hts: An R Package for Forecasting Hierarchical or Grouped Time Series

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Abstract

This paper describes several methods that are currently available in the **hts** package, for forecasting hierarchical time series. The methods included are: top-down, buttom-up, middle-out and optimal combination. The implementation of these methods is illustrated by using regional infant mortality counts in Australia.

Keywords: top-down, bottom-up, middle-out, optimal combination .

Introduction

Advances in data collection and storage have resulted in large numbers of time series that are hierarchical in structure, and clusters of which may be correlated. In many applications the related time series can be organized in a hierarchical structure based on dimensions such as gender, geography or product type. This has led to the problem of hierarchical time series modeling and forecasting. The aim of this article is to describe the R functions that are available in the **hts** package (Hyndman, Ahmed, Shang, and Wang 2014), for modeling and forecasting hierarchical and grouped time series.

Forecasting hierarchical time series has been the subject of increasing attention recently (e.g., Athanasopoulos, Ahmed, and Hyndman 2009), and has application in diverse fields. In macroeconomic forecasting, the national economic account is disaggregated into production, income and outlay, and capital transactions. Production is further classified into production in Britain and production in the rest of world; income and outlay and capital transactions are each further classified into persons, companies, public corporations, general government, and rest of world. In demographic forecasting, the mortality counts in Australia can be disaggregated by gender; within each gender, mortality counts can be further disaggregated by different states in Australia. The first example is referred to as a hierarchical time series, in which the order of disaggregation is unique. By contrast, the second example is a grouped time series, which can be thought of as hierarchical time series without a unique hierarchical structure. In other words, the order by which the series can be grouped is not unique — the mortality counts in Australia can be first disaggregated by states and then by gender, or they can be disaggregated by gender first, and then by states.

Hierarchical forecasting methods allow the forecasts at each level to be summed giving the forecasts at the level above. When the data are grouped, the forecasts of each group must be equal to the forecasts of the individual series making up the group.

In the current statistical literature, existing approaches to hierarchical time-series forecasting usually involve either a top-down method or a bottom-up method or a combination of both methods often referred to as the "middle-out" approach. The top-down method involves forecasting the aggregated series, then disaggregating the forecasts based on the historical or forecast proportions (see Gross and Sohl 1990; Athanasopoulos *et al.* 2009, for possible ways of choosing these proportions). The bottom-up method involves forecasting each of the disaggregated series at the lowest level of the hierarchy, and then using aggregation to obtain forecasts at higher levels of the hierarchy (Kahn 1998). The middle-out method starts at an intermediate level of the hierarchy chosen by the user, and then aggregation is used to obtain forecasts at higher levels and disaggregation is used to obtain forecasts at lower levels. However, none of these methods takes the correlation among the series at each level into account. To address this issue, Hyndman, Ahmed, Athanasopoulos, and Shang (2011) proposed a statistical method for optimal hierarchical forecasting.

This article proceeds as follows. Techniques for modeling and forecasting hierarchical time series are first reviewed and their implementations using the **hts** package are then described. Conclusions are given at the end.

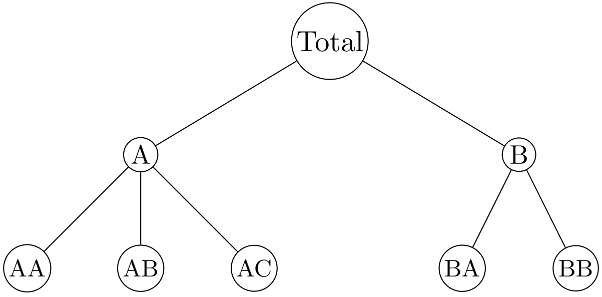
Hierarchical forecasting methods

Notation

Figure 1 shows a $K = 2$-level hierarchy. At the top of the hierarchy, level 0, is the "Total", the most aggregate level of the data. We denote as y_t the tth observation of the "Total" series for $t = 1, \dots, T$. Below this level we denote as $y_{\mathrm{j},t}$ the tth observation of the series which corresponds to node j of the hierarchical tree. The "Total" is disaggregated into two series at level 1 and each of these into three and two series respectively at the bottom level of the hierarchy (level 2). A denotes series A at level 1; AB denotes series B at level 2 within series A at level 1, and so on.

The total number of series in a hierarchy is given by $n = 1 + n_1 + \cdots + n_K$ where n_i is the number of series at level i of the hierarchy. In this case $n = 1 + 2 + 5 = 8$.

Figure 1: A two level hierarchical tree diagram



For any time t, the observations of the bottom level series will aggregate to the observations of the series above. This can be effectively represented using matrix notation. We construct an $n \times n_K$ matrix referred to as the "summing" matrix $\boldsymbol{S}$ which dictates how the bottom level series are aggregated, consistent with the hierarchical structure. For the hierarchy in Figure 1 we can write

$$\begin{bmatrix} y_t \\ y_{\mathrm{A},t} \\ y_{\mathrm{B},t} \\ y_{\mathrm{AA},t} \\ y_{\mathrm{AB},t} \\ y_{\mathrm{AC},t} \\ y_{\mathrm{BA},t} \\ y_{\mathrm{BB},t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{\mathrm{AA},t} \\ y_{\mathrm{AB},t} \\ y_{\mathrm{AC},t} \\ y_{\mathrm{BA},t} \\ y_{\mathrm{BB},t} \end{bmatrix}$$

or in more compact notation

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{y}_{K,t},$$

where $\boldsymbol{y}_t$ is a vector of all the observations in the hierarchy at time t, $\boldsymbol{S}$ is the summing matrix as defined above, and $\boldsymbol{y}_{K,t}$ is a vector of all the observation in the bottom level of the hierarchy at time t.

We are interested in generating forecasts for each series in the hierarchy. We denote as $\hat{y}_{\mathrm{j},h}$ the h-step-ahead forecast generated for the series at node j having observed the time series up to observation T and as $\hat{y}_h$ the h-step-ahead forecast generated for the "Total" series.[1] We refer to these as "base" forecasts. They are independent forecasts generated for each series in the hierarchy using a suitable forecasting method. These base forecasts are then combined to produce final forecasts for the whole hierarchy that aggregate in a manner that is consistent with the structure of the hierarchy. We refer to these as revised forecasts and denote them as $\tilde{y}_{\mathrm{j},h}$ and $\tilde{y}_h$ respectively.

There are a number of ways of combining the base forecasts in order to obtain revised forecasts. The following sections discuss some of the possible combining approaches.

Bottom-up method

A commonly applied method for hierarchical forecasting is the bottom-up approach. This approach involves first generating base independent forecasts for each series at the bottom level of the hierarchy and then aggregating these upwards to produce revised forecasts for the whole hierarchy.

For example, for the hierarchy of Figure 1 we first generate h-step-ahead base forecasts for the bottom level series: $\hat{y}_{\mathrm{AA},h}$, $\hat{y}_{\mathrm{AB},h}$, $\hat{y}_{\mathrm{AC},h}$, $\hat{y}_{\mathrm{BA},h}$ and $\hat{y}_{\mathrm{BB},h}$. Aggregating these up the hierarchy we get h-step-ahead forecasts for the rest of the series: $\tilde{y}_{\mathrm{A},h} = \hat{y}_{\mathrm{AA},h} + \hat{y}_{\mathrm{AB},h} + \hat{y}_{\mathrm{AC},h}$, $\tilde{y}_{\mathrm{B},h} = \hat{y}_{\mathrm{BA},h} + \hat{y}_{\mathrm{BB},h}$ and $\tilde{y}_h = \tilde{y}_{\mathrm{A},h} + \tilde{y}_{\mathrm{B},h}$. Note that for the bottom-up approach the revised forecasts for the bottom level series are equal to the base forecasts.

Using matrix notation we can again employ the summing matrix and write

$$\tilde{\boldsymbol{y}}_h = \boldsymbol{S}\hat{\boldsymbol{y}}_{K,h}.$$

The greatest advantage of this approach is that no information is lost due to aggregation. On the other hand bottom level data can be quite noisy and more challenging to model and forecast (Shlifer and Wolff 1979; Schwarzkopf, Tersine, and Morris 1988).

Top-down method

Top-down approaches involve first generating base forecasts for the "Total" series y_t on the top of the hierarchy and then disaggregating these downwards. We let $p_1, \dots, p_{m_K}$ be a set of proportions which dictate how the base forecasts of the "Total" series are to be distributed to revised forecasts for each series at the bottom level of the hierarchy. Once the bottom level forecasts have been generated we can use the summing matrix to generate forecasts for the rest of the series in the hierarchy. Note that for top-down approaches the top level revised forecasts are equal to the top level base forecasts, that is $\tilde{y}_h = \hat{y}_h$.

When the bottom-level series are noisy, the forecasts of the top-down approach can be more accurate than the bottom-up approach (Grunfeld and Griliches 1960; Fliedner 1999). The performance of the top-down approach depends on three factors: (1) the accuracy of the total series forecasts; (2) the accuracy of the disaggregate proportions; (3) the degree of accuracy

[1] We have simplified the usual notation of $\hat{y}_{T+h|T}$ for brevity.

in the base forecasts. Gross and Sohl (1990) studied the top-down approach extensively, and put forward 21 disaggregation methods. Furthermore, they found two promising methods that are also simple to use, namely average historical proportion and proportions of the historical averages.

Average historical proportions

The average historical proportions can be expressed as

$$p_j = \frac{1}{T}\sum_{t=1}^{T}\frac{y_{j,t}}{y_t}$$

for $j = 1, \ldots, m_K$. Each proportion p_j reflects the average of the historical proportions of the bottom level series $y_{j,t}$ over the period $t = 1, \ldots, T$ relative to the total aggregate y_t.

Proportions of the historical averages

The proportions of the historical averages are expressed by

$$p_j = \sum_{t=1}^{T}\frac{y_{j,t}}{T}\Big/\sum_{t=1}^{T}\frac{y_t}{T}$$

for $j = 1, \ldots, m_K$. Each proportion p_j captures the average historical value of the bottom level series $y_{j,t}$ relative to the average value of the total aggregate y_t.

The greatest attribute of such top-down approaches is their simplicity to apply. One only needs to model and generate forecasts for the most aggregated top level series. In general these approaches seem to produce quite reliable forecasts for the aggregate levels and they are very useful with low count data. On the other hand, their greatest disadvantage is the loss of information due to aggregation. With these top-down approaches, we are unable to capture and take advantage of individual series characteristics such as time dynamics, special events, etc.

Forecasted proportions

An alternative approach that improves on the historical and static nature of the proportions specified above is to use forecasted proportions (Athanasopoulos *et al.* 2009).

To demonstrate the intuition of this method, consider a one level hierarchy. We first generate h-step-ahead base forecasts for all the series independently. At level 1 we calculate the proportion of each h-step-ahead base forecast to the aggregate of all the h-step-ahead base forecasts at this level. We refer to these as the forecasted proportions and we use these to disaggregate the top level forecast and generate revised forecasts for the whole of the hierarchy.

For a K-level hierarchy this process is repeated for each node going from the top to the very bottom level. Applying this process leads to the following general rule for obtaining the forecasted proportions

$$p_j = \prod_{\ell=0}^{K-1}\frac{\hat{y}_{j,h}^{(\ell)}}{\hat{S}_{j,h}^{(\ell+1)}}$$

for $j = 1, 2, \ldots, m_K$. These forecasted proportions disaggregate the h-step-ahead base forecast of the "Total" series to h-step-ahead revised forecasts of the bottom level series. $\hat{y}_{j,h}^{(\ell)}$ is the h-step-ahead base forecast of the series that corresponds to the node which is ℓ levels above j. $\hat{S}_{j,h}^{(\ell)}$ is the sum of the h-step-ahead base forecasts below the node that is ℓ levels above node j and are directly connected to that node.

We will use the hierarchy of Figure 1 to explain this notation and to demonstrate how this general rule is reached. Assume we have generated independent base forecasts for each series in the hierarchy. Remember that for the top level "Total" series, $\tilde{y}_h = \hat{y}_h$. Here are some example using the above notation:

- $\hat{y}^{(1)}_{\mathrm{A},h} = \hat{y}^{(1)}_{\mathrm{B},h} = \tilde{y}_h$
- $\hat{y}^{(1)}_{\mathrm{BA},h} = \hat{y}^{(1)}_{\mathrm{BB},h} = \hat{y}_{\mathrm{B},h}$
- $\hat{y}^{(2)}_{\mathrm{AA},h} = \hat{y}^{(2)}_{\mathrm{AB},h} = \hat{y}^{(2)}_{\mathrm{AC},h} = \hat{y}^{(2)}_{\mathrm{BA},h} = \hat{y}^{(2)}_{\mathrm{BB},h} = \tilde{y}_h$
- $\hat{S}^{(1)}_{\mathrm{BA},h} = \hat{S}^{(1)}_{\mathrm{BB},h} = \hat{y}_{\mathrm{BA},h} + \hat{y}_{\mathrm{BB},h}$
- $\hat{S}^{(2)}_{\mathrm{BA},h} = \hat{S}^{(2)}_{\mathrm{BB},h} = \hat{S}^{(1)}_{\mathrm{A},h} = \hat{S}^{(1)}_{\mathrm{B},h} = \hat{S}_h = \hat{y}_{\mathrm{A},h} + \hat{y}_{\mathrm{B},h}$

Moving down the farthest left branch of the hierarchy the final revised forecasts are

$$\tilde{y}_{\mathrm{A},h} = \left(\frac{\hat{y}_{\mathrm{A},h}}{\hat{S}^{(1)}_{\mathrm{A},h}}\right)\tilde{y}_h = \left(\frac{\hat{y}^{(1)}_{\mathrm{AA},h}}{\hat{S}^{(2)}_{\mathrm{AA},h}}\right)\tilde{y}_h$$

and

$$\tilde{y}_{\mathrm{AA},h} = \left(\frac{\hat{y}_{\mathrm{AA},h}}{\hat{S}^{(1)}_{\mathrm{AA},h}}\right)\tilde{y}_{\mathrm{A},h} = \left(\frac{\hat{y}_{\mathrm{AA},h}}{\hat{S}^{(1)}_{\mathrm{AA},h}}\right)\left(\frac{\hat{y}^{(1)}_{\mathrm{AA},h}}{\hat{S}^{(2)}_{\mathrm{AA},h}}\right)\tilde{y}_h.$$

Consequently,

$$p_1 = \left(\frac{\hat{y}_{\mathrm{AA},h}}{\hat{S}^{(1)}_{\mathrm{AA},h}}\right)\left(\frac{\hat{y}^{(1)}_{\mathrm{AA},h}}{\hat{S}^{(2)}_{\mathrm{AA},h}}\right)\tilde{y}_h$$

The other proportions can be similarly obtained. The greatest disadvantage of the top-down forecasted proportions approach, which is a disadvantage of any top-down approach, is that they do not produce unbiased revised forecasts even if the base forecasts are unbiased.

Middle-out approach

The middle-out approach combines bottom-up and top-down approaches. First the "middle level" is chosen and base forecasts are generated for all the series of this level and the ones below. For the series above the middle level, revised forecasts are generated using the bottom-up approach by aggregating the "middle-level" base forecasts upwards. For the series below the "middle level", revised forecasts are generated using a top-down approach by disaggregating the "middle level" base forecasts downwards.

Optimal forecast combination

This approach involves first generating independent base forecast for each series in the hierarchy. As these base forecasts are independently generated they will not be "aggregate consistent" (i.e., they will not add up according to the hierarchical structure). The optimal combination approach, due to Hyndman *et al.* (2011), optimally combines the independent base forecasts and generates a set of revised forecasts that are as close as possible to the univariate forecasts but also aggregate consistently with the hierarchical structure.

Unlike any other existing method, this approach uses all the information available within a hierarchy. It allows for correlations and interactions between series at each level of the hierarchy, it accounts for ad hoc adjustments of forecasts at any level, and, provided the base forecasts are unbiased, it produces unbiased revised forecasts.

The general idea is derived from the representation of the h-step-ahead base forecasts for the whole of the hierarchy by a linear regression model. We write

$$\hat{\boldsymbol{y}}_h = \boldsymbol{S}\boldsymbol{\beta}_h + \boldsymbol{\varepsilon}_h$$

where $\hat{\boldsymbol{y}}_h$ is a vector of the h-step-ahead base forecasts for the whole hierarchy, $\boldsymbol{\beta}_h$ is the unknown mean of the future values of the bottom level K, and $\boldsymbol{\varepsilon}_h$ has zero mean and covariance matrix $\boldsymbol{\Sigma}_h$. Note that $\boldsymbol{\varepsilon}_h$ represents the error in the above regression and should not be confused with the h-step-ahead forecast error.

In general $\boldsymbol{\Sigma}_h$ in unknown. However, it can be shown that the correlations between series are not important when calculating point forecasts.

Provided the base forecasts approximately satisfy the hierarchical aggregation structure, (which should occur for any reasonable set of forecasts), then the errors should also approximately satisfy the hierarchical aggregation structure. That is, $\boldsymbol{\varepsilon}_h \approx \boldsymbol{S}\boldsymbol{\varepsilon}_{K,h}$ where $\boldsymbol{\varepsilon}_{K,h}$ contains the forecast errors in the bottom level. Under this assumption, Hyndman *et al.* (2011) show that the best linear unbiased estimator for $\boldsymbol{\beta}_h$ is $\hat{\boldsymbol{\beta}}_h = (\boldsymbol{S}'\boldsymbol{S})^{-1}\boldsymbol{S}'\hat{\boldsymbol{y}}_n(h)$. This leads to a set of revised forecasts given by

$$\tilde{\boldsymbol{y}}_h = \boldsymbol{S}(\boldsymbol{S}'\boldsymbol{S})^{-1}\boldsymbol{S}'\hat{\boldsymbol{y}}_h,$$

which does not depend on $\boldsymbol{\Sigma}_h$. However, prediction intervals for these forecasts will depend on $\boldsymbol{\Sigma}_h$ as

$$\mathrm{Var}(\tilde{\boldsymbol{y}}_h) = \boldsymbol{S}(\boldsymbol{S}'\boldsymbol{\Sigma}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}.$$

The hts package

Hierarchical time series forecasting methods described previously can be implemented in the **hts** package (Hyndman *et al.* 2014) for R (R Core Team 2013).

The `hts` function creates a hierarchical time series. The required inputs are the bottom-level time series, and information about the hierarchical structure. For example, the structure shown in Figure 1 is specified as follows:

```
# bts is a time series matrix containing the bottom-level series
# The first three series belong to one group, and the last two
# series belong to a different group
# nodes is a list containing the number of child nodes at each level.
bts <- ts(5 + matrix(sort(rnorm(500)), ncol=5, nrow=100))
y <- hts(bts, nodes=list(2, c(3, 2)))
```

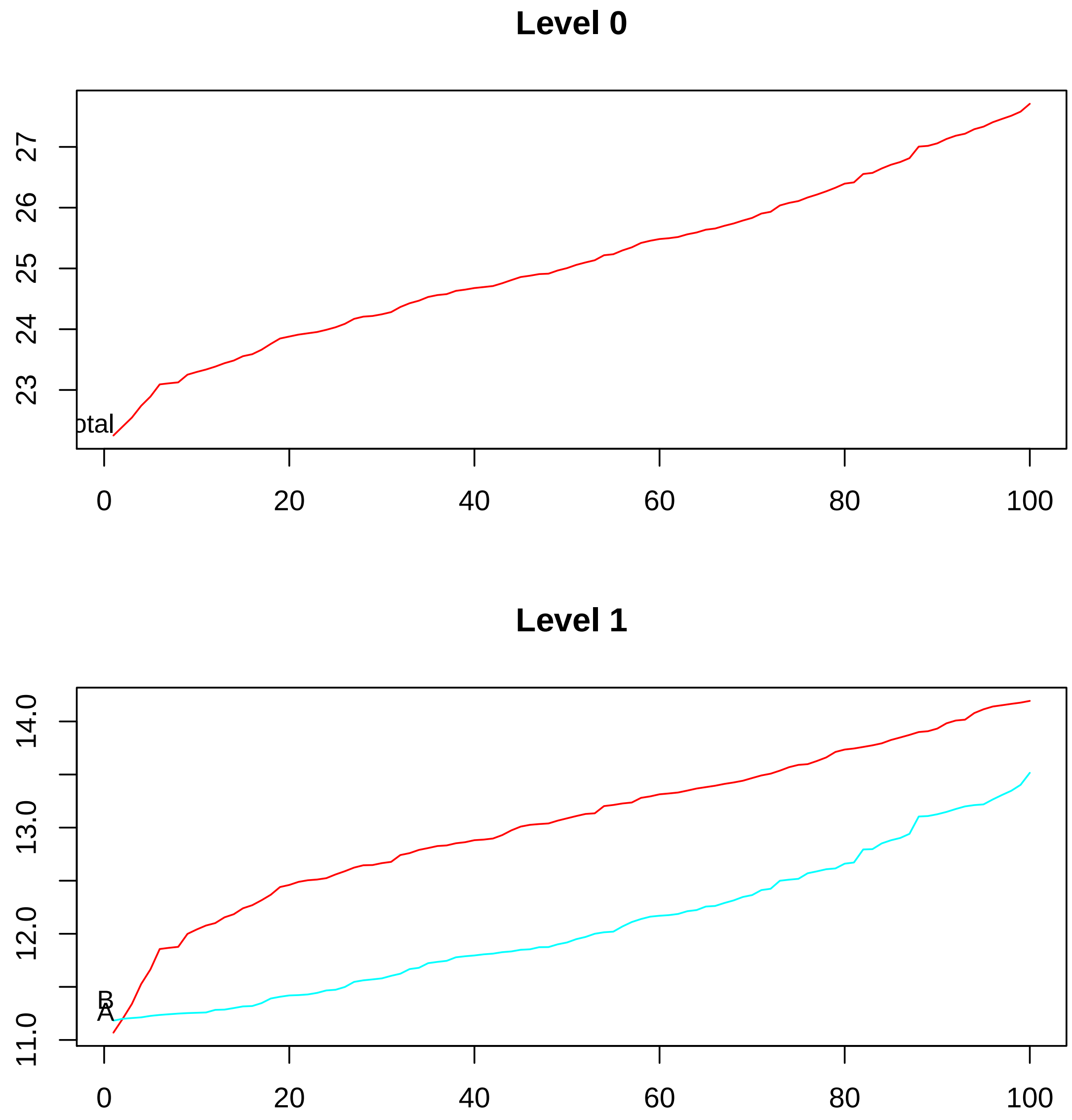
For a grouped but non-hierarchical time series, the `gts` function can be used. If there are more levels, the `groups` argument should be a matrix where each row contains the grouping structure for each level.

The `aggts` function is used to extract time series either across the whole hierarchy or at the selected levels, while the `smatrix` function is used to show the summing matrix as defined in Hyndman *et al.* (2011).

```
ally <-  aggts(y) # Returns all series in the hierarchy
somey <-  aggts(y, levels = c(0, 2)) # Returns time series at levels 0 and 2
S <- smatrix(y) # Returns the summing matrix
```

The `plot.gts` function allows users to display the historical time series for both an `hts` or `gts` object. By default, it displays time series across all hierarchical levels, although users can restrict the plot to the selected levels. For example, we plot the top two levels of an `hts` object in Figure 2.

Figure 2: Hierarchical time-series plots



```
plot(y, levels = c(0, 1))
```

Forecasts are obtained with the `forecast` function. By default, it produces forecasts using the optimal combination approach with exponential smoothing (ETS) models used for the base forecasts. But other forecasting models and disaggregation methods can be specified via the following arguments:

`fmethod` The forecasting method to be used for the base forecasts. Possible values are `"ets"`, `"arima"` and `"rw"`.

`method` The method used for reconciling the base forecasts. It can take the following values:

- `comb` Optimal combination forecasts;
- `bu` Bottom-up forecasts;
- `mo` Middle-out forecasts where the level used is specified by the level argument;
- `tdgsa` Top-down forecasts based on the average historical proportions (Gross-Sohl method A);
- `tdgsf` Top-down forecasts based on the proportion of historical averages (Gross-Sohl method F);
- `tdfp` Top-down forecasts using forecast proportions.

Forecasting regional infant mortality counts in Australia

We consider the infant mortality counts for eight states and territories of Australia: New South Wales (NSW), Victoria (VIC), Queensland (QLD), South Australia (SA), Western Australia (WA), Northern Territory (NT), Australian Capital Territory (ACTOT), and Tasmania (TAS). For each series, we have yearly observations on the number of infant deaths. The available data, from 1901 to 2003, were obtained from the Australian Social Science Data Archive. This data set is also publicly available in the **addb** package (Hyndman 2010) for R. Due to missing values, we use the data from 1933 to 2003 in our analysis. Based on these observations, we are interested in forecasting regional infant mortality counts from 2004 to 2013.

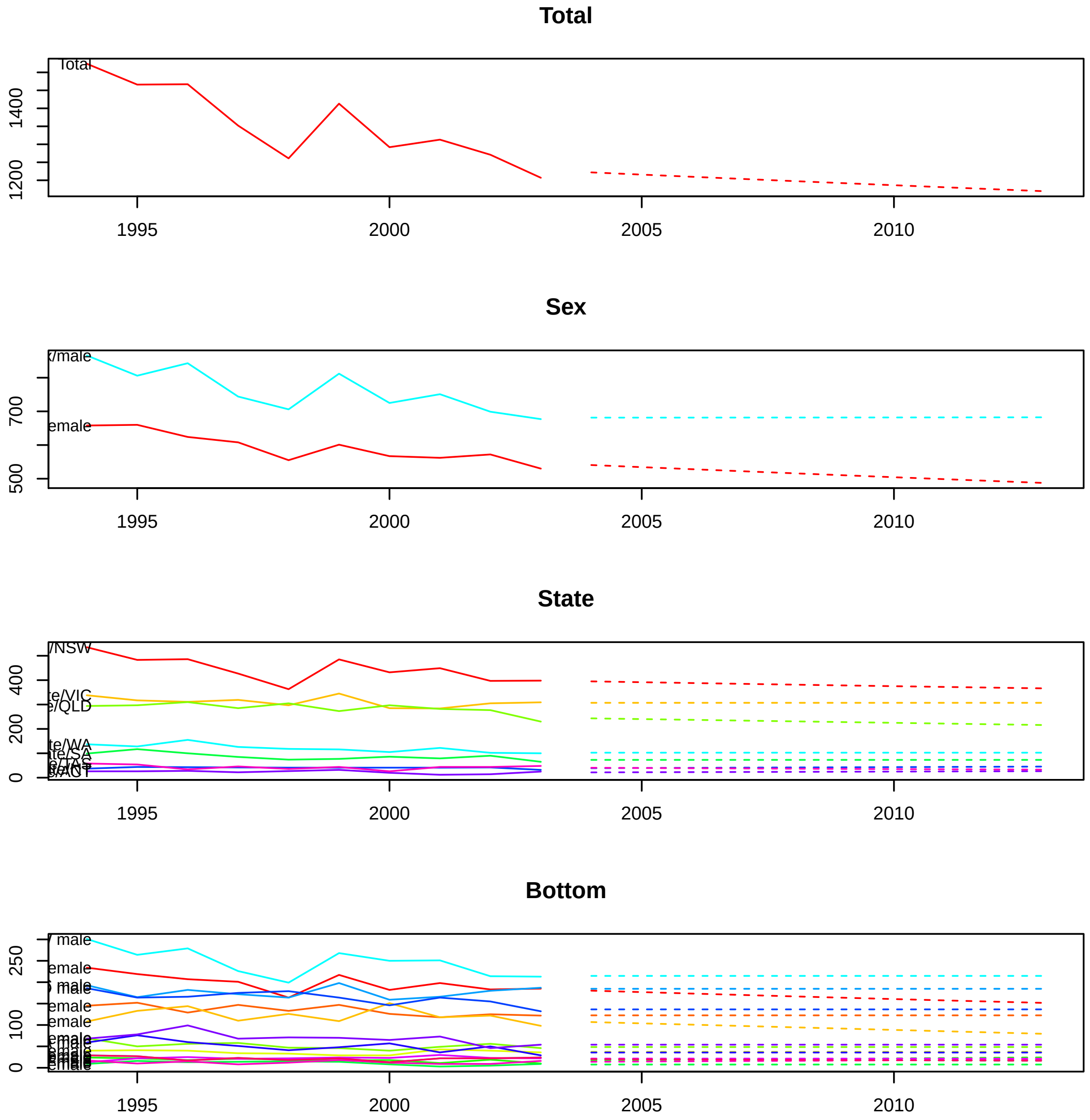
The data are grouped by gender and state, with two genders, eight states, and 16 series at the bottom level.

Figure 3 shows the forecasts of regional infant mortality counts across Australia. The forecasts indicate a continuing decline in infant mortality counts, due to improved health services. Moreover, male mortality counts are higher than female mortality counts. Figure 3 was produced using the following commands.

```
library(hts)
# Forecast 10-step-ahead using the bottom-up method
infantforecast <- forecast(infantgts, h=10, method="bu")
# plot the forecasts including the last ten historical years
plot(infantforecast, include=10)
```

While Figure 3 was produced using the `forecast.gts` function, there is alternative approach to produce forecasts which may give users extra flexibility.

Figure 3: Hierarchical time-series forecasts using the bottom-up approach



```
allts_infant <- aggts(infantgts)
allf <- matrix(, nrow=10, ncol=ncol(allts_infant))
# Users can select their preferred time-series forecasting method
# for each time series
for(i in 1:ncol(allts_infant))
  allf[,i] <- forecast(auto.arima(allts_infant[,i]), h=10, PI=FALSE)$mean
allf <- ts(allf, start=2004)
# combine the forecasts with the group matrix to get a gts object
g <- matrix(c(rep(1, 8), rep(2, 8), rep(1:8, 2)), nrow = 2, byrow = T)
y.f <- combinef(allf, groups = g)
```

In order to test the forecast accuracy, the `accuracy.gts` function can be used.

```
# set up the training sample and testing sample
data <- window(infantgts, start=1933, end=1993)
test <- window(infantgts, start=1994, end=2003)
forecast <- forecast(data, h=10, method="bu")
# calculate ME, RMSE, MAE, MAPE, MPE and MASE
accuracy.gts(forecast, test)
```

Changes from the hts package version 3

Every function in **hts** version 4 has been rewritten in order to improve its efficiency and flexibility. In addition, there are a few new features and changes which will affect users. The most important user-level changes are listed below.

hts Argument g is replaced with nodes, providing information about the hierarchical structure. This is a simpler and more efficient way of representing the hierarchical structure. The new argument nodes requires a list class containing the number of child nodes associated with the level rather than using the old gmatrix concept. The hierarchy presented in Figure 1 requires nodes specified as list(2, c(3, 2)) referring to 2 nodes at level 1, with 3 and 2 nodes respectively at level 2.

A new argument characters has also been added, which allows users to customize names for labelling but in a limited way. To illustrate the usage, we use the Anatomical Therapeutic Chemical (ATC) classification system. As shown in Table 1, the hierarchy has six levels including level 0 and one of the bottom series is named “A10BA02”. In order to construct the labels according to Table 1, we use characters = c(1, 2, 1, 1, 2). This indicates that the bottom level names can be read in segments of 1, 2, 1, 1 and 2 characters. The function can then construct the appropriate names for the upper levels.

A	Alimentary tract and metabolism (1st level, anatomical main group);
A10	Drugs used in diabetes (2nd level, therapeutic subgroup);
A10B	Blood glucose lowering drugs, excl. insulins (3rd level, pharmacological subgroup);
A10BA	Biguanides (4th level, chemical subgroup);
A10BA02	metformin (5th level, chemical substance);

Table 1: ATC classification system

aggts This is a new more flexible version of allts which allows the extraction of specific levels of the hierarchy. For instance, we are interested in observing the regional infant mortality counts across the states and they are obtained by specifying levels = "State" or levels=2. All time series in the hierarchy are returned if levels is omitted. The specification of levels now also works in the plot function in the same way.

```
    # Return time series aggregated for each state.
    statets <- aggts(infantgts, levels = "State")
    # Plot time series aggregated for each state.
    plot(infantgts, levels = "State")
```

forecast Fitted values and residuals at the bottom level are made available to users. These are reconciled in the same way as the forecasts, as long as users set keep.fitted = TRUE and `keep.resid = TRUE`. Furthermore, the **parallel** package has been imported for the support of parallel computing. The `forecast` function generates independent forecasts for each time series and it can take a long time to model and forecast these time series. Producing forecasts in parallel (with `parallel = TRUE`) enables users to save a significant amount of computing time.

Another major change is the new argument `weights`, associated with the optimal combination approach. The function uses ordinary least squares for the unscaled forecasts in the case of `weights = "none"` (as it did in previous versions). But when `weights = "sd"`, it will use weighted least squares for the forecasts scaled by the standard deviation of the forecast errors. A third possibility is `weights = "nseries"`; this case is useful when the bottom level series are on the same scale and the weights are then equal to the inverse of the row sums of $\boldsymbol{S}$.

```
  fcasts <- forecast(htseg1, h = 10, method = "comb", fmethod = "arima",
                     weights = "sd", keep.fitted = TRUE, parallel = TRUE)
```

`accuracy.gts` now returns in-sample error measures at the bottom level, if `keep.fitted = TRUE` in the `forecast` function and the argument `test` is missing.

```
  accuracy.gts(fcasts)  # In-sample error measure
```

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