X_0 = s_i\}$. A state s_i is said to be recurrent if it is visited infinitely often, i.e., $Pr(T_i < +\infty | X_0 = s_i) = 1$. On the opposite, s_i is called transient if there is a positive probability that the chain will never return to s_i, i.e., $Pr(T_i = +\infty | X_0 = s_i) > 0$.

Given a time homogeneous Markov chain with transition matrix P, a stationary distribution z is a stochastic row vector such that $z = z \cdot P$, where $0 \leq z_j \leq 1 \, \forall j$ and $\sum_j z_j = 1$.

If a DTMC $\{X_n\}$ is irreducible and aperiodic, then it has a limit distribution and this distribution is stationary. As a consequence, if P is the $k \times k$ transition matrix of the chain and $z = (z_1, ..., z_k)$ is the eigenvector of P such that $\sum_{i=1}^{k} z_i = 1$, then we get

$$ \underset{n \to \infty}{lim} P^n = Z, \tag{7} $$

where Z is the matrix having all rows equal to z. The stationary distribution of $\{X_n\}$ is represented by z.

2.3. A short example

Consider the following numerical example. Suppose we have a DTMC with a set of 3 possible states $S = \{s_1, s_2, s_3\}$. Let the transition matrix be

$$ P = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.15 & 0.45 & 0.4 \\ 0.25 & 0.35 & 0.4 \end{bmatrix}. \tag{8} $$

In P, $p_{11} = 0.5$ is the probability that $X_1 = s_1$ given that we observed $X_0 = s_1$ is 0.5, and so on. It is easy to see that the chain is irreducible since all the states communicate (it is made by one communicating class only).

Suppose that the current state of the chain is $X_0 = s_2$, i.e., $x^{(0)} = (0 1 0)$, then the probability distribution of states after 1 and 2 steps can be computed as shown in Equations 9 and 10.

$$ x^{(1)} = (0\,1\,0) \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.15 & 0.45 & 0.4 \\ 0.25 & 0.35 & 0.4 \end{bmatrix} = (0.15\,0.45\,0.4)\,. \tag{9} $$

$$ x^{(n)} = x^{(n-1)} P \to (0.15\,0.45\,0.4) \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.15 & 0.45 & 0.4 \\ 0.25 & 0.35 & 0.4 \end{bmatrix} = (0.2425\,0.3725\,0.385)\,. \tag{10} $$

If, f.e., we are interested in the probability of reaching the state s_3 in two steps, then $Pr\,(X_2 = s_3\,|X_0 = s_2\,) = 0.385$.

3. The structure of the package

3.1. Creating markovchain objects

The package is loaded within the R command line as follows:

```
R> library("markovchain")
```

The `markovchain` and `markovchainList` S4 classes (Chambers 2008) are defined within the **markovchain** package as displayed:

```
Class "markovchain" [package "markovchain"]

Slots:

Name:            states            byrow transitionMatrix
Class:        character          logical           matrix

Name:              name
Class:        character

Class "markovchainList" [package "markovchain"]

Slots:

Name:  markovchains         name
Class:         list    character
```

The first class has been designed to handle homogeneous Markov chain processes, while the latter (which is itself a list of `markovchain` objects) has been designed to handle non-homogeneous Markov chains processes.

Any element of `markovchain` class is comprised by following slots:

1. `states`: a character vector, listing the states for which transition probabilities are defined.

2. `byrow`: a logical element, indicating whether transition probabilities are shown by row or by column.

3. `transitionMatrix`: the probabilities of the transition matrix.

4. `name`: optional character element to name the DTMC.

The `markovchainList` objects are defined by following slots:

1. `markovchains`: a list of `markovchain` objects.

2. `name`: optional character element to name the DTMC.

The `markovchain` objects can be created either in a long way, as the following code shows

```
R> weatherStates <- c("sunny", "cloudy", "rain")
R> byRow <- TRUE
R> weatherMatrix <- matrix(data = c(0.70, 0.2, 0.1,
+                          0.3, 0.4, 0.3,
+                          0.2, 0.45, 0.35), byrow = byRow, nrow = 3,
+                        dimnames = list(weatherStates, weatherStates))
R> mcWeather <- new("markovchain", states = weatherStates, byrow = byRow,
+                  transitionMatrix = weatherMatrix, name = "Weather")
```

or in a shorter way, displayed below

```
R> mcWeather <- new("markovchain", states = c("sunny", "cloudy", "rain"),
+                    transitionMatrix = matrix(data = c(0.70, 0.2, 0.1,
+                          0.3, 0.4, 0.3,
+                          0.2, 0.45, 0.35), byrow = byRow, nrow = 3),
+                    name = "Weather")
```

When `new("markovchain")` is called alone, a default Markov chain is created.

```
R> defaultMc <- new("markovchain")
```

The quicker way to create `markovchain` objects is made possible thanks to the implemented `initialize` S4 method that checks that:

- the `transitionMatrix` to be a transition matrix, i.e., all entries to be probabilities and either all rows or all columns to sum up to one.
- the columns and rows names of `transitionMatrix` to be defined and to coincide with `states` vector slot.

The `markovchain` objects can be collected in a list within `markovchainList` S4 objects as following example shows.

```
R> mcList <- new("markovchainList", markovchains = list(mcWeather, defaultMc),
+                   name = "A list of Markov chains")
```

3.2. Handling markovchain objects

Table 1 lists which methods handle and manipulate `markovchain` objects.

The examples that follow shows how operations on `markovchain` objects can be easily performed. For example, using the previously defined matrix we can find what is the probability distribution of expected weather states in two and seven days, given the actual state to be cloudy.

Method	Purpose
`*`	Direct multiplication for transition matrices.
`[`	Direct access to the elements of the transition matrix.
`==`	Equality operator between two transition matrices.
`as`	Operator to convert `markovchain` objects into `data.frame` and `table` object.
`dim`	Dimension of the transition matrix.
`names`	Equal to `states`.
`names<-`	Change the `states` name.
`name`	Get the name of `markovchain object`.
`name<-`	Change the name of `markovchain object`.
`plot`	`plot` method for `markovchain` objects.
`print`	`print` method for `markovchain` objects.
`show`	`show` method for `markovchain` objects.
`states`	Name of the transition states.
`t`	Transposition operator (which switches byrow slot value and modifies the transition matrix coherently).

Table 1: **markovchain** methods for handling `markovchain` objects.

```
R> initialState <- c(0, 1, 0)
R> after2Days <- initialState * (mcWeather * mcWeather)
R> after7Days <- initialState * (mcWeather ^ 7)
R> after2Days

     sunny cloudy  rain
[1,]  0.39  0.355 0.255

R> round(after7Days, 3)

     sunny cloudy  rain
[1,] 0.462  0.319 0.219
```

A similar answer could have been obtained defining the vector of probabilities as a column vector. A column - defined probability matrix could be set up either creating a new matrix or transposing an existing `markovchain` object thanks to the `t` method.

```
R> initialState <- c(0, 1, 0)
R> after2Days <- (t(mcWeather) * t(mcWeather)) * initialState
R> after7Days <- (t(mcWeather) ^ 7) * initialState
R> after2Days

        [,1]
sunny  0.390
cloudy 0.355
rain   0.255
```

```
R> round(after7Days, 3)
```

```
        [,1]
sunny  0.462
cloudy 0.319
rain   0.219
```

The initial state vector previously shown can not necessarily be a probability vector, as the code that follows shows:

```
R> fvals<-function(mchain,initialstate,n) {
+    out<-data.frame()
+    names(initialstate)<-names(mchain)
+    for (i in 0:n)
+    {
+      iteration<-initialstate*mchain^(i)
+      out<-rbind(out,iteration)
+    }
+    out<-cbind(out, i=seq(0,n))
+    out<-out[,c(4,1:3)]
+    return(out)
+  }
R> fvals(mchain=mcWeather,initialstate=c(90,5,5),n=4)
```

```
  i    sunny   cloudy     rain
1 0 90.00000  5.00000  5.00000
2 1 65.50000 22.25000 12.25000
3 2 54.97500 27.51250 17.51250
4 3 50.23875 29.88063 19.88062
5 4 48.10744 30.94628 20.94628
```

Basic methods have been defined for `markovchain` objects to quickly get states and transition matrix dimension.

```
R> states(mcWeather)
```

```
[1] "sunny"  "cloudy" "rain"
```

```
R> names(mcWeather)
```

```
[1] "sunny"  "cloudy" "rain"
```

```
R> dim(mcWeather)
```

```
[1] 3
```

Methods are available to set and get the name of `markovchain` object.

```
R> name(mcWeather)

[1] "Weather"

R> name(mcWeather) <- "New Name"
R> name(mcWeather)

[1] "New Name"
```

A direct access to transition probabilities is provided both by `transitionProbability` method and "[" method.

```
R> transitionProbability(mcWeather, "cloudy", "rain")

[1] 0.3

R> mcWeather[2,3]

[1] 0.3
```

The transition matrix of a `markovchain` object can be displayed using `print` or `show` methods (the latter being less laconic). Similarly, the underlying transition probability diagram can be plotted by the use of `plot` method (as shown in Figure 1) which is based on **igraph** package (Csardi and Nepusz 2006). `plot` method for `markovchain` objects is a wrapper of `plot.igraph` for `igraph` S4 objects defined within the **igraph** package. Additional parameters can be passed to `plot` function to control the network graph layout. There are also **diagram** and **DiagrammeR** ways available for plotting as shown in Figure 2.

```
R> print(mcWeather)

       sunny cloudy rain
sunny    0.7   0.20 0.10
cloudy   0.3   0.40 0.30
rain     0.2   0.45 0.35

R> show(mcWeather)

New Name
 A  3 - dimensional discrete Markov Chain defined by the following states:
 sunny, cloudy, rain
 The transition matrix  (by rows)  is defined as follows:
       sunny cloudy rain
sunny    0.7   0.20 0.10
cloudy   0.3   0.40 0.30
rain     0.2   0.45 0.35
```

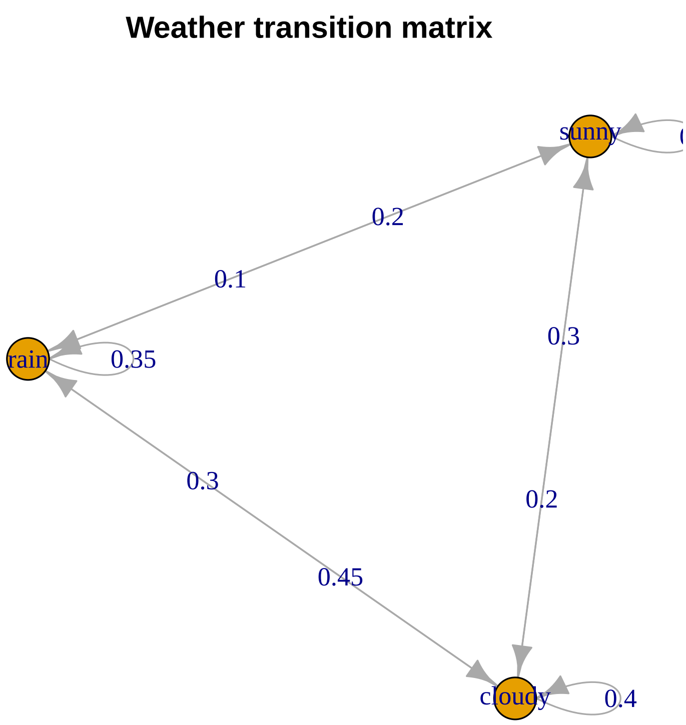

Figure 1: Weather example. Markov chain plot.

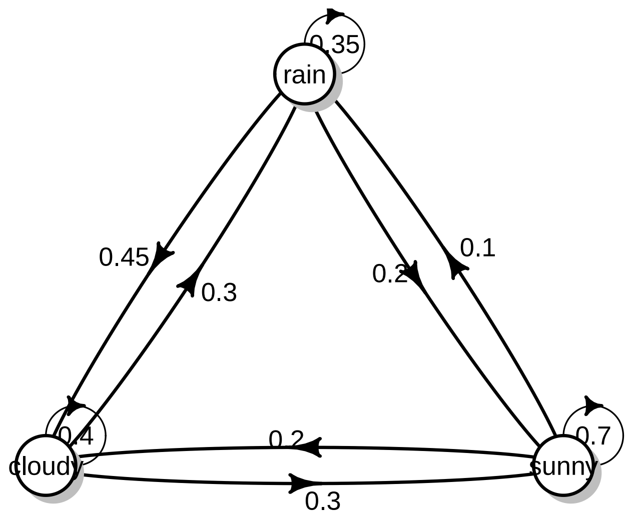

Figure 2: Weather example. Markov chain plot with diagram. plot(mcWeather, package="diagram", box.size = 0.04)

Import and export from some specific classes is possible, as shown in Figure 3 and in the following code.

```
R> mcDf <- as(mcWeather, "data.frame")
R> mcNew <- as(mcDf, "markovchain")
R> mcDf

      t0     t1 prob
1  sunny  sunny 0.70
2  sunny cloudy 0.20
3  sunny   rain 0.10
4 cloudy  sunny 0.30
5 cloudy cloudy 0.40
6 cloudy   rain 0.30
7   rain  sunny 0.20
8   rain cloudy 0.45
9   rain   rain 0.35

R> mcIgraph <- as(mcWeather, "igraph")

R> library(msm)
R> data(cav)
R> Q <- rbind ( c(0, 0.25, 0, 0.25),
+               c(0.166, 0, 0.166, 0.166),
+               c(0, 0.25, 0, 0.25),
+               c(0, 0, 0, 0) )
R> cavmsm <- msm(state ~ years, subject = PTNUM, data = cav, qmatrix = Q, death = 4)
R> msmMc <- as(cavmsm, "markovchain")
R> msmMc

Unnamed Markov chain
 A  4 - dimensional discrete Markov Chain defined by the following states:
 State 1, State 2, State 3, State 4
 The transition matrix  (by rows)  is defined as follows:
            State 1    State 2    State 3    State 4
State 1 0.853958721 0.08836953 0.01475543 0.04291632
State 2 0.155576908 0.56663284 0.20599563 0.07179462
State 3 0.009903994 0.07853691 0.65965727 0.25190183
State 4 0.000000000 0.00000000 0.00000000 1.00000000

R> library(etm)
R> data(sir.cont)
R> sir.cont <- sir.cont[order(sir.cont$id, sir.cont$time), ]
R> for (i in 2:nrow(sir.cont)) {
+    if (sir.cont$id[i]==sir.cont$id[i-1]) {
```

```
+       if (sir.cont$time[i]==sir.cont$time[i-1]) {
+         sir.cont$time[i-1] <- sir.cont$time[i-1] - 0.5
+       }
+     }
+   }
R> tra <- matrix(ncol=3,nrow=3,FALSE)
R> tra[1, 2:3] <- TRUE
R> tra[2, c(1, 3)] <- TRUE
R> tr.prob <- etm(sir.cont, c("0", "1", "2"), tra, "cens", 1)
R> tr.prob

Multistate model with 2 transient state(s)
 and 1 absorbing state(s)

Possible transitions:
 from to
    0  1
    0  2
    1  0
    1  2

Estimate of P(1, 183)
  0 1 2
0 0 0 1
1 0 0 1
2 0 0 1

Estimate of cov(P(1, 183))
    0 0 1 0 2 0 0 1 1 1 2 1           0 2           1 2 2 2
0 0   0   0   0   0   0   0  0.000000e+00  0.000000e+00   0
1 0   0   0   0   0   0   0  0.000000e+00  0.000000e+00   0
2 0   0   0   0   0   0   0  0.000000e+00  0.000000e+00   0
0 1   0   0   0   0   0   0  0.000000e+00  0.000000e+00   0
1 1   0   0   0   0   0   0  0.000000e+00  0.000000e+00   0
2 1   0   0   0   0   0   0  0.000000e+00  0.000000e+00   0
0 2   0   0   0   0   0   0 -2.864030e-20 -3.557538e-20   0
1 2   0   0   0   0   0   0 -4.785736e-20  2.710505e-19   0
2 2   0   0   0   0   0   0  0.000000e+00  0.000000e+00   0

R> etm2mc<-as(tr.prob, "markovchain")
R> etm2mc

Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 0, 1, 2
 The transition matrix  (by rows)  is defined as follows:
          0         1         2
```

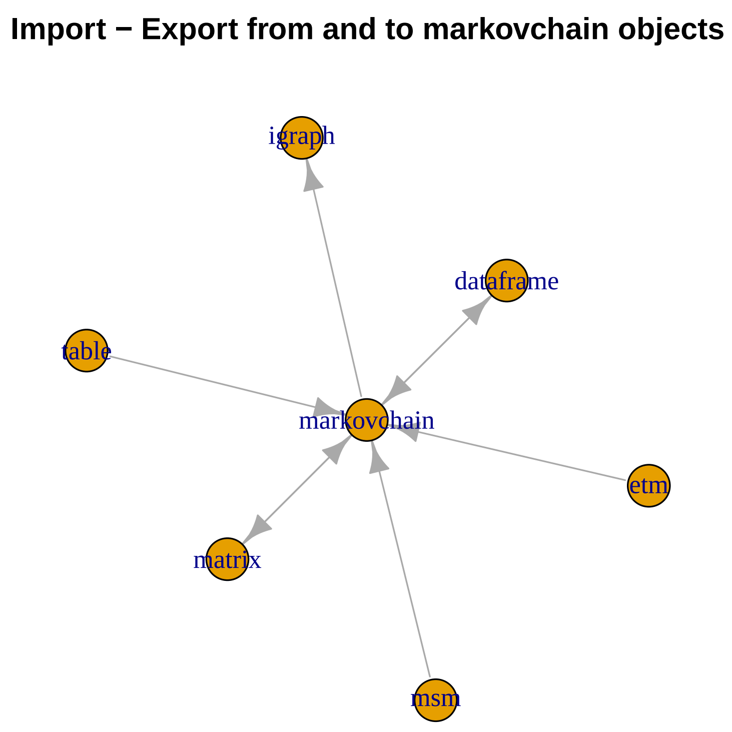

Figure 3: The **markovchain** methods for import and export.

```
0 0.0000000 0.5000000 0.5000000
1 0.5000000 0.0000000 0.5000000
2 0.3333333 0.3333333 0.3333333
```

Coerce from `matrix` method, as the code below shows, represents another approach to create a `markovchain` method starting from a given squared probability matrix.

```
R> myMatr<-matrix(c(.1,.8,.1,.2,.6,.2,.3,.4,.3), byrow=TRUE, ncol=3)
R> myMc<-as(myMatr, "markovchain")
R> myMc
```

```
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 s1, s2, s3
 The transition matrix  (by rows)  is defined as follows:
    s1  s2  s3
```

```
s1 0.1 0.8 0.1
s2 0.2 0.6 0.2
s3 0.3 0.4 0.3
```

Non-homogeneous Markov chains can be created with the aid of `markovchainList` object. The example that follows arises from health insurance, where the costs associated to patients in a Continuous Care Health Community (CCHC) are modelled by a non-homogeneous Markov Chain, since the transition probabilities change by year. Methods explicitly written for `markovchainList` objects are: `print`, `show`, `dim` and `[`.

```
R> stateNames = c("H", "I", "D")
R> Q0 <- new("markovchain", states = stateNames,
+          transitionMatrix =matrix(c(0.7, 0.2, 0.1,0.1, 0.6, 0.3,0, 0, 1),
+          byrow = TRUE, nrow = 3), name = "state t0")
R> Q1 <- new("markovchain", states = stateNames,
+          transitionMatrix = matrix(c(0.5, 0.3, 0.2,0, 0.4, 0.6,0, 0, 1),
+          byrow = TRUE, nrow = 3), name = "state t1")
R> Q2 <- new("markovchain", states = stateNames,
+          transitionMatrix = matrix(c(0.3, 0.2, 0.5,0, 0.2, 0.8,0, 0, 1),
+          byrow = TRUE,nrow = 3), name = "state t2")
R> Q3 <- new("markovchain", states = stateNames,
+            transitionMatrix = matrix(c(0, 0, 1, 0, 0, 1, 0, 0, 1),
+          byrow = TRUE, nrow = 3), name = "state t3")
R> mcCCRC <- new("markovchainList",markovchains = list(Q0,Q1,Q2,Q3),
+        name = "Continuous Care Health Community")
R> print(mcCCRC)
```

```
Continuous Care Health Community  list of Markov chain(s)
Markovchain  1
state t0
 A  3 - dimensional discrete Markov Chain defined by the following states:
 H, I, D
 The transition matrix  (by rows)  is defined as follows:
    H   I   D
H 0.7 0.2 0.1
I 0.1 0.6 0.3
D 0.0 0.0 1.0

Markovchain  2
state t1
 A  3 - dimensional discrete Markov Chain defined by the following states:
 H, I, D
 The transition matrix  (by rows)  is defined as follows:
    H   I   D
H 0.5 0.3 0.2
I 0.0 0.4 0.6
D 0.0 0.0 1.0
```

```
Markovchain  3
state t2
 A  3 - dimensional discrete Markov Chain defined by the following states:
 H, I, D
 The transition matrix  (by rows)  is defined as follows:
    H   I   D
H 0.3 0.2 0.5
I 0.0 0.2 0.8
D 0.0 0.0 1.0

Markovchain  4
state t3
 A  3 - dimensional discrete Markov Chain defined by the following states:
 H, I, D
 The transition matrix  (by rows)  is defined as follows:
  H I D
H 0 0 1
I 0 0 1
D 0 0 1
```

It is possible to perform direct access to `markovchainList` elements, as well as to determine the number of `markovchain` objects by which a `markovchainList` object is composed.

```
R> mcCCRC[[1]]
```

```
state t0
 A  3 - dimensional discrete Markov Chain defined by the following states:
 H, I, D
 The transition matrix  (by rows)  is defined as follows:
    H   I   D
H 0.7 0.2 0.1
I 0.1 0.6 0.3
D 0.0 0.0 1.0
```

```
R> dim(mcCCRC)
```

```
[1] 4
```

The `markovchain` package contains some data found in the literature related to DTMC models (see Section 6). Table 2 lists datasets and tables included within the current release of the package.

Finally, Table 3 lists the demos included in the demo directory of the package.

Dataset	Description
`blanden`	Mobility across income quartiles, Jo Blanden and Machin (2005).
`craigsendi`	CD4 cells, B. A. Craig and A. A. Sendi (2002).
`preproglucacon`	Preproglucacon DNA basis, P. J. Avery and D. A. Henderson (1999).
`rain`	Alofi Island rains, P. J. Avery and D. A. Henderson (1999).
`holson`	Individual states trajectiories.
`sales`	Sales of six beverages in Hong Kong.
Ching, Ng, and Fung (2008).	

Table 2: The **markovchain** `data.frame` and `table`.

R Code Filee	Description
`bard.R`	Structural analysis of Markov chains from Bard PPT.
`examples.R`	Notable Markov chains, e.g., The Gambler Ruin chain.
`quickStart.R`	Generic examples.
`extractMatrices.R`	Generic examples.

Table 3: The **markovchain** demos.

4. Probability with markovchain objects

The **markovchain** package contains functions to analyse DTMC from a probabilistic perspective. For example, the package provides methods to find stationary distributions and identifying absorbing and transient states. Many of these methods come from MATLAB listings that have been ported into R. For a full description of the underlying theory and algorithm the interested reader can overview the original MATLAB listings, Feres (2007) and Montgomery (2009).

Table 4 shows methods that can be applied on `markovchain` objects to perform probabilistic analysis.

Method	Returns
`absorbingStates`	the absorbing states of the transition matrix, if any.
`steadyStates`	the vector(s) of steady state(s) in matrix form.
`communicatingClasses`	list of communicating classes.
	s_j, given actual state s_i.
`canonicForm`	the transition matrix into canonic form.
`is.accessible`	verification if a state j is reachable from state i.
`is.irreducible`	verification whether a DTMC is irreducible.
`period`	the period of an irreducible DTMC.
`recurrentClasses`	list of recurrent classes.
`steadyStates`	the vector(s) of steady state(s) in matrix form.
`summary`	DTMC summary.
`transientStates`	the transient states of the transition matrix, if any.

Table 4: **markovchain** methods: statistical operations.

The conditional distribution of weather states, given that current day's weather is sunny, is given by following code.

```
R> conditionalDistribution(mcWeather, "sunny")

 sunny cloudy   rain
   0.7    0.2    0.1
```

A stationary (steady state) vector is a probability vector such that Equation 11

$$\begin{array}{c} 0 \leq \pi_j \leq 1 \\ \sum_{j \in S} \pi_j = 1 \\ \pi * P = \pi \end{array} \tag{11}$$

Steady states are associated to P eigenvalues equal to one. Therefore the steady states vector can be identified by the following:

1. decompose the transition matrix in eigenvalues and eigenvectors;
2. consider only eigenvectors corresponding to eigenvalues equal to one;
3. normalize such eigenvalues so that the sum of their components is one.

Numeric issue (negative values) can arise when the Markov Chain contains more closed classes. If negative values are found in the initial solution, the above described algorithm is performed on the submatrix corresponding to recurrent P classes. Another vignette in the package focuses on this issue.

The result is returned in matrix form.

```
R> steadyStates(mcWeather)
```

```
         sunny    cloudy      rain
[1,] 0.4636364 0.3181818 0.2181818
```

It is possible for a Markov chain to have more than one stationary distribution, as the gambler ruin example shows.

```
R> gamblerRuinMarkovChain <- function(moneyMax, prob = 0.5) {
+    require(matlab)
+    matr <- zeros(moneyMax + 1)
+    states <- as.character(seq(from = 0, to = moneyMax, by = 1))
+    rownames(matr) = states; colnames(matr) = states
+    matr[1,1] = 1; matr[moneyMax + 1,moneyMax + 1] = 1
+    for(i in 2:moneyMax)
+    { matr[i,i-1] = 1 - prob; matr[i, i + 1] = prob   }
+    out <- new("markovchain",
+              transitionMatrix = matr,
+              name = paste("Gambler ruin", moneyMax, "dim", sep = " ")
+              )
+    return(out)
+  }
R> mcGR4 <- gamblerRuinMarkovChain(moneyMax = 4, prob = 0.5)
R> steadyStates(mcGR4)
```

```
     0 1 2 3 4
[1,] 1 0 0 0 0
[2,] 0 0 0 0 1
```

Absorbing states are determined by means of `absorbingStates` method.

```
R> absorbingStates(mcGR4)
```

```
[1] "0" "4"
```

```
R> absorbingStates(mcWeather)
```

```
character(0)
```

The key function used within Feres (2007) (and **markovchain**'s derived functions) is `.commclassKernel`, that is called below.

```
R> .commclassesKernel <- function(P){
+    m <- ncol(P)
+          stateNames <- rownames(P)
+          T <- zeros(m)
+          i <- 1
+          while (i <= m) {
+                  a <- i
+                  b <- zeros(1,m)
+                  b[1,i] <- 1
+                  old <- 1
+                  new <- 0
+                  while (old != new) {
+                          old <- sum(find(b > 0))
+                          n <- size(a)[2]
+                          matr <- matrix(as.numeric(P[a,]), ncol = m,
+                       nrow = n)
+                          c <- colSums(matr)
+                          d <- find(c)
+                          n <- size(d)[2]
+                          b[1,d] <- ones(1,n)
+                          new <- sum(find(b>0))
+                          a <- d
+                  }
+                  T[i,] <- b
+                  i <- i+1 }
+          F <- t(T)
+          C <- (T > 0)&(F > 0)
+          v <- (apply(t(C) == t(T), 2, sum) == m)
+          colnames(C) <- stateNames
+          rownames(C) <- stateNames
+          names(v) <- stateNames
+          out <- list(C = C, v = v)
+          return(out)
+  }
```

The `.commclassKernel` function gets a transition matrix of dimension n and return a list of two items:

1. `C`, an adjacency matrix showing for each state s_j (in the row) which states lie in the same communicating class of s_j (flagged with 1).

2. `v`, a binary vector indicating whether the state s_j is transient (0) or not (1).

These functions are used by two other internal functions on which the `summary` method for `markovchain` objects works.

The example matrix used in Feres (2007) well exemplifies the purpose of the function.

```
R> P <- matlab::zeros(10)
R> P[1, c(1, 3)] <- 1/2;
R> P[2, 2] <- 1/3; P[2,7] <- 2/3;
R> P[3, 1] <- 1;
R> P[4, 5] <- 1;
R> P[5, c(4, 5, 9)] <- 1/3;
R> P[6, 6] <- 1;
R> P[7, 7] <- 1/4; P[7,9] <- 3/4;
R> P[8, c(3, 4, 8, 10)] <- 1/4;
R> P[9, 2] <- 1;
R> P[10, c(2, 5, 10)] <- 1/3;
R> rownames(P) <- letters[1:10]
R> colnames(P) <- letters[1:10]
R> probMc <- new("markovchain", transitionMatrix = P,
+                name = "Probability MC")
R> .commclassesKernel(P)

$C
      a     b     c     d     e     f     g     h     i     j
a  TRUE FALSE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
b FALSE  TRUE FALSE FALSE FALSE FALSE  TRUE FALSE  TRUE FALSE
c  TRUE FALSE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
d FALSE FALSE FALSE  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE
e FALSE FALSE FALSE  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE
f FALSE FALSE FALSE FALSE FALSE  TRUE FALSE FALSE FALSE FALSE
g FALSE  TRUE FALSE FALSE FALSE FALSE  TRUE FALSE  TRUE FALSE
h FALSE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE FALSE
i FALSE  TRUE FALSE FALSE FALSE FALSE  TRUE FALSE  TRUE FALSE
j FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE

$v
    a     b     c     d     e     f     g     h     i     j
 TRUE  TRUE  TRUE FALSE FALSE  TRUE  TRUE FALSE  TRUE FALSE

R> summary(probMc)

Probability MC  Markov chain that is composed by:
Closed classes:
a c
b g i
f
Recurrent classes:
{a,c},{b,g,i},{f}
Transient classes:
{d,e},{h},{j}
The Markov chain is not irreducible
The absorbing states are: f
```

All states that pertain to a transient class are named "transient" and a specific method has been written to elicit them.

```
R> transientStates(probMc)

[1] "d" "e" "h" "j"
```

Listings from Feres (2007) have been adapted into `canonicForm` method that turns a Markov chain into canonic form.

```
R> probMcCanonic <- canonicForm(probMc)
R> probMc

Probability MC
 A  10 - dimensional discrete Markov Chain defined by the following states:
 a, b, c, d, e, f, g, h, i, j
 The transition matrix  (by rows)  is defined as follows:
    a         b    c         d         e f         g    h         i
a 0.5 0.0000000 0.50 0.0000000 0.0000000 0 0.0000000 0.00 0.0000000
b 0.0 0.3333333 0.00 0.0000000 0.0000000 0 0.6666667 0.00 0.0000000
c 1.0 0.0000000 0.00 0.0000000 0.0000000 0 0.0000000 0.00 0.0000000
d 0.0 0.0000000 0.00 0.0000000 1.0000000 0 0.0000000 0.00 0.0000000
e 0.0 0.0000000 0.00 0.3333333 0.3333333 0 0.0000000 0.00 0.3333333
f 0.0 0.0000000 0.00 0.0000000 0.0000000 1 0.0000000 0.00 0.0000000
g 0.0 0.0000000 0.00 0.0000000 0.0000000 0 0.2500000 0.00 0.7500000
h 0.0 0.0000000 0.25 0.2500000 0.0000000 0 0.0000000 0.25 0.0000000
i 0.0 1.0000000 0.00 0.0000000 0.0000000 0 0.0000000 0.00 0.0000000
j 0.0 0.3333333 0.00 0.0000000 0.3333333 0 0.0000000 0.00 0.0000000
          j
a 0.0000000
b 0.0000000
c 0.0000000
d 0.0000000
e 0.0000000
f 0.0000000
g 0.0000000
h 0.2500000
i 0.0000000
j 0.3333333

R> probMcCanonic

Probability MC
 A  10 - dimensional discrete Markov Chain defined by the following states:
 a, c, b, g, i, f, d, e, h, j
 The transition matrix  (by rows)  is defined as follows:
```

```
    a    c         b         g         i f         d         e    h
a 0.5 0.50 0.0000000 0.0000000 0.0000000 0 0.0000000 0.0000000 0.00
c 1.0 0.00 0.0000000 0.0000000 0.0000000 0 0.0000000 0.0000000 0.00
b 0.0 0.00 0.3333333 0.6666667 0.0000000 0 0.0000000 0.0000000 0.00
g 0.0 0.00 0.0000000 0.2500000 0.7500000 0 0.0000000 0.0000000 0.00
i 0.0 0.00 1.0000000 0.0000000 0.0000000 0 0.0000000 0.0000000 0.00
f 0.0 0.00 0.0000000 0.0000000 0.0000000 1 0.0000000 0.0000000 0.00
d 0.0 0.00 0.0000000 0.0000000 0.0000000 0 0.0000000 1.0000000 0.00
e 0.0 0.00 0.0000000 0.0000000 0.3333333 0 0.3333333 0.3333333 0.00
h 0.0 0.25 0.0000000 0.0000000 0.0000000 0 0.2500000 0.0000000 0.25
j 0.0 0.00 0.3333333 0.0000000 0.0000000 0 0.0000000 0.3333333 0.00
          j
a 0.0000000
c 0.0000000
b 0.0000000
g 0.0000000
i 0.0000000
f 0.0000000
d 0.0000000
e 0.0000000
h 0.2500000
j 0.3333333
```

The function `is.accessible` permits to investigate whether a state s_j is accessible from state s_i, that is whether the probability to eventually reach s_j starting from s_i is greater than zero.

```
R> is.accessible(object = probMc, from = "a", to = "c")

[1] TRUE

R> is.accessible(object = probMc, from = "g", to = "c")

[1] FALSE
```

In Section 2.2 we observed that, if a DTMC is irreducible, all its states share the same periodicity. Then, the `period` function returns the periodicity of the DTMC, provided that it is irreducible. The example that follows shows how to find if a DTMC is reducible or irreducible by means of the function `is.irreducible` and, in the latter case, the method `period` is used to compute the periodicity of the chain.

```
R> E <- matrix(0, nrow = 4, ncol = 4)
R> E[1, 2] <- 1
R> E[2, 1] <- 1/3; E[2, 3] <- 2/3
R> E[3,2] <- 1/4; E[3, 4] <- 3/4
R> E[4, 3] <- 1
R> mcE <- new("markovchain", states = c("a", "b", "c", "d"),
```

```
+                   transitionMatrix = E,
+                   name = "E")
R> is.irreducible(mcE)
```

```
[1] TRUE
```

```
R> period(mcE)
```

```
[1] 2
```

The example Markov chain found in Mathematica web site (Wolfram Research 2013a) has been used, and is plotted in Figure 4.

```
R> require(matlab)
R> mathematicaMatr <- zeros(5)
R> mathematicaMatr[1,] <- c(0, 1/3, 0, 2/3, 0)
R> mathematicaMatr[2,] <- c(1/2, 0, 0, 0, 1/2)
R> mathematicaMatr[3,] <- c(0, 0, 1/2, 1/2, 0)
R> mathematicaMatr[4,] <- c(0, 0, 1/2, 1/2, 0)
R> mathematicaMatr[5,] <- c(0, 0, 0, 0, 1)
R> statesNames <- letters[1:5]
R> mathematicaMc <- new("markovchain", transitionMatrix = mathematicaMatr,
+                       name = "Mathematica MC", states = statesNames)
```

```
Mathematica MC  Markov chain that is composed by:
Closed classes:
c d
e
Recurrent classes:
{c,d},{e}
Transient classes:
{a,b}
The Markov chain is not irreducible
The absorbing states are: e
```

Feres (2007) provides code to compute first passage time (within $1, 2, \ldots, n$ steps) given the initial state to be i. The MATLAB listings translated into R on which the `firstPassage` function is based are

```
R> .firstpassageKernel <- function(P, i, n){
+    G <- P
+    H <- P[i,]
+    E <- 1 - diag(size(P)[2])
+    for (m in 2:n) {
+      G <- P %*% (G * E)
```

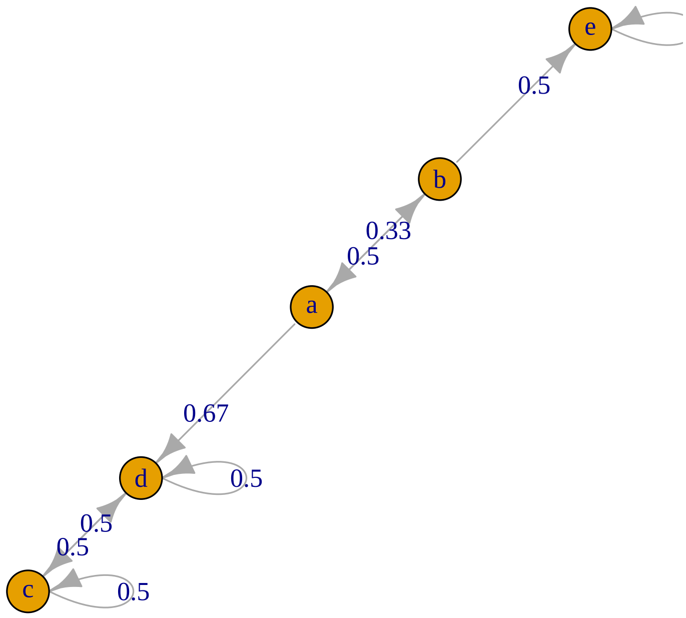

Figure 4: Mathematica 9 example. Markov chain plot.

```
+       H <- rbind(H, G[i,])
+     }
+     return(H)
+   }
```

We conclude that the probability for the first rainy day to be the third one, given that the current state is sunny, is given by

```
R> firstPassagePdF <- firstPassage(object = mcWeather, state = "sunny",
+                                  n = 10)
R> firstPassagePdF[3, 3]
```

```
[1] 0.121
```

5. Statistical analysis

Table 5 lists the functions and methods implemented within the package which help to fit, simulate and predict DTMC.

Function	Purpose
`markovchainFit`	Function to return fitted Markov chain for a given sequence.
`predict`	Method to calculate predictions from `markovchain` or `markovchainList` objects.
`rmarkovchain`	Function to sample from `markovchain` or `markovchainList` objects.

Table 5: The **markovchain** statistical functions.

5.1. Simulation

Simulating a random sequence from an underlying DTMC is quite easy thanks to the function `rmarkovchain`. The following code generates a year of weather states according to `mcWeather` underlying stochastic process.

```
R> weathersOfDays <- rmarkovchain(n = 365, object = mcWeather, t0 = "sunny")
R> weathersOfDays[1:30]
```

```
 [1] "sunny"  "sunny"  "sunny"  "sunny"  "sunny"  "sunny"  "sunny"
 [8] "sunny"  "sunny"  "sunny"  "sunny"  "sunny"  "sunny"  "cloudy"
[15] "sunny"  "sunny"  "cloudy" "rain"   "cloudy" "sunny"  "sunny"
[22] "cloudy" "sunny"  "sunny"  "sunny"  "cloudy" "sunny"  "cloudy"
[29] "rain"   "sunny"
```

Similarly, it is possible to simulate one or more sequences from a non-homogeneous Markov chain, as the following code (applied on CCHC example) exemplifies.

```
R> patientStates <- rmarkovchain(n = 5, object = mcCCRC, t0 = "H",
+                                include.t0 = TRUE)
R> patientStates[1:10,]

   iteration values
1          1      H
2          1      I
3          1      D
4          1      D
5          1      D
6          2      H
7          2      H
8          2      H
9          2      I
10         2      D
```

Two advance parameters are availabe to the `rmarkovchain` method which helps you decide which implementation to use. There are four options available : R, R in parallel, C++ and C++ in parallel. Two boolean parameters `useRcpp` and `parallel` will decide which implementation will be used. Default is `useRcpp = TRUE` and `parallel = FALSE` i.e. C++ implementation. The C++ implementation is generally faster than the `R` implementation. If you have multicore processors then you can take advantage of `parallel` parameter by setting it to `TRUE`. When both `Rcpp=TRUE` and `parallel=TRUE` the parallelization has been carried out using **RcppParallel** package (Allaire, Francois, Ushey, Vandenbrouck, Geelnard, and Intel 2016).

5.2. Estimation

A time homogeneous Markov chain can be fit from given data. Four methods have been implemented within current version of **markovchain** package: maximum likelihood, maximum likelihood with Laplace smoothing, Bootstrap approach, maximum a posteriori.

Equation 12 shows the maximum likelihood estimator (MLE) of the p_{ij} entry, where the n_{ij} element consists in the number sequences $(X_t = s_i, X_{t+1} = s_j)$ found in the sample, that is

$$\hat{p}_{ij}^{MLE} = \frac{n_{ij}}{\sum\limits_{u=1}^{k} n_{iu}}. \tag{12}$$

Equation 13 shows the `standardError` of the MLE (Skuriat-Olechnowska 2005).

$$SE_{ij} = \frac{\hat{p}_{ij}^{MLE}}{\sqrt{n_{ij}}} \tag{13}$$

```
R> weatherFittedMLE <- markovchainFit(data = weathersOfDays, method = "mle",
+                                    name = "Weather MLE")
R> weatherFittedMLE$estimate
```

```
Weather MLE
 A  3 - dimensional discrete Markov Chain defined by the following states:
 cloudy, rain, sunny
 The transition matrix  (by rows)  is defined as follows:
          cloudy       rain     sunny
cloudy 0.3790323 0.29838710 0.3225806
rain   0.4871795 0.34615385 0.1666667
sunny  0.2407407 0.08641975 0.6728395
```

```
R> weatherFittedMLE$standardError
```

```
           cloudy       rain      sunny
cloudy 0.05528754 0.04905454 0.05100448
rain   0.07903095 0.06661734 0.04622502
sunny  0.03854937 0.02309665 0.06444634
```

The Laplace smoothing approach is a variation of the MLE, where the n_{ij} is substituted by $n_{ij} + \alpha$ (see Equation 14), being α an arbitrary positive stabilizing parameter.

$$\hat{p}^{LS}_{ij} = \frac{n_{ij} + \alpha}{\sum_{u=1}^{k} (n_{iu} + \alpha)} \tag{14}$$

```
R> weatherFittedLAPLACE <- markovchainFit(data = weathersOfDays,
+                                       method = "laplace", laplacian = 0.01,
+                                       name = "Weather LAPLACE")
R> weatherFittedLAPLACE$estimate
```

```
Weather LAPLACE
 A  3 - dimensional discrete Markov Chain defined by the following states:
 cloudy, rain, sunny
 The transition matrix  (by rows)  is defined as follows:
          cloudy       rain     sunny
cloudy 0.3790212 0.29839555 0.3225832
rain   0.4871203 0.34614892 0.1667307
sunny  0.2407579 0.08646547 0.6727766
```

Both MLE and Laplace approach are based on the `createSequenceMatrix` functions that converts a data (character) sequence into a contingency table, showing the $(X_t = i, X_{t+1} = j)$ distribution within the sample, as code below shows.

```
R> createSequenceMatrix(stringchar = weathersOfDays)
```

```
       cloudy rain sunny
cloudy     47   37    40
rain       38   27    13
sunny      39   14   109
```

An issue occurs when the sample contains only one realization of a state (say X_β) which is located at the end of the data sequence, since it yields to a row of zero (no sample to estimate the conditional distribution of the transition). In this case the estimated transition matrix is corrected assuming $p_{\beta,j} = 1/k$, being k the possible states.

Create sequence matrix can also be used to obtain raw count transition matrices from a given $n * 2$ matrix as the following example shows:

```
R> myMatr<-matrix(c("a","b","b","a","a","b","b","b","b","a","a","a","b","a"),ncol=2)
R> createSequenceMatrix(stringchar = myMatr,toRowProbs = TRUE)

          a         b
a 0.6666667 0.3333333
b 0.5000000 0.5000000
```

A bootstrap estimation approach has been developed within the package in order to provide an indication of the variability of $\hat{p}_{ij}$ estimates. The bootstrap approach implemented within the **markovchain** package follows these steps:

1. bootstrap the data sequences following the conditional distributions of states estimated from the original one. The default bootstrap samples is 10, as specified in `nboot` parameter of `markovchainFit` function.
2. apply MLE estimation on bootstrapped data sequences that are saved in `bootStrapSamples` slot of the returned list.
3. the ${p^{BOOTSTRAP}}_{ij}$ is the average of all ${p^{MLE}}_{ij}$ across the `bootStrapSamples` list, normalized by row. A `standardError` of ${p^{\hat{MLE}}}_{ij}$ estimate is provided as well.

```
R> weatherFittedBOOT <- markovchainFit(data = weathersOfDays,
+                                      method = "bootstrap", nboot = 20)
R> weatherFittedBOOT$estimate

BootStrap Estimate
 A  3 - dimensional discrete Markov Chain defined by the following states:
 cloudy, rain, sunny
 The transition matrix  (by rows)  is defined as follows:
          cloudy       rain     sunny
cloudy 0.3884175 0.29402251 0.3175600
rain   0.4723063 0.35580253 0.1718912
sunny  0.2555405 0.07612772 0.6683318

R> weatherFittedBOOT$standardError

            cloudy        rain       sunny
cloudy 0.007832050 0.012821710 0.011835006
rain   0.010778315 0.009417920 0.009383170
sunny  0.008420141 0.005095549 0.008787744
```

The bootstrapping process can be done in parallel thanks to **RcppParallel** package (Allaire *et al.* 2016). Parallelized implementation is definitively suggested when the data sample size or the required number of bootstrap runs is high.

```
R> weatherFittedBOOTParallel <- markovchainFit(data = weathersOfDays,
+                                      method = "bootstrap", nboot = 20,
+                                      parallel = TRUE)
R> weatherFittedBOOTParallel$estimate
R> weatherFittedBOOTParallel$standardError
```

The parallel bootstrapping uses all the available cores on a machine by default. However, it is also possible to tune the number of threads used. Note that this should be done in R before calling the `markovchainFit` function. For example, the following code will set the number of threads to 4.

```
R> RcppParallel::setNumThreads(2)
```

For more details, please refer to **RcppParallel** web site.

For all the fitting methods, the `logLikelihood` (Skuriat-Olechnowska 2005) denoted in Equation 15 is provided.

$$LLH = \sum_{i,j} n_{ij} * log(p_{ij}) \tag{15}$$

where n_{ij} is the entry of the frequency matrix and p_{ij} is the entry of the transition probability matrix.

```
R> weatherFittedMLE$logLikelihood

[1] -347.8699

R> weatherFittedBOOT$logLikelihood

[1] -348.1088
```

Confidence matrices of estimated parameters (parametric for MLE, non - parametric for BootStrap) are available as well. The `confidenceInterval` is provided with the two matrices: `lowerEndpointMatrix` and `upperEndpointMatrix`. The confidence level (CL) is 0.95 by default and can be given as an argument of the function `markovchainFit`. This is used to obtain the standard score (z-score). Equations 16 and 17 (Skuriat-Olechnowska 2005) show the `confidenceInterval` of a fitting. Note that each entry of the matrices is bounded between 0 and 1.

$$LowerEndpoint_{ij} = p_{ij} - zscore(CL) * SE_{ij} \tag{16}$$

$$UpperEndpoint_{ij} = p_{ij} + zscore(CL) * SE_{ij} \tag{17}$$

```
R> weatherFittedMLE$confidenceInterval

NULL

R> weatherFittedBOOT$confidenceInterval

$confidenceLevel
[1] 0.95

$lowerEndpointMatrix
          cloudy       rain     sunny
cloudy 0.3755349 0.27293267 0.2980931
rain   0.4545775 0.34031143 0.1564572
sunny  0.2416906 0.06774629 0.6538772

$upperEndpointMatrix
          cloudy       rain     sunny
cloudy 0.4013001 0.31511234 0.3370269
rain   0.4900350 0.37129363 0.1873251
sunny  0.2693904 0.08450916 0.6827863
```

A special function, `multinomialConfidenceIntervals`, has been written in order to obtain multinomial wise confidence intervals. The code has been based on and Rcpp translation of package's **MultinomialCI** functions Villacorta (2012) that were themselves based on the Sison and Glaz (1995) paper.

```
R> multinomialConfidenceIntervals(transitionMatrix =
+          weatherFittedMLE$estimate@transitionMatrix,
+          countsTransitionMatrix = createSequenceMatrix(weathersOfDays))

$confidenceLevel
[1] 0.95

$lowerEndpointMatrix
          cloudy       rain      sunny
cloudy 0.2903226 0.20967742 0.23387097
rain   0.3846154 0.24358974 0.06410256
sunny  0.1728395 0.01851852 0.60493827

$upperEndpointMatrix
          cloudy      rain     sunny
cloudy 0.4798274 0.3991822 0.4233758
rain   0.6130492 0.4720236 0.2925364
sunny  0.3155597 0.1612387 0.7476585
```

The functions for fitting DTMC have mostly been rewritten in C++ using **Rcpp** Eddelbuettel (2013) since version 0.2.

Is is also possible to fit a DTMC object from `matrix` or `data.frame` objects as shown in following code.

```
R> data(holson)
R> singleMc<-markovchainFit(data=holson[,2:12],name="holson")
```

The same applies for `markovchainList`.

```
R> mcListFit<-markovchainListFit(data=holson[,2:6],name="holson")
R> mcListFit$estimate

holson  list of Markov chain(s)
Markovchain  1
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  2
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  3
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  4
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  5
```

```
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  6
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  7
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  8
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  9
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  10
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
```

```
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  11
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  12
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  13
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.5000000 0.5000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  14
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  15
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  16
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 0.7500000 0.2500000 0.0000000

Markovchain  17
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  18
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  19
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  20
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5
```

```
Markovchain  21
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  22
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  23
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  24
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  25
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  26
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.7500000 0.2500000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  27
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  28
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  29
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  30
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  31
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  32
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  33
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  34
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  35
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  36
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  37
Unnamed Markov chain
```

```
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 0 1
2 0 1

Markovchain  38
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  39
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  40
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  41
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  42
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  43
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  44
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  45
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  46
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  47
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  48
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
```

```
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  49
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  50
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  51
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  52
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  53
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  54
```

```
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  55
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  56
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  57
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  58
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  59
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  60
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  61
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  62
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  63
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  64
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  65
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  66
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  67
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  68
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  69
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  70
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  71
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  72
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  73
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  74
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  75
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  76
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  77
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  78
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  79
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  80
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  81
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
```

```
1 0.8 0.2
2 0.5 0.5

Markovchain  82
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  83
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  84
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  85
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  86
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  87
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  88
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6666667 0.3333333
2 0.0000000 0.5000000 0.5000000
3 0.3333333 0.3333333 0.3333333

Markovchain  89
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  90
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  91
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  92
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  93
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  94
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  95
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  96
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  97
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5
```

```
Markovchain  98
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.75 0.25
2 1.00 0.00

Markovchain  99
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  100
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  101
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  102
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  103
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  104
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  105
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  106
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  107
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  108
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5
```

```
Markovchain  109
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  110
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  111
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  112
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  113
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.7500000 0.2500000 0.0000000
3 1.0000000 0.0000000 0.0000000
```

```
Markovchain  114
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  115
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  116
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  117
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  118
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  119
Unnamed Markov chain
```

```
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.6666667 0.3333333 0.0000000
3 0.0000000 1.0000000 0.0000000

Markovchain  120
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  121
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  122
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 2
 The transition matrix  (by rows)  is defined as follows:
  2
2 1

Markovchain  123
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  124
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
```

```
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  125
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  126
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  127
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  128
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  129
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  130
```

```
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  131
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  132
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.8 0.2
3 0.5 0.5

Markovchain  133
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.7500000 0.2500000 0.0000000
2 0.3333333 0.3333333 0.3333333
3 1.0000000 0.0000000 0.0000000

Markovchain  134
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  135
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  136
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  137
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  138
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.6666667 0.0000000
2 0.0000000 0.5000000 0.5000000
3 0.3333333 0.3333333 0.3333333

Markovchain  139
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 0 0 1
2 0 0 1
3 0 0 1

Markovchain  140
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
```

```
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 0.6000000 0.4000000 0.0000000

Markovchain  141
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  142
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  143
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  144
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  145
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  146
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  147
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  148
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  149
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  150
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 0.2000000 0.4000000 0.4000000

Markovchain  151
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  152
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  153
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  154
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  155
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
```

```
3 0.6000000 0.4000000 0.0000000

Markovchain  156
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  157
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  158
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  159
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  160
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  161
Unnamed Markov chain
```

```
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  162
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  163
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  164
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  165
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  166
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
```

```
1 0.5 0.5
3 1.0 0.0

Markovchain  167
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  168
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  169
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  170
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  171
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  172
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  173
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  174
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  175
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  176
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  177
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  178
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  179
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  180
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  181
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  182
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
```

```
2 1 0

Markovchain  183
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  184
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 0.2 0.8

Markovchain  185
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
     2    3
2 0.00 1.00
3 0.75 0.25

Markovchain  186
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 0 1
3 0 1

Markovchain  187
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0
```

```
Markovchain  188
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  189
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  190
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 0.2 0.8

Markovchain  191
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 0 1
3 0 1

Markovchain  192
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  193
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  194
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  195
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  196
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  197
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  198
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5
```

```
Markovchain  199
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  200
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  201
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  202
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  203
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  204
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  205
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  206
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  207
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  208
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.75 0.25
2 1.00 0.00

Markovchain  209
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0
```

```
Markovchain  210
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  211
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  212
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  213
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  214
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  215
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  216
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  217
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  218
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  219
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  220
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  221
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
```

```
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  222
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  223
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  224
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  225
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  226
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  227
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  228
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  229
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  230
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  231
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  232
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
```

```
1 0.5 0.5
3 1.0 0.0

Markovchain  233
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  234
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  235
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  236
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  237
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.2000000 0.6000000
2 0.3333333 0.3333333 0.3333333
```

```
3 0.3333333 0.3333333 0.3333333

Markovchain  238
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  239
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  240
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  241
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  242
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  243
```

```
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  244
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  245
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  246
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  247
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  248
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
```

```
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  249
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  250
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  251
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  252
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 0.0 1.0

Markovchain  253
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333
```

```
Markovchain  254
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  255
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  256
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.6000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  257
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  258
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.4000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333
```

```
Markovchain  259
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  260
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  261
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  262
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  263
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5
```

```
Markovchain  264
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  265
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 0.8000000 0.2000000 0.0000000

Markovchain  266
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  267
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  268
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 0.6666667 0.3333333
2 0.5000000 0.5000000
```

```
Markovchain  269
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  270
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  271
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  272
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  273
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  274
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  275
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.7500000 0.2500000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  276
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  277
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  278
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.0 1.0
3 0.5 0.5

Markovchain  279
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  280
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  281
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  282
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  283
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  284
```

```
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  285
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  286
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.6666667 0.0000000
3 0.0000000 1.0000000 0.0000000

Markovchain  287
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  288
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  289
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  290
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  291
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  292
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 0 1
2 0 1

Markovchain  293
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  294
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
```

```
2 0.5 0.5

Markovchain  295
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 1.0000000 0.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  296
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
     2    3
2 1.00 0.00
3 0.75 0.25

Markovchain  297
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  298
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  299
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
```

```
2 1 0

Markovchain  300
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  301
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  302
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  303
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  304
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  305
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  306
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  307
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  308
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2500000 0.7500000 0.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  309
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  310
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  311
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  312
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  313
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 0 1
3 0 1

Markovchain  314
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 0.6 0.4

Markovchain  315
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
```

```
3 1.0000000 0.0000000 0.0000000

Markovchain  316
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  317
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  318
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  319
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  320
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  321
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
```

```
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  322
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  323
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  324
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 0 1
3 0 1

Markovchain  325
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  326
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
```

```
  3
3 1

Markovchain  327
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  328
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  329
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  330
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  331
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  332
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  333
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  334
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  335
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 0 1
2 1 0

Markovchain  336
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2500000 0.7500000
2 0.0000000 1.0000000 0.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  337
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.3333333 0.3333333 0.3333333
2 0.5000000 0.5000000 0.0000000
3 0.0000000 0.6666667 0.3333333

Markovchain  338
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2 3
1 0.0000000 1.0000000 0
2 0.6666667 0.3333333 0
3 1.0000000 0.0000000 0

Markovchain  339
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  340
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  341
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  342
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
```

```
3 1.0 0.0

Markovchain  343
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  344
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  345
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  346
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  347
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  348
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 0 1
3 0 1

Markovchain  349
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  350
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  351
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  352
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  353
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 0 1
```

```
2 0 1

Markovchain  354
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 0.4 0.6

Markovchain  355
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 1.0000000 0.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  356
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
          2         3
2 1.0000000 0.0000000
3 0.6666667 0.3333333

Markovchain  357
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.2500000 0.7500000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  358
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
```

```
2 1 0

Markovchain  359
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  360
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  361
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 0.2 0.8

Markovchain  362
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 0.2500000 0.7500000 0.0000000

Markovchain  363
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 1.0000000 0.0000000
2 0.6666667 0.3333333
```

```
Markovchain  364
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  365
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  366
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  367
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 1.0 0.0

Markovchain  368
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  369
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  370
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  371
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  372
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.4000000 0.4000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  373
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
    1   2 3
1 0.5 0.5 0
2 1.0 0.0 0
3 1.0 0.0 0

Markovchain  374
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
```

```
1 0.25 0.75
2 0.00 1.00

Markovchain  375
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.00 1.00
2 0.75 0.25

Markovchain  376
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.3333333 0.6666667
2 0.5000000 0.5000000 0.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  377
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  378
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  379
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 0.4000000 0.4000000 0.2000000

Markovchain  380
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  381
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  382
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  383
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  384
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  385
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  386
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  387
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  388
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  389
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0
```

```
Markovchain  390
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  391
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 0.2 0.8

Markovchain  392
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  393
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  394
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  395
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  396
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  397
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  398
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  399
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  400
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 2
 The transition matrix  (by rows)  is defined as follows:
  2
2 1
```

```
Markovchain  401
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  402
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  403
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  404
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  405
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
```

```
3 0.3333333 0.3333333 0.3333333

Markovchain  406
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  407
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  408
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 0.0 1.0

Markovchain  409
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.5000000 0.5000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  410
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1   2   3
1 1 0.0 0.0
2 0 1.0 0.0
```

```
3 0 0.5 0.5

Markovchain  411
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  412
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  413
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  414
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.6000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  415
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 0 1 0
2 0 1 0
3 0 0 1
```

```
Markovchain  416
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 1 0
3 1 0

Markovchain  417
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.2 0.8
3 0.5 0.5

Markovchain  418
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  419
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  420
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0
```

```
Markovchain  421
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  422
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  423
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.75 0.25
2 0.00 1.00

Markovchain  424
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  425
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.5000000 0.5000000
3 0.3333333 0.3333333 0.3333333

Markovchain  426
Unnamed Markov chain
```

```
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  427
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  428
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  429
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  430
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  431
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  432
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  433
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  434
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  435
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  436
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  437
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  438
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  439
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  440
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  441
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  442
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  443
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  444
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  445
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  446
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  447
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  448
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  449
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  450
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  451
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  452
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  453
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  454
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  455
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  456
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  457
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  458
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.5000000 0.5000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  459
```

```
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  460
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  461
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  462
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  463
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  464
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
```

```
  1 2
1 1 0
2 1 0

Markovchain  465
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  466
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  467
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 0.6666667 0.3333333
2 1.0000000 0.0000000

Markovchain  468
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 0 1

Markovchain  469
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0
```

```
Markovchain  470
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  471
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  472
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  473
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  474
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2500000 0.5000000 0.2500000
2 0.0000000 1.0000000 0.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  475
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  476
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  477
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  478
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  479
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  480
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  481
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  482
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  483
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  484
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  485
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  486
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
```

```
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.00 1.00
2 0.25 0.75

Markovchain  487
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 1.0000000 0.0000000
2 0.0000000 0.5000000 0.5000000
3 0.3333333 0.3333333 0.3333333

Markovchain  488
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  489
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  490
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  491
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
```

```
  1 2
1 1 0
2 1 0

Markovchain  492
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  493
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  494
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  495
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.6000000 0.2000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  496
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1         2         3
1 0 0.6666667 0.3333333
```

```
2 0 0.0000000 1.0000000
3 0 0.0000000 1.0000000

Markovchain  497
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  498
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  499
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.5000000 0.5000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  500
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  501
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  502
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  503
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  504
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  505
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  506
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  507
Unnamed Markov chain
```

```
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.6000000 0.2000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  508
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  509
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  510
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  511
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  512
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
```

```
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  513
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  514
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  515
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  516
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  517
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.8 0.2
3 0.5 0.5

Markovchain  518
```

```
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
  1 3
1 0 1
3 0 1

Markovchain  519
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  520
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  521
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  522
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  523
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
```

```
  1 2
1 1 0
2 1 0

Markovchain  524
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  525
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 0.0000000 0.5000000 0.5000000

Markovchain  526
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
    1   2 3
1 1.0 0.0 0
2 1.0 0.0 0
3 0.5 0.5 0

Markovchain  527
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.75 0.25
2 1.00 0.00

Markovchain  528
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
```

```
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 0 1
2 0 1

Markovchain  529
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  530
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  531
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  532
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  533
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  534
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  535
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  536
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 0.8000000 0.2000000 0.0000000

Markovchain  537
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  538
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
```

```
3 1.0 0.0

Markovchain  539
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  540
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  541
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  542
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  543
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  544
Unnamed Markov chain
```

```
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  545
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  546
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  547
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  548
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  549
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  550
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  551
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  552
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  553
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.4000000 0.4000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  554
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0
```

```
Markovchain  555
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  556
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  557
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  558
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  559
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  560
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  561
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  562
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  563
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  564
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  565
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  566
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  567
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  568
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 0.0 1.0

Markovchain  569
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  570
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  571
```

```
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  572
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  573
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  574
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  575
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  576
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
```

```
  1 2
1 1 0
2 1 0

Markovchain  577
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  578
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  579
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  580
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  581
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
```

```
3 0.3333333 0.3333333 0.3333333

Markovchain  582
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  583
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  584
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  585
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  586
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  587
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  588
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.6000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  589
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  590
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  591
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.0000000 0.0000000 1.0000000
3 0.5000000 0.2500000 0.2500000

Markovchain  592
```

```
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  593
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  594
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  595
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  596
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  597
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  598
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  599
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  600
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  601
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  602
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  603
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  604
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  605
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  606
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  607
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  608
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  609
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  610
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  611
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  612
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  613
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  614
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  615
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  616
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 1.0 0.0

Markovchain  617
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.2500000 0.5000000 0.2500000
3 0.0000000 1.0000000 0.0000000

Markovchain  618
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  619
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  620
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  621
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  622
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 1.00 0.00
2 0.75 0.25

Markovchain  623
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  624
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  625
Unnamed Markov chain
```

```
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  626
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  627
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  628
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  629
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  630
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
```

```
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  631
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  632
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  633
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  634
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  635
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  636
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.4000000 0.4000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  637
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  638
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  639
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 1 0
3 1 0

Markovchain  640
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  641
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  642
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  643
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  644
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.2000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  645
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  646
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
```

```
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  647
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  648
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  649
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  650
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  651
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
```

```
3 1.0000000 0.0000000 0.0000000

Markovchain  652
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  653
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  654
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.2 0.8
3 0.5 0.5

Markovchain  655
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  656
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5
```

```
Markovchain  657
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  658
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  659
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  660
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  661
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 1.0000000 0.0000000
2 0.6666667 0.3333333

Markovchain  662
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
```

```
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  663
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  664
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  665
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  666
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  667
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  668
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  669
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  670
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  671
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  672
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0
```

```
Markovchain  673
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  674
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  675
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  676
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  677
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  678
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
```

```
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  679
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  680
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  681
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  682
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  683
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  684
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  685
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  686
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  687
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  688
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  689
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  690
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  691
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  692
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  693
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  694
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2500000 0.5000000 0.2500000
```

```
2 0.0000000 1.0000000 0.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  695
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 0 1 0
2 0 1 0
3 1 0 0

Markovchain  696
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  697
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  698
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  699
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  700
```

```
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  701
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  702
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  703
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  704
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  705
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  706
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  707
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  708
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  709
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  710
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  711
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  712
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  713
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  714
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  715
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  716
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  717
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  718
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  719
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  720
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  721
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  722
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  723
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  724
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.4000000 0.4000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  725
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  726
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  727
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  728
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  729
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  730
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  731
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  732
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2500000 0.7500000 0.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  733
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
```

```
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  734
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  735
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  736
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  737
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  738
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  739
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  740
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  741
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  742
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  743
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
```

```
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  744
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 0.2 0.8

Markovchain  745
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
  2 3
2 1 0
3 1 0

Markovchain  746
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  747
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  748
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  749
```

```
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  750
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  751
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  752
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  753
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  754
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
```

```
1 0.2 0.8
2 0.5 0.5

Markovchain  755
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  756
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  757
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  758
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  759
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  760
Unnamed Markov chain
```

```
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  761
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 0.6666667 0.3333333
2 0.5000000 0.5000000

Markovchain  762
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  763
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  764
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 0.5000000 0.5000000
2 0.6666667 0.3333333

Markovchain  765
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
```

```
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  766
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  767
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  768
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  769
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  770
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
```

```
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  771
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.0000000 1.0000000 0.0000000
3 0.7500000 0.2500000 0.0000000

Markovchain  772
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 0.6666667 0.3333333
2 0.5000000 0.5000000

Markovchain  773
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  774
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  775
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
```

```
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  776
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  777
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  778
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 0 1
2 0 1

Markovchain  779
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.6 0.4
3 0.5 0.5

Markovchain  780
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
          2         3
2 0.6666667 0.3333333
```

```
3 0.5000000 0.5000000

Markovchain  781
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  782
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  783
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  784
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  785
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  786
```

```
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  787
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  788
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  789
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  790
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  791
```

```
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  792
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  793
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  794
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  795
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  796
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  797
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  798
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  799
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  800
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  801
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
```

```
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  802
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  803
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  804
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  805
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  806
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.4000000 0.4000000
```

```
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  807
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  808
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  809
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  810
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.0 1.0
3 0.5 0.5

Markovchain  811
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0
```

```
Markovchain  812
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  813
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  814
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  815
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  816
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  817
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
```

```
1 1

Markovchain  818
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  819
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  820
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  821
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  822
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  823
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  824
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  825
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  826
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  827
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  828
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  829
```

```
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  830
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  831
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  832
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  833
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  834
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.3333333 0.6666667
2 1.0000000 0.0000000 0.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  835
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  836
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  837
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 0.2000000 0.8000000 0.0000000

Markovchain  838
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  839
Unnamed Markov chain
```

```
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  840
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  841
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2000000 0.6000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  842
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  843
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  844
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
```

```
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  845
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  846
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  847
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  848
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  849
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  850
```

```
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  851
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  852
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  853
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  854
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  855
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.8000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  856
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  857
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  858
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  859
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  860
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  861
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  862
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  863
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  864
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  865
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  866
```

```
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  867
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  868
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  869
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  870
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 1.0 0.0

Markovchain  871
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
```

```
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  872
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  873
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  874
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  875
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  876
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  877
```

```
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  878
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  879
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  880
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  881
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  882
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.75 0.25
2 1.00 0.00

Markovchain  883
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  884
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  885
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  886
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.4000000 0.4000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  887
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
```

```
1 1 0 0
2 1 0 0
3 0 1 0

Markovchain  888
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  889
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  890
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  891
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
          1         2
1 0.3333333 0.6666667
2 0.0000000 1.0000000

Markovchain  892
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0
```

```
Markovchain  893
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.2 0.8
3 0.5 0.5

Markovchain  894
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 1.0000000 0.0000000
2 0.3333333 0.3333333 0.3333333
3 0.7500000 0.2500000 0.0000000

Markovchain  895
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  896
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 3
 The transition matrix  (by rows)  is defined as follows:
  3
3 1

Markovchain  897
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0
```

```
Markovchain  898
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  899
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  900
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  901
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  902
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  903
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
```

```
1 0.5 0.5
3 1.0 0.0

Markovchain  904
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  905
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  906
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  907
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.5 0.5
2 0.8 0.2

Markovchain  908
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.0000000 1.0000000
2 0.0000000 0.0000000 1.0000000
```

```
3 0.3333333 0.3333333 0.3333333

Markovchain  909
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 0.4000000 0.6000000 0.0000000

Markovchain  910
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  911
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  912
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  913
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0
```

```
Markovchain  914
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  915
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  916
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  917
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  918
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.2500000 0.0000000 0.7500000
2 0.0000000 0.0000000 1.0000000
3 0.3333333 0.3333333 0.3333333

Markovchain  919
Unnamed Markov chain
```

```
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 1.0000000 0.0000000 0.0000000
2 0.3333333 0.3333333 0.3333333
3 0.7500000 0.2500000 0.0000000

Markovchain  920
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  921
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.6000000 0.2000000 0.2000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  922
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 0 0 1
2 0 1 0
3 0 1 0

Markovchain  923
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.5000000 0.5000000 0.0000000
3 0.0000000 1.0000000 0.0000000
```

```
Markovchain  924
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  925
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  926
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  927
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  928
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
     1    2
1 0.75 0.25
2 1.00 0.00

Markovchain  929
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
```

```
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  930
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  931
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  932
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  933
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  934
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0
```

```
Markovchain  935
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  936
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  937
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 1.0000000 0.0000000
2 0.0000000 0.2500000 0.7500000
3 0.3333333 0.3333333 0.3333333

Markovchain  938
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  939
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  940
Unnamed Markov chain
```

```
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  941
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 0.3333333 0.3333333 0.3333333
3 1.0000000 0.0000000 0.0000000

Markovchain  942
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
  1 2 3
1 1 0 0
2 1 0 0
3 1 0 0

Markovchain  943
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.6 0.4
2 0.5 0.5

Markovchain  944
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  945
```

```
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  946
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  947
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  948
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  949
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  950
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  951
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.2 0.8
2 0.5 0.5

Markovchain  952
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  953
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.4 0.6
2 0.5 0.5

Markovchain  954
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  955
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.0 1.0
2 0.5 0.5

Markovchain  956
Unnamed Markov chain
```

```
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.0 1.0
3 0.5 0.5

Markovchain  957
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  958
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  959
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  960
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  961
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
```

```
1 1 0
2 1 0

Markovchain  962
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  963
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  964
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  965
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  966
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  967
Unnamed Markov chain
```

```
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  968
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.6000000 0.4000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  969
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  970
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  971
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  972
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
```

```
    1   2
1 0.8 0.2
2 0.5 0.5

Markovchain  973
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
1 1 0
2 1 0

Markovchain  974
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  975
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.4000000 0.6000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  976
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 0.6666667 0.3333333 0.0000000

Markovchain  977
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 2
 The transition matrix  (by rows)  is defined as follows:
  1 2
```

```
1 1 0
2 1 0

Markovchain  978
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  979
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 2, 3
 The transition matrix  (by rows)  is defined as follows:
    2   3
2 0.5 0.5
3 0.4 0.6

Markovchain  980
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  981
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  982
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1
```

```
Markovchain  983
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  984
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  985
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  986
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5

Markovchain  987
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.5 0.5
3 1.0 0.0

Markovchain  988
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
```

```
  1
1 1

Markovchain  989
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  990
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  991
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  992
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  993
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  994
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
```

```
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  995
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  996
Unnamed Markov chain
 A  1 - dimensional discrete Markov Chain defined by the following states:
 1
 The transition matrix  (by rows)  is defined as follows:
  1
1 1

Markovchain  997
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.0000000 0.2000000 0.8000000
2 0.3333333 0.3333333 0.3333333
3 0.3333333 0.3333333 0.3333333

Markovchain  998
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 1, 2, 3
 The transition matrix  (by rows)  is defined as follows:
          1         2         3
1 0.3333333 0.3333333 0.3333333
2 1.0000000 0.0000000 0.0000000
3 1.0000000 0.0000000 0.0000000

Markovchain  999
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
```

```
 1, 3
 The transition matrix  (by rows)  is defined as follows:
    1   3
1 0.0 1.0
3 0.5 0.5
```

Finally, given a `list` object, it is possible to fit a `markovchain` object or to obtain the raw transition matrix.

```
R> c1<-c("a","b","a","a","c","c","a")
R> c2<-c("b")
R> c3<-c("c","a","a","c")
R> c4<-c("b","a","b","a","a","c","b")
R> c5<-c("a","a","c","b")
R> c6<-c("b","c","b","c","a")
R> mylist<-list(c1,c2,c3,c4,c5,c6)
R> mylistMc<-markovchainFit(data=mylist)
R> mylistMc

$estimate
MLE Fit
 A  3 - dimensional discrete Markov Chain defined by the following states:
 a, b, c
 The transition matrix  (by rows)  is defined as follows:
          a         b         c
a 0.4000000 0.2000000 0.4000000
b 0.6000000 0.0000000 0.4000000
c 0.4285714 0.4285714 0.1428571


$standardError
          a         b         c
a 0.2000000 0.1414214 0.2000000
b 0.3464102 0.0000000 0.2828427
c 0.2474358 0.2474358 0.1428571

$confidenceLevel
[1] 0.95

$lowerEndpointMatrix
           a          b          c
a 0.07102927 0.00000000 0.07102927
b 0.03020599 0.00000000 0.00000000
c 0.02157571 0.02157571 0.00000000

$upperEndpointMatrix
          a         b         c
```

```
a 0.7289707 0.4326174 0.7289707
b 1.0000000 0.0000000 0.8652349
c 0.8355672 0.8355672 0.3778362
```

The same works for `markovchainFitList`.

```
R> markovchainListFit(data=mylist)

$estimate
  list of Markov chain(s)
Markovchain  1
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 a, b, c
 The transition matrix  (by rows)  is defined as follows:
    a   b   c
a 0.5 0.5 0.0
b 0.5 0.0 0.5
c 1.0 0.0 0.0

Markovchain  2
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 a, b, c
 The transition matrix  (by rows)  is defined as follows:
          a         b         c
a 0.3333333 0.3333333 0.3333333
b 1.0000000 0.0000000 0.0000000
c 0.0000000 1.0000000 0.0000000

Markovchain  3
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 a, b, c
 The transition matrix  (by rows)  is defined as follows:
    a b   c
a 0.5 0 0.5
b 0.5 0 0.5
c 0.0 1 0.0

Markovchain  4
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 a, c
 The transition matrix  (by rows)  is defined as follows:
    a   c
a 0.5 0.5
```

```
c 1.0 0.0

Markovchain  5
Unnamed Markov chain
 A  2 - dimensional discrete Markov Chain defined by the following states:
 a, c
 The transition matrix  (by rows)  is defined as follows:
  a c
a 0 1
c 0 1

Markovchain  6
Unnamed Markov chain
 A  3 - dimensional discrete Markov Chain defined by the following states:
 a, b, c
 The transition matrix  (by rows)  is defined as follows:
          a         b         c
a 0.3333333 0.3333333 0.3333333
b 0.3333333 0.3333333 0.3333333
c 0.5000000 0.5000000 0.0000000
```

5.3. Prediction

The n-step forward predictions can be obtained using the `predict` methods explicitly written for `markovchain` and `markovchainList` objects. The prediction is the mode of the conditional distribution of X_{t+1} given $X_t = s_j$, being s_j the last realization of the DTMC (homogeneous or non-homogeneous).

Predicting from a markovchain object

The 3-days forward predictions from `markovchain` object can be generated as follows, assuming that the last two days were respectively "cloudy" and "sunny".

```
R> predict(object = weatherFittedMLE$estimate, newdata = c("cloudy", "sunny"),
+          n.ahead = 3)

[1] "sunny" "sunny" "sunny"
```

Predicting from a markovchainList object

Given an initial two year Healty status, the 5-year ahead prediction of any CCRC guest is

```
R> predict(mcCCRC, newdata = c("H", "H"), n.ahead = 5)

[1] "H" "D" "D"
```

The prediction has stopped at time sequence since the underlying non-homogeneous Markov chain has a length of four. In order to continue five years ahead, the `continue=TRUE` parameter setting makes the `predict` method keeping to use the last `markovchain` in the sequence list.

```
R> predict(mcCCRC, newdata = c("H", "H"), n.ahead = 5, continue = TRUE)

[1] "H" "D" "D" "D" "D"
```

5.4. Statistical Tests

In this section, we describe the statistical tests: assessing the Markov property (`verifyMarkovProperty`), the order (`assessOrder`), the statinarity (`assessStationarity`) of a Markov chain sequence, and the divergence test for empirically estimated transition matrices (`divergenceTest`). For the first three tests, we use the χ^2 statistics (Anderson and Goodman 1957; Skuriat-Olechnowska 2005). As an example, we use the following sequence.

```
R> sequence<-c("a", "b", "a", "a", "a", "a", "b", "a", "b", "a",
+              "b", "a", "a", "b", "b", "b", "a")
```

Assessing the Markov property of a Markov chain sequence

The `verifyMarkovProperty` function invloves verifying whether the given chain holds the Markov property for which the future state is independent of the past states and depends only on the current state.

$$\Pr\{X_{t+1} = m | X_t = j, X_{t-1} = i\} = \Pr\{X_{t+1} = m | X_t = j\} \tag{18}$$

We first construct a contingency table in which the columns are the frequency of past→present→future (PPF) (or state transition sequence (STS)), present→future (PF) (or two-state occurrences (STO)) and (PF - PPF), and the rows are the possible three state transitions as shown in Table 6 for each possible present to future transition.

Transitions (p→p→f)	PPF (p→p→f)	PF (p→f)	PF - PPF
a → a → b	2 (a → a → b)	4 (a→a)	2
b → a → b	2 (b → a → b)	5 (b→a)	3

Table 6: Contingency table for Markov property of the transition from the present state a to the future state b (a → b).

Using the table, the function performs the χ^2 test by calling the `chisq.test` function. This test returns a list of the chi-squared value, the degrees of freedom, and the p-value for each possible transition. If the p-value is greater than the given significance level, we cannot reject the hypothesis that the Markov property holds for the specific transition.

```
R> verifyMarkovProperty(sequence)
```

```
$bb
$bb$statistic
  X-squared
1.55307e-31

$bb$parameter
df
 1

$bb$p.value
[1] 1

$bb$method
[1] "Pearson's Chi-squared test with Yates' continuity correction"

$bb$data.name
[1] "table"

$bb$observed
  SSO TSO-SSO
a   1       4
b   1       1

$bb$expected
        SSO  TSO-SSO
a 1.4285714 3.571429
b 0.5714286 1.428571

$bb$residuals
         SSO    TSO-SSO
a -0.3585686  0.2267787
b  0.5669467 -0.3585686

$bb$stdres
         SSO    TSO-SSO
a -0.7937254  0.7937254
b  0.7937254 -0.7937254

$bb$table
  SSO TSO
a   1   5
b   1   2
```

Assessing the order of a Markov chain sequence

The `assessOrder` function checks whether the given chain is of first order or of second order. For each possible present state, we construct a contingency table of the frequency of the future state for each past to present state transition as shown in Table 7.

past	present	future a	future b
a	a	2	2
b	a	2	2

Table 7: Contingency table to assess the order for the present state a.

Using the table, the function performs the χ^2 test by calling the `chisq.test` function. This test returns a list of the chi-squared value and the p-value. If the p-value is greater than the given significance level, we cannot reject the hypothesis that the sequence is of first order.

```
R> data(rain)
R> assessOrder(rain$rain)

The assessOrder test statistic is:  26.09575  the Chi-Square d.f. are:  12  the p-value is
$statistic
[1] 26.09575

$p.value
[1] 0.01040395
```

Assessing the stationarity of a Markov chain sequence

The `assessStationarity` function assesses if the transition probabilities of the given chain change over time. To be more specific, the chain is stationary if the following condition meets.

$$p_{ij}(t) = p_{ij} \quad \text{for all} \quad t \tag{19}$$

For each possible state, we construct a contingency table of the estimated transition probabilities over time as shown in Table 8.

time (t)	probability of transition to a	probability of transition to b
1	0	1
2	0	1
.	.	.
.	.	.
.	.	.
16	0.44	0.56

Table 8: Contingency table to assess the stationarity of the state a.

Using the table, the function performs the χ^2 test by calling the `chisq.test` function. This test returns a list of the chi-squared value and the p-value. If the p-value is greater than the given significance level, we cannot reject the hypothesis that the sequence is stationary.

```
R> assessStationarity(rain$rain, 10)
```

```
The assessStationarity test statistic is:  4.181815  the Chi-Square d.f. are:  54  the p-v
$statistic
[1] 4.181815

$p.value
[1] 1
```

Divergence test for empirically estimated transition matrices

In order to test if two empirically estimated transition matrices $m1$ and $m2$ are different, we use the ϕ-divergence test (Pardo 2005) that is given by

$$T_n^{\phi}(m_1, m_2, mc) = \frac{2n}{\phi''(1)} \sum_{i=1}^{M} \frac{v_{i*}}{n} \sum_{j=1}^{M} m_2(i,j)\phi\left(\frac{m_1(i,j)}{m_2(i,j)}\right) \tag{20}$$

where mc is the markov chain sequence, n is the length of the sequence, M is the number of states, v_{i*} is the number of transitions starting from the state i, and ϕ is

$$\phi(x) = x \log x - x + 1. \tag{21}$$

This test returns a list of the chi-squared value and the p-value.

```
R> mcFit<-markovchainFit(data=sequence,byrow=FALSE)
R> divergenceTest(sequence, mcFit$estimate@transitionMatrix)
```

```
The Divergence test statistic is:  0.6349206  the Chi-Square d.f. are:  2  the p-value is:
$statistic
[1] 0.6349206

$p.value
[1] 0.7279956
```

5.5. Continuous Times Markov Chains

Intro

The **markovchain** package provides functionality for continuous time Markov chains (CTMCs). CTMCs are a generalisation of discrete time Markov chains (DTMCs) in that we allow time to be continuous. We assume a finite state space S (for an infinite state space wouldn't fit in memory). We can think of CTMCs as Markov chains in which state transitions can happen at any time.

More formally, we would like our CTMCs to satisfy the following two properties:

- The Markov property - let $F_{X(s)}$ denote the information about X upto time s. Let $j \in S$ and $s \leq t$. Then, $P(X(t) = j | F_{X(s)}) = P(X(t) = j | X(s))$.

- Time homogenity - $P(X(t) = j|X(s) = k) = P(X(t-s) = j|X(0) = k)$.

If both the above properties are satisfied, it is referred to as a time-homogeneous CTMC. If a transition occurs at time t, then $X(t)$ denotes the new state and $X(t) \neq X(t-)$.

Now, let $X(0) = x$ and let T_x be the time a transition occurs from this state. We are interested in the distribution of T_x. For $s, t \geq 0$, it can be shown that $P(T_x > s+t|T_x > s) = P(T_x > t)$

This is the memory less property that only the exponential random variable exhibits. Therefore, this is the sought distribution, and each state $s \in S$ has an exponential holding parameter $\lambda(s)$. Since $\mathrm{E}T_x = \frac{1}{\lambda(x)}$, higher the rate $\lambda(x)$, smaller the expected time of transitioning out of the state x.

However, specifying this parameter alone for each state would only paint an incomplete picture of our CTMC. To see why, consider a state x that may transition to either state y or z. The holding parameter enables us to predict when a transition may occur if we start off in state x, but tells us nothing about which state will be next.

To this end, we also need transition probabilities associated with the process, defined as follows (for $y \neq x$) - $p_{xy} = P(X(T_s) = y|X(0) = x)$ Note that $\sum_{y \neq x} p_{xy} = 1$. Let Q denote this transition matrix ($Q_{ij} = p_{ij}$). What is key here is that T_x and the state y are independent random variables. Let's define $\lambda(x, y) = \lambda(x)p_{xy}$

We now look at Kolmogorov's backward equation. Let's define $P_{ij}(t) = P(X(t) = j|X(0) = i)$ for $i, j \in S$. The backward equation is given by (it can be proved) $P_{ij}(t) = \delta_{ij}e^{-\lambda(i)t} + \int_0^t \lambda(i)e^{-\lambda(i)t} \sum_{k \neq i} Q_{ik}P_{kj}(t-s)ds$ Basically, the first term is non-zero if and only if $i = j$ and represents the probability that the first transition from state i occurs after time t. This would mean that at t, the state is still i. The second term accounts for any transitions that may occur before time t and denotes the probability that at time t, when the smoke clears, we are in state j.

This equation can be represented compactly as follows $P'(t) = AP(t)$ where A is the *generator* matrix. $A(i, j) = \begin{cases} \lambda(i, j) & \text{if } i \neq j \\ -\lambda(i) & \text{else.} \end{cases}$ Observe that the sum of each row is 0. A CTMC can be completely specified by the generator matrix.

Stationary Distributions

The following theorem guarantees the existence of a unique stationary distribution for CTMCs. Note that $X(t)$ being irreducible and recurrent is the same as $X_n(t)$ being irreducible and recurrent.

Suppose that $X(t)$ is irreducible and recurrent. Then $X(t)$ has an invariant measure η, which is unique up to multiplicative factors. Moreover, for each $k \in S$, we have

$\eta_k = \frac{\pi_k}{\lambda(k)}$

where π is the unique invariant measure of the embedded discrete time Markov chain Xn. Finally, η satisfies

$0 < \eta_j < \infty, \forall j \in S$

and if $\sum_i \eta_i < \infty$ then η can be normalised to get a stationary distribution.

Estimation

Let the data set be $D = \{(s_0, t_0), (s_1, t_1), ..., (s_{N-1}, t_{N-1})\}$ where $N = |D|$. Each s_i is a state from the state space S and during the time $[t_i, t_{i+1}]$ the chain is in state s_i. Let the parameters be represented by $\theta = \{\lambda, P\}$ where λ is the vector of holding parameters for each state and P the transition matrix of the embedded discrete time Markov chain.

Then the probability is given by $Pr(D|\theta) \propto \lambda(s_0)e^{-\lambda(s_0)(t_1-t_0)}Pr(s_1|s_0) \;.\; \lambda(s_1)e^{-\lambda(s_1)(t_2-t_1)}Pr(s_2|s_1) \;...\; \lambda(s_{N-}$

Let $n(j|i)$ denote the number of i-¿j transitions in D, and $n(i)$ the number of times s_i occurs in D. Let $t(s_i)$ denote the total time the chain spends in state s_i.

Then the MLEs are given by $\hat{\lambda(s)} = \frac{n(s)}{t(s)}, \hat{Pr(j|i)} = \frac{n(j|i)}{n(i)}$

Examples

To create a CTMC object, you need to provide a valid generator matrix, say Q. The CTMC object has the following slots - states, generator, byrow, name (look at the documentation object for further details). Consider the following example in which we aim to model the transition of a molecule from the σ state to the σ^* state. When in the former state, if it absorbs sufficient energy, it can make the jump to the latter state and remains there for some time before transitioning back to the original state. Let us model this by a CTMC:

```
R> energyStates <- c("sigma", "sigma_star")
R> byRow <- TRUE
R> gen <- matrix(data = c(-3, 3,
+                         1, -1), nrow = 2,
+                byrow = byRow, dimnames = list(energyStates, energyStates))
R> molecularCTMC <- new("ctmc", states = energyStates,
+                    byrow = byRow, generator = gen,
+                    name = "Molecular Transition Model")
```

To generate random CTMC transitions, we provide an initial distribution of the states. This must be in the same order as the dimnames of the generator. The output can be returned either as a list or a data frame.

```
R> statesDist <- c(0.8, 0.2)
R> rctmc(n = 3, ctmc = molecularCTMC, initDist = statesDist, out.type = "df", include.T0 =

      states              time
1      sigma 0.839708235072804
2 sigma_star 0.932185871656202
3      sigma  2.45310400803475
```

n represents the number of samples to generate. There is an optional argument T for `rctmc`. It represents the time of termination of the simulation. To use this feature, set n to a very high value, say `Inf` (since we do not know the number of transitions before hand) and set T accordingly.

```
R> statesDist <- c(0.8, 0.2)
R> rctmc(n = Inf, ctmc = molecularCTMC, initDist = statesDist, T = 2)
```

```
[[1]]
[1] "sigma"      "sigma_star" "sigma"      "sigma_star" "sigma"
[6] "sigma_star"

[[2]]
[1] 0.0000000 0.1439937 0.3352246 1.2313218 1.2889825 1.4448601
```

To obtain the stationary distribution simply invoke the `steadyStates` function

```
R> steadyStates(molecularCTMC)

     sigma sigma_star
[1,]  0.25       0.75
```

For fitting, use the `ctmcFit` function. It returns the MLE values for the parameters along with the confidence intervals.

```
R> data <- list(c("a", "b", "c", "a", "b", "a", "c", "b", "c"),
+               c(0, 0.8, 2.1, 2.4, 4, 5, 5.9, 8.2, 9))
R> ctmcFit(data)

$estimate
An object of class "ctmc"
Slot "states":
[1] "a" "b" "c"

Slot "byrow":
[1] TRUE

Slot "generator":
           a          b          c
a -0.9090909  0.6060606  0.3030303
b  0.3225806 -0.9677419  0.6451613
c  0.3846154  0.3846154 -0.7692308

Slot "name":
[1] ""



$errors
$errors$dtmcConfidenceInterval
$errors$dtmcConfidenceInterval$confidenceLevel
[1] 0.95

$errors$dtmcConfidenceInterval$lowerEndpointMatrix
  a b c
a 0 0 0
```

```
b 0 0 0
c 0 0 0

$errors$dtmcConfidenceInterval$upperEndpointMatrix
          a b         c
a 0.0000000 1 0.8816179
b 0.8816179 0 1.0000000
c 1.0000000 1 0.0000000


$errors$lambdaConfidenceInterval
$errors$lambdaConfidenceInterval$lowerEndpointVector
[1] 0.04576665 0.04871934 0.00000000

$errors$lambdaConfidenceInterval$upperEndpointVector
[1] 1 1 1
```

One approach to obtain the generator matrix is to apply the `logm` function from the **expm** package on a transition matrix. Numeric issues arise, see Israel, Rosenthal, and Wei (2001). For example, applying the standard `method` ('Higham08') on `mcWeather` raises an error, whilst the alternative method (eigenvalue decomposition) is ok. The following code estimates the generator matrix of the `mcWeather` transition matrix.

```
R> mcWeatherQ <- expm::logm(mcWeather@transitionMatrix,method='Eigen')
R> mcWeatherQ
```

```
           sunny     cloudy       rain
sunny  -0.863221   2.428723  -1.565502
cloudy  4.284592 -20.116312  15.831720
rain   -4.414019  24.175251 -19.761232
```

Therefore, the "half - day" transition probability for mcWeather DTMC is

```
R> mcWeatherHalfDayTM <- expm::expm(mcWeatherQ*.5)
R> mcWeatherHalfDay <- new("markovchain",transitionMatrix=mcWeatherHalfDayTM,name="Half Da
R> mcWeatherHalfDay
```

```
Half Day Weather Transition Matrix
 A  3 - dimensional discrete Markov Chain defined by the following states:
 sunny, cloudy, rain
 The transition matrix  (by rows)  is defined as follows:
            sunny    cloudy       rain
sunny  0.81598647 0.1420068 0.04200677
cloudy 0.21970167 0.4401492 0.34014916
rain   0.07063048 0.5146848 0.41468476
```

5.6. Bayesian Estimation

The **markovchain** package provides functionality for maximum a posteriori (MAP) estimation of the chain parameters (at the time of writing this document, only first order models are supported) by Bayesian inference. It also computes the probability of observing a new data set, given a (different) data set. This vignette provides the mathematical description for the methods employed by the package.

Notation and set-up

The data is denoted by D, the model parameters (transition matrix) by θ. The object of interest is $P(\theta|D)$ (posterior density). $\mathcal{A}$ represents an alphabet class, each of whose members represent a state of the chain. Therefore

$$D = s_0 s_1 ... s_{N-1}, s_t \in \mathcal{A}$$

where N is the length of the data set. Also,

$$\theta = \{p(s|u), s \in \mathcal{A}, u \in \mathcal{A}\}$$

where $\sum_{s \in \mathcal{A}} p(s|u) = 1$ for each $u \in \mathcal{A}$.
Our objective is to find θ which maximises the posterior. That is, if our solution is denoted by $\hat{\theta}$, then

$$\hat{\theta} = \underset{\theta}{argmax} P(\theta|D)$$

where the search space is the set of right stochastic matrices of dimension $|\mathcal{A}|x|\mathcal{A}|$.
$n(u, s)$ denotes the number of times the word us occurs in D and $n(u) = \sum_{s \in \mathcal{A}} n(u, s)$. The hyperparameters are similarly denoted by $\alpha(u, s)$ and $\alpha(u)$ respectively.

Methods

Given D, its likelihood conditioned on the observed initial state in D is given by

$$P(D|\theta) = \prod_{s \in \mathcal{A}} \prod_{u \in \mathcal{A}} p(s|u)^{n(u,s)}$$

Conjugate priors are used to model the prior $P(\theta)$. The reasons are two fold:

1. Exact expressions can be derived for the MAP estimates, expectations and even variances
2. Model order selection/comparison can be implemented easily (available in a future release of the package)

The hyperparameters determine the form of the prior distribution, which is a product of Dirichlet distributions

$$P(\theta) = \prod_{u \in \mathcal{A}} \left\{ \frac{\Gamma(\alpha(u))}{\prod_{s \in \mathcal{A}} \Gamma(\alpha(u,s))} \prod_{s \in \mathcal{A}} p(s|u)^{\alpha(u,s))-1} \right\}$$

where $\Gamma(.)$ is the Gamma function. The hyperparameters are specified using the `hyperparam` argument in the `markovchainFit` function. If this argument is not specified, then a default value of 1 is assigned to each hyperparameter resulting in the prior distribution of each chain parameter to be uniform over $[0, 1]$.

Given the likelihood and the prior as described above, the evidence $P(D)$ is simply given by

$$P(D) = \int P(D|\theta)P(\theta)d\theta$$

which simplifies to

$$P(D) = \prod_{u \in \mathcal{A}} \left\{ \frac{\Gamma(\alpha(u))}{\prod_{s \in \mathcal{A}} \Gamma(\alpha(u,s))} \frac{\prod_{s \in \mathcal{A}} \Gamma(n(u,s) + \alpha(u,s))}{\Gamma(\alpha(u) + n(u))} \right\}$$

Using Bayes' theorem, the posterior now becomes (thanks to the choice of conjugate priors)

$$P(\theta|D) = \prod_{u \in \mathcal{A}} \left\{ \frac{\Gamma(n(u) + \alpha(u))}{\prod_{s \in \mathcal{A}} \Gamma(n(u,s) + \alpha(u,s))} \prod_{s \in \mathcal{A}} p(s|u)^{n(u,s)+\alpha(u,s))-1} \right\}$$

Since this is again a product of Dirichlet distributions, the marginalised distribution of a particular parameter $P(s|u)$ of our chain is given by

$$P(s|u) \sim Beta(n(u,s) + \alpha(u,s), n(u) + \alpha(u) - n(u,s) - \alpha(u,s))$$

Thus, the MAP estimate $\hat{\theta}$ is given by

$$\hat{\theta} = \left\{ \frac{n(u,s) + \alpha(u,s) - 1}{n(u) + \alpha(u) - |\mathcal{A}|}, s \in \mathcal{A}, u \in \mathcal{A} \right\}$$

The function also returns the expected value, given by

$$\mathrm{E}_{\mathrm{post}} p(s|u) = \left\{ \frac{n(u,s) + \alpha(u,s)}{n(u) + \alpha(u)}, s \in \mathcal{A}, u \in \mathcal{A} \right\}$$

The variance is given by

$$\mathrm{Var}_{\mathrm{post}} p(s|u) = \frac{n(u,s) + \alpha(u,s)}{(n(u) + \alpha(u))^2} \frac{n(u) + \alpha(u) - n(u,s) - \alpha(u,s)}{n(u) + \alpha(u) + 1}$$

The square root of this quantity is the standard error, which is returned by the function.

The confidence intervals are constructed by computing the inverse of the beta integral.

Predictive distribution

Given the old data set, the probability of observing new data is $P(D'|D)$ where D' is the new data set. Let $m(u,s), m(u)$ denote the corresponding counts for the new data. Then,

$$P(D'|D) = \int P(D'|\theta)P(\theta|D)d\theta$$

We already know the expressions for both quantities in the integral and it turns out to be similar to evaluating the evidence

$$P(D'|D) = \prod_{u \in \mathcal{A}} \left\{ \frac{\Gamma(\alpha(u))}{\prod_{s \in \mathcal{A}} \Gamma(\alpha(u,s))} \frac{\prod_{s \in \mathcal{A}} \Gamma(n(u,s) + m(u,s) + \alpha(u,s))}{\Gamma(\alpha(u) + n(u) + m(u))} \right\}$$

Choosing the hyperparameters

The hyperparameters model the shape of the parameters' prior distribution. These must be provided by the user. The package offers functionality to translate a given prior belief transition matrix into the hyperparameter matrix. It is assumed that this belief matrix corresponds to the mean value of the parameters. Since the relation

$$\mathrm{E}_{\mathrm{prior}} p(s|u) = \frac{\alpha(u,s)}{\alpha(u)}$$

holds, the function accepts as input the belief matrix as well as a scaling vector (serves as a proxy for $\alpha(.)$) and proceeds to compute $\alpha(.,.)$.

Alternatively, the function accepts a data sample and infers the hyperparameters from it. Since the mode of a parameter (with respect to the prior distribution) is proportional to one less than the corresponding hyperparameter, we set

$$\alpha(u,s) - 1 = m(u,s)$$

where $m(u,s)$ is the $u- > s$ transition count in the data sample. This is regarded as a 'fake count' which helps $\alpha(u,s)$ to reflect knowledge of the data sample.

Usage and examples

```
R> weatherStates <- c("sunny", "cloudy", "rain")
R> byRow <- TRUE
R> weatherMatrix <- matrix(data = c(0.7, 0.2, 0.1,
+                                   0.3, 0.4, 0.3,
+                                   0.2, 0.4, 0.4),
+                          byrow = byRow, nrow = 3,
+                          dimnames = list(weatherStates, weatherStates))
R> mcWeather <- new("markovchain", states = weatherStates,
+                   byrow = byRow, transitionMatrix = weatherMatrix,
+                   name = "Weather")
R> weathersOfDays <- rmarkovchain(n = 365, object = mcWeather, t0 = "sunny")
```

For the purpose of this section, we shall continue to use the weather of days example introduced in the main vignette of the package (reproduced above for convenience).

Let us invoke the fit function to estimate the MAP parameters with 92% confidence bounds and hyperparameters as shown below, based on the first 200 days of the weather data. Additionally, let us find out what the probability is of observing the weather data for the next 165 days. The usage would be as follows

```
R> hyperMatrix<-matrix(c(1, 1, 2,
+                        3, 2, 1,
+                        2, 2, 3),
+                      nrow = 3, byrow = TRUE,
+                      dimnames = list(weatherStates,weatherStates))
R> markovchainFit(weathersOfDays[1:200], method = "map",
+                 confidencelevel = 0.92, hyperparam = hyperMatrix)

$estimate
Bayesian Fit
 A  3 - dimensional discrete Markov Chain defined by the following states:
 cloudy, rain, sunny
 The transition matrix  (by rows)  is defined as follows:
          cloudy       rain     sunny
cloudy 0.4307692 0.33846154 0.2307692
rain   0.2833333 0.48333333 0.2333333
sunny  0.2317073 0.09756098 0.6707317



$expectedValue
          cloudy      rain     sunny
cloudy 0.4264706 0.3382353 0.2352941
rain   0.2857143 0.4761905 0.2380952
sunny  0.2352941 0.1058824 0.6588235

$standardError
           [,1]       [,2]       [,3]
[1,] 0.05953849 0.05695564 0.05106557
[2,] 0.05646924 0.06242910 0.05323971
[3,] 0.04574078 0.03317874 0.05112400

$confidenceInterval
$confidenceInterval$confidenceLevel
[1] 0.92

$confidenceInterval$lowerEndpointMatrix
          [,1]      [,2]      [,3]
[1,] 0.3405624 0.2485242 0.1431451
[2,] 0.1913584 0.3883767 0.1423703
[3,] 0.1530883 0.0000000 0.5856034

$confidenceInterval$upperEndpointMatrix
          [,1]     [,2]      [,3]
[1,] 0.5754446 0.451546 0.3209113
[2,] 0.3885752 1.000000 0.3275604
[3,] 0.3124846 0.155173 1.0000000
```

```
$logLikelihood
[1] -190.4026

R> predictiveDistribution(weathersOfDays[1:200],
+                          weathersOfDays[201:365],hyperparam = hyperMatrix)

[1] -161.8695
```

The results should not change after permuting the dimensions of the matrix.

```
R> hyperMatrix2<- hyperMatrix[c(2,3,1), c(2,3,1)]
R> markovchainFit(weathersOfDays[1:200], method = "map",
+                 confidencelevel = 0.92, hyperparam = hyperMatrix2)

$estimate
Bayesian Fit
 A  3 - dimensional discrete Markov Chain defined by the following states:
 cloudy, rain, sunny
 The transition matrix  (by rows)  is defined as follows:
          cloudy       rain     sunny
cloudy 0.4307692 0.33846154 0.2307692
rain   0.2833333 0.48333333 0.2333333
sunny  0.2317073 0.09756098 0.6707317


$expectedValue
          cloudy      rain     sunny
cloudy 0.4264706 0.3382353 0.2352941
rain   0.2857143 0.4761905 0.2380952
sunny  0.2352941 0.1058824 0.6588235

$standardError
           [,1]       [,2]       [,3]
[1,] 0.05953849 0.05695564 0.05106557
[2,] 0.05646924 0.06242910 0.05323971
[3,] 0.04574078 0.03317874 0.05112400

$confidenceInterval
$confidenceInterval$confidenceLevel
[1] 0.92

$confidenceInterval$lowerEndpointMatrix
          [,1]      [,2]      [,3]
[1,] 0.3405624 0.2485242 0.1431451
[2,] 0.1913584 0.3883767 0.1423703
```

```
[3,] 0.1530883 0.0000000 0.5856034

$confidenceInterval$upperEndpointMatrix
          [,1]     [,2]      [,3]
[1,] 0.5754446 0.451546 0.3209113
[2,] 0.3885752 1.000000 0.3275604
[3,] 0.3124846 0.155173 1.0000000


$logLikelihood
[1] -190.4026
```

```
R> predictiveDistribution(weathersOfDays[1:200],
+                        weathersOfDays[201:365],hyperparam = hyperMatrix2)

[1] -161.8695

R>
```

Note that the predictive probability is very small. However, this can be useful when comparing model orders.

Suppose we have an idea of the (prior) transition matrix corresponding to the expected value of the parameters, and have a data set from which we want to deduce the MAP estimates. We can infer the hyperparameters from this known transition matrix itself, and use this to obtain our MAP estimates.

```
R> inferHyperparam(transMatr = weatherMatrix, scale = c(10, 10, 10))

$scaledInference
       cloudy rain sunny
cloudy      4    3     3
rain        4    4     2
sunny       2    1     7
```

Alternatively, we can use a data sample to infer the hyperparameters.

```
R> inferHyperparam(data = weathersOfDays[1:15])

$dataInference
       cloudy rain sunny
cloudy      1    3     1
rain        2    5     3
sunny       3    2     3
```

In order to use the inferred hyperparameter matrices, we do

```
R> hyperMatrix3 <- inferHyperparam(transMatr = weatherMatrix, scale = c(10, 10, 10))
R> hyperMatrix3 <- hyperMatrix3$scaledInference
R> hyperMatrix4 <- inferHyperparam(data = weathersOfDays[1:15])
R> hyperMatrix4 <- hyperMatrix4$dataInference
```

Now we can safely use `hyperMatrix3` and `hyperMatrix4` with `markovchainFit` (in the `hyperparam` argument).

Supposing we don't provide any hyperparameters, then the prior is uniform. This is the same as maximum likelihood.

```
R> data(preproglucacon)
R> preproglucacon <- preproglucacon[[2]]
R> MLEest <- markovchainFit(preproglucacon, method = "mle")
R> MAPest <- markovchainFit(preproglucacon, method = "map")
R> MLEest$estimate

MLE Fit
 A  4 - dimensional discrete Markov Chain defined by the following states:
 A, C, G, T
 The transition matrix  (by rows)  is defined as follows:
          A         C          G         T
A 0.3585271 0.1434109 0.16666667 0.3313953
C 0.3840304 0.1558935 0.02281369 0.4372624
G 0.3053097 0.1991150 0.15044248 0.3451327
T 0.2844523 0.1819788 0.17667845 0.3568905

R> MAPest$estimate

Bayesian Fit
 A  4 - dimensional discrete Markov Chain defined by the following states:
 A, C, G, T
 The transition matrix  (by rows)  is defined as follows:
          A         C          G         T
A 0.3585271 0.1434109 0.16666667 0.3313953
C 0.3840304 0.1558935 0.02281369 0.4372624
G 0.3053097 0.1991150 0.15044248 0.3451327
T 0.2844523 0.1819788 0.17667845 0.3568905
```

5.7. Higher Order Markov Chains

Continuous time Markov chains are discussed in the CTMC vignette which is a part of the package.

An experimental `fitHigherOrder` function has been written in order to fit a higher order Markov chain (Ching, Huang, Ng, and Siu (2013); Ching *et al.* (2008)). It takes a sequence and the order as arguments and returns the frequency probability vector X of the given sequence and the parameters λ_i with the transition probability matrices Q_i for each order i. Its quadratic programming problem is solved using `solnp` function of **Rsolnp**, Ghalanos and Theussl (2014).

```
R> library(Rsolnp)
R> data(rain)
R> fitHigherOrder(rain$rain, 2)

$lambda
[1] 0.5 0.5

$Q
$Q[[1]]
            0       1-5        6+
0   0.6605839 0.4625850 0.1976285
1-5 0.2299270 0.3061224 0.3122530
6+  0.1094891 0.2312925 0.4901186

$Q[[2]]
            0       1-5        6+
0   0.6021898 0.4489796 0.3412698
1-5 0.2445255 0.2687075 0.3214286
6+  0.1532847 0.2823129 0.3373016


$X
        0       1-5        6+
0.5000000 0.2691606 0.2308394

R> fitHigherOrder(rain$rain, 3)

$lambda
[1] 0.3333333 0.3333333 0.3333333

$Q
$Q[[1]]
            0       1-5        6+
0   0.6605839 0.4625850 0.1976285
1-5 0.2299270 0.3061224 0.3122530
6+  0.1094891 0.2312925 0.4901186

$Q[[2]]
            0       1-5        6+
0   0.6021898 0.4489796 0.3412698
1-5 0.2445255 0.2687075 0.3214286
6+  0.1532847 0.2823129 0.3373016
```

```
$Q[[3]]
             0       1-5        6+
0   0.5693431 0.4455782 0.4183267
1-5 0.2536496 0.2891156 0.2749004
6+  0.1770073 0.2653061 0.3067729


$X
        0       1-5        6+
0.5000000 0.2691606 0.2308394
```

5.8. Higher Order Multivariate Markov Chains

Introduction

HOMMC model is used for modeling behaviour of multiple categorical sequences generated by similar sources. The main reference is Ching *et al.* (2008). Assume that there are s categorical sequences and each has possible states in M. In nth order MMC the state probability distribution of the jth sequence at time $t = r + 1$ depend on the state probability distribution of all the sequences (including itself) at times $t = r, r-1, ..., r-n+1$.

$$x_{r+1}^{(j)} = \sum_{k=1}^{s}\sum_{h=1}^{n} \lambda_{jk}^{(h)} P_h^{(jk)} x_{r-h+1}^{(k)}, j = 1, 2, ..., s, r = n-1, n, ... \tag{22}$$

with initial distribution $x_0^{(k)}, x_1^{(k)}, ..., x_{n-1}^{(k)} (k = 1, 2, ..., s)$. Here

$$\lambda_{jk}^{(h)} \geq 0, 1 \leq j, k \leq s, 1 \leq h \leq n \ \ and \ \ \sum_{k=1}^{s}\sum_{h=1}^{n} \lambda_{jk}^{(h)} = 1, j = 1, 2, 3, ..., s. \tag{23}$$

Now we will see the simpler representation of the model which will help us understand the result of `fitHighOrderMultivarMC` method.

Let $X_r^{(j)} = ((x_r^{(j)})^T, (x_{r-1}^{(j)})^T, ..., (x_{r-n+1}^{(j)})^T)^T for \ \ j = 1, 2, 3, ..., s$. Then

$$\begin{pmatrix} X_{r+1}^{(1)} \\ X_{r+1}^{(2)} \\ . \\ . \\ . \\ X_{r+1}^{(s)} \end{pmatrix} = \begin{pmatrix} B^{11} & B^{12} & . & . & B^{1s} \\ B^{21} & B^{22} & . & . & B^{2s} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ B^{s1} & B^{s2} & . & . & B^{ss} \end{pmatrix} \begin{pmatrix} X_r^{(1)} \\ X_r^{(2)} \\ . \\ . \\ . \\ X_r^{(s)} \end{pmatrix} where$$

$$B^{ii} = \begin{pmatrix} \lambda_{ii}^{(1)} P_1^{(ii)} & \lambda_{ii}^{(2)} P_2^{(ii)} & . & . & \lambda_{ii}^{(n)} P_n^{(ii)} \\ I & 0 & . & . & 0 \\ 0 & I & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & I & 0 \end{pmatrix}_{mn*mn} \quad and$$

$$B^{ij} = \begin{pmatrix} \lambda_{ij}^{(1)} P_1^{(ij)} & \lambda_{ij}^{(2)} P_2^{(ij)} & . & . & \lambda_{ij}^{(n)} P_n^{(ij)} \\ 0 & 0 & . & . & 0 \\ 0 & 0 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & 0 & 0 \end{pmatrix}_{mn*mn} \quad when \;\; i \neq j.$$

Represenation of parameters in the code

$P_h^{(ij)}$ is represented as $Ph(i,j)$ and $\lambda_{ij}^{(h)}$ as Lambdah(i,j). For example: $P_2^{(13)}$ as $P2(1,3)$ and $\lambda_{45}^{(3)}$ as Lambda3(4,5).

Definition of HOMMC class

```
Class "hommc" [package "markovchain"]

Slots:

Name:       order    states          P    Lambda     byrow      name
Class:    numeric character      array   numeric   logical character
```

Any element of `hommc` class is comprised by following slots:

1. states: a character vector, listing the states for which transition probabilities are defined.
2. byrow: a logical element, indicating whether transition probabilities are shown by row or by column.
3. order: order of Multivariate Markov chain.
4. P: an array of all transition matrices.
5. Lambda: a vector to store the weightage of each transition matrix.
6. name: optional character element to name the HOMMC

How to create an object of class HOMMC

```
R> states <- c('a', 'b')
R> P <- array(dim = c(2, 2, 4), dimnames = list(states, states))
R> P[ , , 1] <- matrix(c(1/3, 2/3, 1, 0), byrow = FALSE, nrow = 2, ncol = 2)
R> P[ , , 2] <- matrix(c(0, 1, 1, 0), byrow = FALSE, nrow = 2, ncol = 2)
R> P[ , , 3] <- matrix(c(2/3, 1/3, 0, 1), byrow = FALSE, nrow = 2, ncol = 2)
R> P[ , , 4] <- matrix(c(1/2, 1/2, 1/2, 1/2), byrow = FALSE, nrow = 2, ncol = 2)
R> Lambda <- c(.8, .2, .3, .7)
R> hob <- new("hommc", order = 1, Lambda = Lambda, P = P, states = states,
+             byrow = FALSE, name = "FOMMC")
R> hob

Order of multivariate markov chain = 1
states = a b

List of Lambda's and the corresponding transition matrix (by cols) :
Lambda1(1,1) : 0.8
P1(1,1) :
          a b
a 0.3333333 1
b 0.6666667 0

Lambda1(1,2) : 0.2
P1(1,2) :
  a b
a 0 1
b 1 0

Lambda1(2,1) : 0.3
P1(2,1) :
          a b
a 0.6666667 0
b 0.3333333 1

Lambda1(2,2) : 0.7
P1(2,2) :
    a   b
a 0.5 0.5
b 0.5 0.5
```

Fit HOMMC

`fitHighOrderMultivarMC` method is available to fit HOMMC. Below are the 3 parameters of this method.

1. seqMat: a character matrix or a data frame, each column represents a categorical sequence.

2. order: order of Multivariate Markov chain. Default is 2.

3. Norm: Norm to be used. Default is 2.

6. Applications

This section shows applications of DTMC in various fields.

6.1. Weather forecasting

Markov chains provide a simple model to predict the next day's weather given the current meteorological condition. The first application herewith shown is the "Land of Oz example" from J. G. Kemeny, J. L.Snell, and G. L. Thompson (1974), the second is the "Alofi Island Rainfall" from P. J. Avery and D. A. Henderson (1999).

Land of Oz

The Land of Oz is acknowledged not to have ideal weather conditions at all: the weather is snowy or rainy very often and, once more, there are never two nice days in a row. Consider three weather states: rainy, nice and snowy. Let the transition matrix be as in the following:

```
R> mcWP <- new("markovchain", states = c("rainy", "nice", "snowy"),
+           transitionMatrix = matrix(c(0.5, 0.25, 0.25,
+                                       0.5, 0, 0.5,
+                                       0.25,0.25,0.5), byrow = T, nrow = 3))
```

Given that today it is a nice day, the corresponding stochastic row vector is $w_0 = (0\ ,1\ ,0)$ and the forecast after 1, 2 and 3 days are given by

```
R> W0 <- t(as.matrix(c(0, 1, 0)))
R> W1 <- W0 * mcWP; W1

     rainy nice snowy
[1,]   0.5    0   0.5

R> W2 <- W0 * (mcWP ^ 2); W2

     rainy nice snowy
[1,] 0.375 0.25 0.375

R> W3 <- W0 * (mcWP ^ 3); W3

       rainy   nice   snowy
[1,] 0.40625 0.1875 0.40625
```

As can be seen from w_1, if in the Land of Oz today is a nice day, tomorrow it will rain or snow with probability 1. One week later, the prediction can be computed as

```
R> W7 <- W0 * (mcWP ^ 7)
R> W7

         rainy      nice     snowy
[1,] 0.4000244 0.1999512 0.4000244
```

The steady state of the chain can be computed by means of the `steadyStates` method.

```
R> q <- steadyStates(mcWP)
R> q

     rainy nice snowy
[1,]   0.4  0.2   0.4
```

Note that, from the seventh day on, the predicted probabilities are substantially equal to the steady state of the chain and they don't depend from the starting point, as the following code shows.

```
R> R0 <- t(as.matrix(c(1, 0, 0)))
R> R7 <- R0 * (mcWP ^ 7); R7

         rainy      nice     snowy
[1,] 0.4000244 0.2000122 0.3999634

R> S0 <- t(as.matrix(c(0, 0, 1)))
R> S7 <- S0 * (mcWP ^ 7); S7

         rainy      nice     snowy
[1,] 0.3999634 0.2000122 0.4000244
```

Alofi Island Rainfall

Alofi Island daily rainfall data were recorded from January 1st, 1987 until December 31st, 1989 and classified into three states: "0" (no rain), "1-5" (from non zero until 5 mm) and "6+" (more than 5mm). The corresponding dataset is provided within the **markovchain** package.

```
R> data("rain", package = "markovchain")
R> table(rain$rain)

  0 1-5  6+
548 295 253
```

The underlying transition matrix is estimated as follows.

```
R> mcAlofi <- markovchainFit(data = rain$rain, name = "Alofi MC")$estimate
R> mcAlofi
```

```
Alofi MC
 A  3 - dimensional discrete Markov Chain defined by the following states:
 0, 1-5, 6+
 The transition matrix  (by rows)  is defined as follows:
            0       1-5        6+
0   0.6605839 0.2299270 0.1094891
1-5 0.4625850 0.3061224 0.2312925
6+  0.1976285 0.3122530 0.4901186
```

The long term daily rainfall distribution is obtained by means of the `steadyStates` method.

```
R> steadyStates(mcAlofi)
```

```
             0       1-5        6+
[1,] 0.5008871 0.2693656 0.2297473
```

6.2. Finance and Economics

Other relevant applications of DTMC can be found in Finance and Economics.

Finance

Credit ratings transitions have been successfully modelled with discrete time Markov chains. Some rating agencies publish transition matrices that show the empirical transition probabilities across credit ratings. The example that follows comes from **CreditMetrics** R package (Wittmann 2007), carrying Standard & Poor's published data.

```
R> rc <- c("AAA", "AA", "A", "BBB", "BB", "B", "CCC", "D")
R> creditMatrix <- matrix(c(90.81, 8.33, 0.68, 0.06, 0.08, 0.02, 0.01, 0.01,
+  0.70, 90.65, 7.79, 0.64, 0.06, 0.13, 0.02, 0.01,
+  0.09, 2.27, 91.05, 5.52, 0.74, 0.26, 0.01, 0.06,
+  0.02, 0.33, 5.95, 85.93, 5.30, 1.17, 1.12, 0.18,
+  0.03, 0.14, 0.67, 7.73, 80.53, 8.84, 1.00, 1.06,
+  0.01, 0.11, 0.24, 0.43, 6.48, 83.46, 4.07, 5.20,
+  0.21, 0, 0.22, 1.30, 2.38, 11.24, 64.86, 19.79,
+  0, 0, 0, 0, 0, 0, 0, 100
+  )/100, 8, 8, dimnames = list(rc, rc), byrow = TRUE)
```

It is easy to convert such matrices into `markovchain` objects and to perform some analyses

```
R> creditMc <- new("markovchain", transitionMatrix = creditMatrix,
+                  name = "S&P Matrix")
R> absorbingStates(creditMc)
```

```
[1] "D"
```

Economics

For a recent application of **markovchain** in Economic, see Jacob (2014).

A dynamic system generates two kinds of economic effects (Bard 2000):

1. those incurred when the system is in a specified state, and
2. those incurred when the system makes a transition from one state to another.

Let the monetary amount of being in a particular state be represented as a m-dimensional column vector c^{S}, while let the monetary amount of a transition be embodied in a C^R matrix in which each component specifies the monetary amount of going from state i to state j in a single step. Henceforth, Equation 24 represents the monetary of being in state i.

$$c_i = c_i^{\mathrm{S}} + \sum_{j=1}^{m} C_{ij}^{\mathrm{R}} p_{ij}. \tag{24}$$

Let $\bar{c} = [c_i]$ and let e_i be the vector valued 1 in the initial state and 0 in all other, then, if f_n is the random variable representing the economic return associated with the stochastic process at time n, Equation 25 holds:

$$E\left[f_n\left(X_n\right)|X_0 = i\right] = e_i P^n \bar{c}. \tag{25}$$

The following example assumes that a telephone company models the transition probabilities between customer/non-customer status by matrix P and the cost associated to states by matrix M.

```
R> statesNames <- c("customer", "non customer")
R> P <- zeros(2); P[1, 1] <- .9; P[1, 2] <- .1; P[2, 2] <- .95; P[2, 1] <- .05;
R> rownames(P) <- statesNames; colnames(P) <- statesNames
R> mcP <- new("markovchain", transitionMatrix = P, name = "Telephone company")
R> M <- zeros(2); M[1, 1] <- -20; M[1, 2] <- -30; M[2, 1] <- -40; M[2, 2] <- 0
```

If the average revenue for existing customer is +100, the cost per state is computed as follows.

```
R> c1 <- 100 + conditionalDistribution(mcP, state = "customer") %*% M[1,]
R> c2 <- 0 + conditionalDistribution(mcP, state = "non customer") %*% M[2,]
```

For an existing customer, the expected gain (loss) at the fifth year is given by the following code.

```
R> as.numeric((c(1, 0)* mcP ^ 5) %*% (as.vector(c(c1, c2))))

[1] 48.96009
```

6.3. Marketing

We tried to replicate the example found in Ching *et al.* (2008) for an application of HOMMC. A soft-drink company in Hong Kong is facing an in-house problem of production planning and inventory control. A pressing issue is the storage space of its central warehouse, which often finds itself in the state of overflow or near capacity. The company is thus in urgent needs to study the interplay between the storage space requirement and the overall growing sales demand. The product can be classified into six possible states (1, 2, 3, 4, 5, 6) according to their sales volumes. All products are labeled as 1 = no sales volume, 2 = very slow-moving (very low sales volume), 3 = slow-moving, 4 = standard, 5 = fast-moving or 6 = very fast-moving (very high sales volume). Such labels are useful from both marketing and production planning points of view. The data is cointaind in `sales` object.

```
R> data(sales)
R> head(sales)
```

```
     A   B   C   D   E
[1,] "6" "1" "6" "6" "6"
[2,] "6" "6" "6" "2" "2"
[3,] "6" "6" "6" "2" "2"
[4,] "6" "1" "6" "2" "2"
[5,] "2" "6" "6" "2" "2"
[6,] "6" "1" "6" "3" "3"
```

The company would also like to predict sales demand for an important customer in order to minimize its inventory build-up. More importantly, the company can understand the sales pattern of this customer and then develop a marketing strategy to deal with this customer. Customer's sales demand sequences of five important products of the company for a year. We expect sales demand sequences generated by the same customer to be correlated to each other. Therefore by exploring these relationships, one can obtain a better higher-order multivariate Markov model for such demand sequences, hence obtain better prediction rules.

In Ching *et al.* (2008) application, they choose the order arbitrarily to be eight, i.e., n = 8. We first estimate all the transition probability matrices $P_h^{(ij)}$ and we also have the estimates of the stationary probability distributions of the five products:.

$$\widehat{\boldsymbol{x}}^{(1)} = \begin{pmatrix}0.0818 & 0.4052 & 0.0483 & 0.0335 & 0.0037 & 0.4275\end{pmatrix}^{\boldsymbol{T}}$$

$$\widehat{\boldsymbol{x}}^{(2)} = \begin{pmatrix}0.3680 & 0.1970 & 0.0335 & 0.0000 & 0.0037 & 0.3978\end{pmatrix}^{\boldsymbol{T}}$$

$$\widehat{\boldsymbol{x}}^{(3)} = \begin{pmatrix}0.1450 & 0.2045 & 0.0186 & 0.0000 & 0.0037 & 0.6283\end{pmatrix}^{\boldsymbol{T}}$$

$$\widehat{\boldsymbol{x}}^{(4)} = \begin{pmatrix}0.0000 & 0.3569 & 0.1338 & 0.1896 & 0.0632 & 0.2565\end{pmatrix}^{\boldsymbol{T}}$$

$$\widehat{\boldsymbol{x}}^{(5)} = \begin{pmatrix}0.0000 & 0.3569 & 0.1227 & 0.2268 & 0.0520 & 0.2416\end{pmatrix}^{\boldsymbol{T}}$$

By solving the corresponding linear programming problems, we obtain the following higher-order multivariate Markov chain model:

$$\boldsymbol{x}_{r+1}^{(1)} = \boldsymbol{P}_1^{(12)} \boldsymbol{x}_r^{(2)}$$
$$\boldsymbol{x}_{r+1}^{(2)} = 0.6364 \boldsymbol{P}_1^{(22)} \boldsymbol{x}_r^{(2)} + 0.3636 \boldsymbol{P}_3^{(22)} \boldsymbol{x}_r^{(2)}$$

$$\boldsymbol{x}_{r+1}^{(3)} = \boldsymbol{P}_1^{(35)}\boldsymbol{x}_r^{(5)}$$
$$\boldsymbol{x}_{r+1}^{(4)} = 0.2994\boldsymbol{P}_8^{(42)}\boldsymbol{x}_r^{(2)} + 0.4324\boldsymbol{P}_1^{(45)}\boldsymbol{x}_r^{(5)} + 0.2681\boldsymbol{P}_2^{(45)}\boldsymbol{x}_r^{(5)}$$
$$\boldsymbol{x}_{r+1}^{(5)} = 0.2718\boldsymbol{P}_8^{(52)}\boldsymbol{x}_r^{(2)} + 0.6738\boldsymbol{P}_1^{(54)}\boldsymbol{x}_r^{(4)} + 0.0544\boldsymbol{P}_2^{(55)}\boldsymbol{x}_r^{(5)}$$

According to the constructed 8th order multivariate Markov model, Products A and B are closely related. In particular, the sales demand of Product A depends strongly on Product B. The main reason is that the chemical nature of Products A and B is the same, but they have different packaging for marketing purposes. Moreover, Products B, C, D and E are closely related. Similarly, products C and E have the same product flavor, but different packaging. In this model, it is interesting to note that both Product D and E quite depend on Product B at order of 8, this relationship is hardly to be obtained in conventional Markov model owing to huge amount of parameters. The results show that higher-order multivariate Markov model is quite significant to analyze the relationship of sales demand.

```
R> # fit 8th order multivariate markov chain
R> object <- fitHighOrderMultivarMC(sales, order = 8, Norm = 2)
```

We choose to show only results shown in the paper. We see that λ values are quite close, but not equal, to those shown in the original paper.

```
Order of multivariate markov chain = 8

states = 1 2 3 4 5 6



List of Lambda's and the corresponding transition matrix (by cols) :

Lambda1(1,2) : 0.9999989
P1(1,2) :
           1         2         3         4 5          6
1 0.06060606 0.1509434 0.0000000 0.1666667 0 0.07547170
2 0.44444444 0.4716981 0.4444444 0.1666667 1 0.33018868
3 0.01010101 0.1320755 0.2222222 0.1666667 0 0.02830189
4 0.01010101 0.0754717 0.2222222 0.1666667 0 0.01886792
5 0.01010101 0.0000000 0.0000000 0.1666667 0 0.00000000
6 0.46464646 0.1698113 0.1111111 0.1666667 0 0.54716981

Lambda1(2,2) : 0.4783695
P1(2,2) :
           1          2         3         4 5           6
1 0.40404040 0.20754717 0.0000000 0.1666667 1 0.433962264
2 0.11111111 0.47169811 0.3333333 0.1666667 0 0.132075472
3 0.02020202 0.05660377 0.3333333 0.1666667 0 0.009433962
4 0.00000000 0.00000000 0.0000000 0.1666667 0 0.000000000
```

```
5 0.00000000 0.00000000 0.1111111 0.1666667 0 0.000000000
6 0.46464646 0.26415094 0.2222222 0.1666667 0 0.424528302

Lambda3(2,2) : 0.3881378
P3(2,2) :
           1          2         3         4 5          6
1 0.40404040 0.16981132 0.3333333 0.1666667 0 0.44230769
2 0.18181818 0.33962264 0.2222222 0.1666667 0 0.14423077
3 0.03030303 0.05660377 0.0000000 0.1666667 0 0.02884615
4 0.00000000 0.00000000 0.0000000 0.1666667 0 0.00000000
5 0.00000000 0.00000000 0.1111111 0.1666667 0 0.00000000
6 0.38383838 0.43396226 0.3333333 0.1666667 1 0.38461538

Lambda1(3,5) : 0.6740322
P1(3,5) :
          1          2         3         4          5          6
1 0.1666667 0.09473684 0.1515152 0.1639344 0.07142857 0.21538462
2 0.1666667 0.18947368 0.2727273 0.2295082 0.14285714 0.18461538
3 0.1666667 0.04210526 0.0000000 0.0000000 0.00000000 0.01538462
4 0.1666667 0.00000000 0.0000000 0.0000000 0.00000000 0.00000000
5 0.1666667 0.01052632 0.0000000 0.0000000 0.00000000 0.00000000
6 0.1666667 0.66315789 0.5757576 0.6065574 0.78571429 0.58461538

Lambda8(4,2) : 0.2740362
P8(4,2) :
           1          2         3         4 5          6
1 0.00000000 0.00000000 0.0000000 0.1666667 0 0.00000000
2 0.34343434 0.18867925 0.6666667 0.1666667 0 0.42424242
3 0.10101010 0.16981132 0.0000000 0.1666667 1 0.14141414
4 0.20202020 0.22641509 0.1111111 0.1666667 0 0.17171717
5 0.08080808 0.09433962 0.1111111 0.1666667 0 0.03030303
6 0.27272727 0.32075472 0.1111111 0.1666667 0 0.23232323

Lambda1(4,5) : 0.2300251
P1(4,5) :
          1          2          3          4          5          6
1 0.1666667 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
2 0.1666667 0.47368421 0.21212121 0.03278689 0.00000000 0.64615385
3 0.1666667 0.10526316 0.21212121 0.19672131 0.07142857 0.09230769
4 0.1666667 0.00000000 0.24242424 0.54098361 0.57142857 0.03076923
5 0.1666667 0.01052632 0.03030303 0.18032787 0.28571429 0.00000000
6 0.1666667 0.41052632 0.30303030 0.04918033 0.07142857 0.23076923

Lambda2(4,5) : 0.2978897
P2(4,5) :
          1          2          3          4          5          6
1 0.1666667 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
```

```
2 0.1666667 0.55319149 0.36363636 0.06557377 0.00000000 0.41538462
3 0.1666667 0.13829787 0.09090909 0.21311475 0.28571429 0.04615385
4 0.1666667 0.05319149 0.24242424 0.40983607 0.64285714 0.06153846
5 0.1666667 0.02127660 0.06060606 0.16393443 0.07142857 0.03076923
6 0.1666667 0.23404255 0.24242424 0.14754098 0.00000000 0.44615385

Lambda8(5,2) : 0.228813
P8(5,2) :
           1          2         3         4 5          6
1 0.00000000 0.00000000 0.0000000 0.1666667 0 0.00000000
2 0.35353535 0.20754717 0.6666667 0.1666667 1 0.39393939
3 0.10101010 0.15094340 0.0000000 0.1666667 0 0.13131313
4 0.22222222 0.30188679 0.2222222 0.1666667 0 0.20202020
5 0.09090909 0.03773585 0.0000000 0.1666667 0 0.03030303
6 0.23232323 0.30188679 0.1111111 0.1666667 0 0.24242424

Lambda1(5,4) : 0.2266793
P1(5,4) :
          1          2          3          4          5          6
1 0.1666667 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
2 0.1666667 0.48421053 0.16666667 0.01960784 0.05882353 0.60869565
3 0.1666667 0.10526316 0.16666667 0.15686275 0.05882353 0.11594203
4 0.1666667 0.00000000 0.44444444 0.62745098 0.64705882 0.02898551
5 0.1666667 0.01052632 0.02777778 0.15686275 0.23529412 0.00000000
6 0.1666667 0.40000000 0.19444444 0.03921569 0.00000000 0.24637681

Lambda2(5,5) : 0.5377752
P2(5,5) :
          1          2          3          4          5          6
1 0.1666667 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
2 0.1666667 0.52127660 0.42424242 0.04918033 0.07142857 0.43076923
3 0.1666667 0.12765957 0.03030303 0.19672131 0.21428571 0.07692308
4 0.1666667 0.05319149 0.33333333 0.54098361 0.50000000 0.07692308
5 0.1666667 0.02127660 0.03030303 0.11475410 0.21428571 0.01538462
6 0.1666667 0.27659574 0.18181818 0.09836066 0.00000000 0.40000000
```

6.4. Actuarial science

Markov chains are widely applied in the field of actuarial science. Two classical applications are policyholders' distribution across Bonus Malus classes in Motor Third Party Liability (MTPL) insurance (Section 6.4.1) and health insurance pricing and reserving (Section 6.4.2).

MPTL Bonus Malus

Bonus Malus (BM) contracts grant the policyholder a discount (enworsen) as a function of the number of claims in the experience period. The discount (enworsen) is applied on a premium that already allows for known (a priori) policyholder characteristics (Denuit, Maréchal,

Pitrebois, and Walhin 2007) and it usually depends on vehicle, territory, the demographic profile of the policyholder, and policy coverages deep (deductible and policy limits).
Since the proposed BM level depends on the claim on the previous period, it can be modelled by a discrete Markov chain. A very simplified example follows. Assume a BM scale from 1 to 5, where 4 is the starting level. The evolution rules are shown in Equation 26:

$$bm_{t+1} = \max\left(1, bm_t - 1\right) * \left(\tilde{N} = 0\right) + \min\left(5, bm_t + 2 * \tilde{N}\right) * \left(\tilde{N} \geq 1\right). \qquad (26)$$

Tthe number of claim $\tilde{N}$ is a random variable that is assumed to be Poisson distributed.

```
R> getBonusMalusMarkovChain <- function(lambda)
+  {
+          bmMatr <- zeros(5)
+          bmMatr[1, 1] <- dpois(x = 0, lambda)
+          bmMatr[1, 3] <- dpois(x = 1, lambda)
+          bmMatr[1, 5] <- 1 - ppois(q = 1, lambda)
+
+          bmMatr[2, 1] <- dpois(x = 0, lambda)
+          bmMatr[2, 4] <- dpois(x = 1, lambda)
+          bmMatr[2, 5] <- 1 - ppois(q = 1, lambda)
+
+          bmMatr[3, 2] <- dpois(x = 0, lambda)
+          bmMatr[3, 5] <- 1 - dpois(x=0, lambda)
+
+          bmMatr[4, 3] <- dpois(x = 0, lambda)
+          bmMatr[4, 5] <- 1 - dpois(x = 0, lambda)
+
+          bmMatr[5, 4] <- dpois(x = 0, lambda)
+          bmMatr[5, 5] <- 1 - dpois(x = 0, lambda)
+          stateNames <- as.character(1:5)
+          out <- new("markovchain", transitionMatrix = bmMatr,
+               states = stateNames, name = "BM Matrix")
+          return(out)
+  }
R>
```

Assuming that the a-priori claim frequency per car-year is 0.05 in the class (being the class the group of policyholders that share the same common characteristics), the underlying BM transition matrix and its underlying steady state are as follows.

```
R> bmMc <- getBonusMalusMarkovChain(0.05)
R> as.numeric(steadyStates(bmMc))

[1] 0.895836079 0.045930498 0.048285405 0.005969247 0.003978772
```

If the underlying BM coefficients of the class are 0.5, 0.7, 0.9, 1.0, 1.25, this means that the average BM coefficient applied on the long run to the class is given by

```
R> sum(as.numeric(steadyStates(bmMc)) * c(0.5, 0.7, 0.9, 1, 1.25))
```

```
[1] 0.534469
```

This means that the average premium paid by policyholders in the portfolio almost halves in the long run.

Health insurance example

Actuaries quantify the risk inherent in insurance contracts evaluating the premium of insurance contract to be sold (therefore covering future risk) and evaluating the actuarial reserves of existing portfolios (the liabilities in terms of benefits or claims payments due to policyholder arising from previously sold contracts). Key quantities of actuarial interest are: the expected present value of future benefits, $PVFB$, the (periodic) benefit premium, P, and the present value of future premium $PVFP$. A level benefit premium could be set equating at the beginning of the contract $PVFB = PVFP$. After the beginning of the contract the benefit reserve is the difference between $PVFB$ and $PVFP$. The example comes from Deshmukh (2012). The interest rate is 5%, benefits are payable upon death (1000) and disability (500). Premiums are payable at the beginning of period only if the policyholder is active. The contract term is three years.

```
R> mcHI <- new("markovchain", states = c("active", "disable", "withdrawn",
+                                        "death"),
+            transitionMatrix = matrix(c(0.5, .25, .15, .1,
+                                       0.4, 0.4, 0.0, 0.2,
+                                       0, 0, 1, 0,
+                                       0, 0, 0, 1), byrow = TRUE, nrow = 4))
R> benefitVector <- as.matrix(c(0, 0, 500, 1000))
```

The policyholders is active at T_0. Therefore the expected states at $T_1, \ldots T_3$ are calculated in the following.

```
R> T0 <- t(as.matrix(c(1, 0, 0, 0)))
R> T1 <- T0 * mcHI
R> T2 <- T1 * mcHI
R> T3 <- T2 * mcHI
```

The present value of future benefit at T0 is given by

```
R> PVFB <- T0 %*% benefitVector * 1.05 ^ -0 +
+    T1 %*% benefitVector * 1.05 ^ -1+
+    T2 %*% benefitVector * 1.05 ^ -2 + T3 %*% benefitVector * 1.05 ^ -3
```

The yearly premium payable whether the insured is alive is as follows.

```
R> P <- PVFB / (T0[1] * 1.05 ^- 0 + T1[1] * 1.05 ^ -1 + T2[1] * 1.05 ^ -2)
```

The reserve at the beginning of the second year, in the case of the insured being alive, is as follows.

```
R> PVFB <- T2 %*% benefitVector * 1.05 ^ -1 + T3 %*% benefitVector * 1.05 ^ -2
R> PVFP <- P*(T1[1] * 1.05 ^ -0 + T2[1] * 1.05 ^ -1)
R> V <- PVFB - PVFP
R> V
```

```
         [,1]
[1,] 300.2528
```

6.5. Sociology

Markov chains have been actively used to model progressions and regressions between social classes. The first study was performed by Glass and Hall (1954), while a more recent application can be found in Jo Blanden and Machin (2005). The table that follows shows the income quartile of the father when the son was 16 (in 1984) and the income quartile of the son when aged 30 (in 2000) for the 1970 cohort.

```
R> data("blanden")
R> mobilityMc <- as(blanden, "markovchain")
R> mobilityMc
```

```
Unnamed Markov chain
 A  4 - dimensional discrete Markov Chain defined by the following states:
 Bottom, 2nd, 3rd, Top
 The transition matrix  (by rows)  is defined as follows:
             2nd       3rd    Bottom       Top
Bottom 0.2900000 0.2200000 0.3800000 0.1100000
2nd    0.2772277 0.2574257 0.2475248 0.2178218
3rd    0.2626263 0.2828283 0.2121212 0.2424242
Top    0.1700000 0.2500000 0.1600000 0.4200000
```

The underlying transition graph is plotted in Figure 5.

The steady state distribution is computed as follows. Since transition across quartiles are shown, the probability function is evenly 0.25.

```
R> round(steadyStates(mobilityMc), 2)
```

```
     Bottom  2nd  3rd  Top
[1,]   0.25 0.25 0.25 0.25
```

6.6. Genetics and Medicine

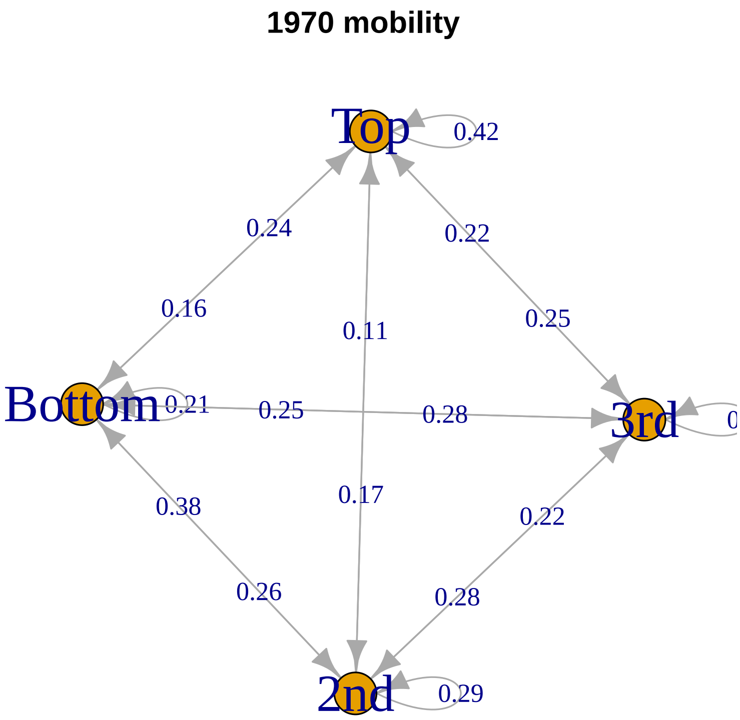

Figure 5: 1970 UK cohort mobility data.

This section contains two examples: the first shows the use of Markov chain models in genetics, the second shows an application of Markov chains in modelling diseases' dynamics.

Genetics

P. J. Avery and D. A. Henderson (1999) discusses the use of Markov chains in model Preproglucacon gene protein bases sequence. The `preproglucacon` dataset in **markovchain** contains the dataset shown in the package.

```
R> data("preproglucacon", package = "markovchain")
```

It is possible to model the transition probabilities between bases as shown in the following code.

```
R> mcProtein <- markovchainFit(preproglucacon$preproglucacon,
+                              name = "Preproglucacon MC")$estimate
R> mcProtein

Preproglucacon MC
 A  4 - dimensional discrete Markov Chain defined by the following states:
 A, C, G, T
 The transition matrix  (by rows)  is defined as follows:
          A         C          G         T
A 0.3585271 0.1434109 0.16666667 0.3313953
C 0.3840304 0.1558935 0.02281369 0.4372624
G 0.3053097 0.1991150 0.15044248 0.3451327
T 0.2844523 0.1819788 0.17667845 0.3568905
```

Medicine

Discrete-time Markov chains are also employed to study the progression of chronic diseases. The following example is taken from B. A. Craig and A. A. Sendi (2002). Starting from six month follow-up data, the maximum likelihood estimation of the monthly transition matrix is obtained. This transition matrix aims to describe the monthly progression of CD4-cell counts of HIV infected subjects.

```
R> craigSendiMatr <- matrix(c(682, 33, 25,
+                  154, 64, 47,
+                  19, 19, 43), byrow = T, nrow = 3)
R> hivStates <- c("0-49", "50-74", "75-UP")
R> rownames(craigSendiMatr) <- hivStates
R> colnames(craigSendiMatr) <- hivStates
R> craigSendiTable <- as.table(craigSendiMatr)
R> mcM6 <- as(craigSendiTable, "markovchain")
R> mcM6@name <- "Zero-Six month CD4 cells transition"
R> mcM6
```

```
Zero-Six month CD4 cells transition
 A  3 - dimensional discrete Markov Chain defined by the following states:
 0-49, 50-74, 75-UP
 The transition matrix  (by rows)  is defined as follows:
           0-49      50-74      75-UP
0-49  0.9216216 0.04459459 0.03378378
50-74 0.5811321 0.24150943 0.17735849
75-UP 0.2345679 0.23456790 0.53086420
```

As shown in the paper, the second passage consists in the decomposition of $M_6 = V \cdot D \cdot V^{-1}$ in order to obtain M_1 as $M_1 = V \cdot D^{1/6} \cdot V^{-1}$.

```
R> eig <- eigen(mcM6@transitionMatrix)
R> D <- diag(eig$values)

R> V <- eig$vectors
R> V %*% D %*% solve(V)

          [,1]       [,2]       [,3]
[1,] 0.9216216 0.04459459 0.03378378
[2,] 0.5811321 0.24150943 0.17735849
[3,] 0.2345679 0.23456790 0.53086420

R> d <- D ^ (1/6)
R> M <- V %*% d %*% solve(V)
R> mcM1 <- new("markovchain", transitionMatrix = M, states = hivStates)
```

7. Discussion, issues and future plans

The **markovchain** package has been designed in order to provide easily handling of DTMC and communication with alternative packages.

Some numerical issues have been found when working with matrix algebra using R internal linear algebra kernel (the same calculations performed with MATLAB gave a more accurate result). Some temporary workarounds have been implemented. For example, the condition for row/column sums to be equal to one is valid up to fifth decimal. Similarly, when extracting the eigenvectors only the real part is taken.

Such limitations are expected to be overcome in future releases. Similarly, future versions of the package are expected to improve the code in terms of numerical accuracy and rapidity. An intitial rewriting of internal function in C++ by means of **Rcpp** package (Eddelbuettel 2013) has been started.

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Affiliation:

Giorgio Alfredo Spedicato
Ph.D C.Stat ACAS
UnipolSai R&D
Via Firenze 11, Paderno Dugnano 20037 Italy
E-mail: spedygiorgio@gmail.com
URL: www.statisticaladvisor.com

Tae Seung Kang
Ph.D student
Computer & Information Science & Engineering
University of Florida
Gainesville, FL, USA
E-mail: tskang3@gmail.com

Sai Bhargav Yalamanchi
B-Tech student
Electrical Engineering
Indian Institute of Technology, Bombay
Mumbai - 400 076, India
E-mail: bhargavcoolboy@gmail.com

Deepak Yadav
B-Tech student
Computer Science and Engineering
Indian Institute of Technology, Varanasi
Uttar Pradesh - 221 005, India
E-mail: deepakyadav.iitbhu@gmail.com