0.1 `ls.mixed`: Mixed effects Linear Regression

Use multi-level linear regression if you have covariates that are grouped according to one or more classification factors and a continuous dependent variable.

While generally called multi-level models in the social sciences, this class of models is often referred to as mixed-effects models in the statistics literature and as hierarchical models in a Bayesian setting. This general class of models consists of linear models that are expressed as a function of both *fixed effects*, parameters corresponding to an entire population or certain repeatable levels of experimental factors, and *random effects*, parameters corresponding to individual experimental units drawn at random from a population.

Syntax

```
z.out <- zelig(formula= y ~ x1 + x2 + tag(z1 + z2 | g),
               data=mydata, model="lm.multi")

z.out <- zelig(formula= list(mu=y ~ xl + x2 + tag(z1, gamma | g),
               gamma= ~ tag(w1 + w2 | g)), data=mydata, model="lm.multi")
```

Inputs

`zelig()` takes the following arguments for `multi`:

- `formula`: a two-sided linear formula object describing the systematic component of the model, with the response on the left of a ˜ operator and the fixed effects terms, separated by `+` operators, on the right. Any random effects terms are included with the notation `tag(z1 + ... + zn | g)` with `z1 + ... + zn` specifying the model for the random effects and `g` the grouping structure. Random intercept terms are included with the notation `tag(1 | g)`.
  Alternatively, `formula` may be a list where the first entry, `mu`, is a two-sided linear formula object describing the systematic component of the model, with the repsonse on the left of a ˜ operator and the fixed effects terms, separated by `+` operators, on the right. Any random effects terms are included with the notation `tag(z1, gamma | g)` with `z1` specifying the individual level model for the random effects, `g` the grouping structure and `gamma` references the second equation in the list. The `gamma` equation is one-sided linear formula object with the group level model for the random effects on the right side of a ˜ operator. The model is specified with the notation `tag(w1 + ... + wn | g)` with `w1 + ... + wn` specifying the group level model and `g` the grouping structure.

Additional Inputs

In addition, `zelig()` accepts the following additional arguments for model specification:

- `data`: An optional data frame containing the variables named in `formula`. By default, the variables are taken from the environment from which `zelig()` is called.
- `family`: A GLM family, see `glm` and `family` in the `stats` package. If `family` is missing then a linear mixed model is fit; otherwise a generalized linear mixed model is fit. In the later case only `gaussian` family with `"log"` link is supported at the moment.
- `na.action`: A function that indicates what should happen when the data contain `NAs`. The default action (`na.fail`) causes `zelig()` to print an error message and terminate if there are any incomplete observations.

Additionally, users may wish to refer to `lmer` in the package `lme4` for more information, including control parameters for the estimation algorithm and their defaults.

Examples

1. Basic Example with First Differences

   Attach sample data:

```
> data(voteincome)
```

   Estimate model:

```
> z.out1 <- zelig(income ~ education + age + female + tag(1 | state),
+     data = voteincome, model = "ls.mixed")
```

   Summarize regression coefficients and estimated variance of random effects:

```
> summary(z.out1)
```

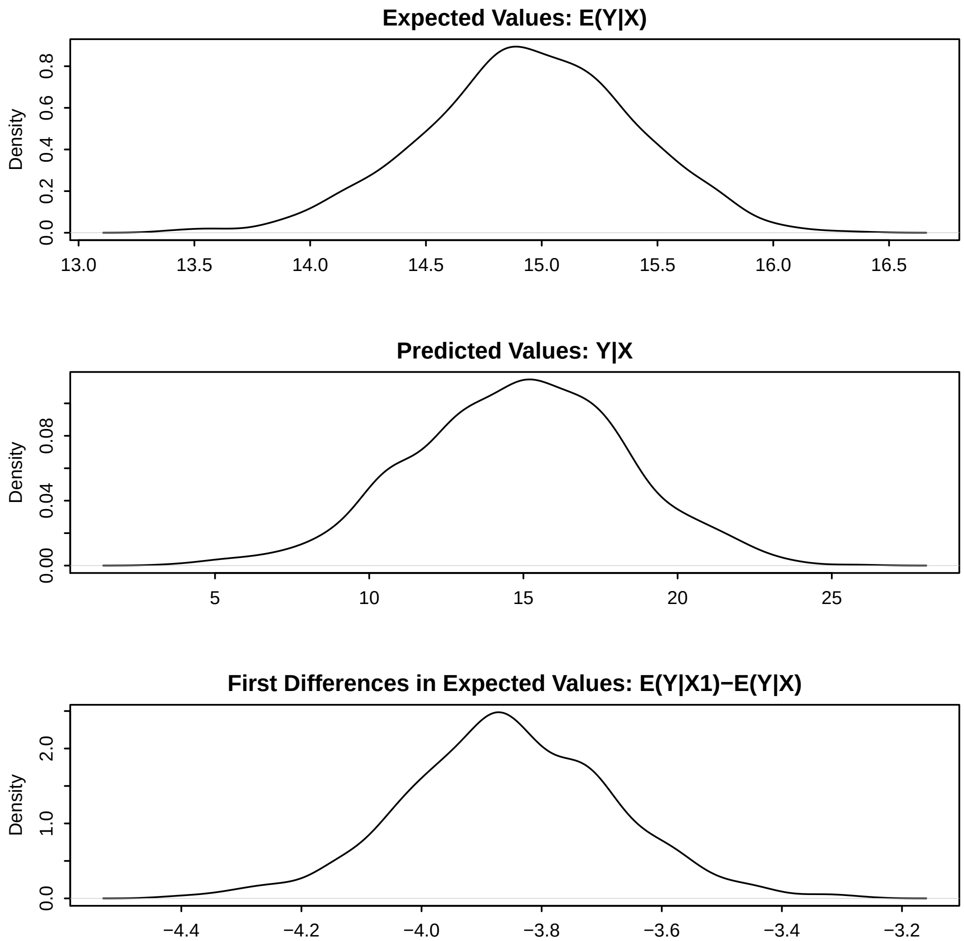
   Set explanatory variables to their default values, with high (80th percentile) and low (20th percentile) values for education:

```
> x.high <- setx(z.out1, education = quantile(voteincome$education,
+     0.8))
> x.low <- setx(z.out1, education = quantile(voteincome$education,
+     0.2))
```

   Generate first differences for the effect of high versus low education on income:

```
> s.out1 <- sim(z.out1, x = x.high, x1 = x.low)
> summary(s.out1)
```

```
> plot(s.out1)
```



Mixed effects linear regression model

Let Y_{ij} be the continuous dependent variable, realized for observation j in group i as y_{ij}, for $i = 1, \ldots, M$, $j = 1, \ldots, n_i$.

- The *stochastic component* is described by a univariate normal model with a vector of means μ_{ij} and scalar variance σ^2.

$$Y_{ij} \sim \text{Normal}(y_{ij} | \mu_{ij}, \sigma^2)$$

- The q-dimensional vector of *random effects*, b_i, is restricted to be mean zero, and therefore is completely characterized by the variance covarance matrix Ψ, a $(q \times q)$ symmetric positive semi-definite matrix.

$$b_i \sim Normal(0, \Psi)$$

- The *systematic component* is

$$\mu_{ij} \equiv X_{ij}\beta + Z_{ij}b_i$$

  where X_{ij} is the $(n_i \times p \times M)$ array of known fixed effects explanatory variables, β is the p-dimensional vector of fixed effects coefficients, Z_{ij} is the $(n_i \times q \times M)$ array of known random effects explanatory variables and b_i is the q-dimensional vector of random effects.

Quantities of Interest

- The predicted values (`qi$pr`) are draws from the normal distribution defined by mean μ_{ij} and variance σ^2,

$$\mu_{ij} = X_{ij}\beta + Z_{ij}b_i$$

  given X_{ij} and Z_{ij} and simulations of β and b_i from their posterior distributions. The estimated variance covariance matrices are taken as correct and are themselves not simulated.

- The expected values (`qi$ev`) are averaged over the stochastic components and are given by

$$E(Y_{ij}|X_{ij}) = X_{ij}\beta.$$

- The first difference (`qi$fd`) is given by the difference in expected values, conditional on X_{ij} and X'_{ij}, representing different values of the explanatory variables.

$$FD(Y_{ij}|X_{ij}, X'_{ij}) = E(Y_{ij}|X_{ij}) - E(Y_{ij}|X'_{ij})$$

- In conditional prediction models, the average predicted treatment effect (`qi$att.pr`) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^{M}\sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^{M} \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij}=1) - Y_{ij}\widehat{(t_{ij}=0)}\},$$

  where t_{ij} is a binary explanatory variable defining the treatment ($t_{ij}=1$) and control ($t_{ij}=0$) groups. Variation in the simulations is due to uncertainty in simulating $Y_{ij}(t_{ij}=0)$, the counterfactual predicted value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_{ij}=0$.

- In conditional prediction models, the average expected treatment effect (`qi$att.ev`) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^{M}\sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^{M} \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij}=1) - E[Y_{ij}(t_{ij}=0)]\},$$

where t_{ij} is a binary explanatory variable defining the treatment ($t_{ij} = 1$) and control ($t_{ij} = 0$) groups. Variation in the simulations is due to uncertainty in simulating $E[Y_{ij}(t_{ij} = 0)]$, the counterfactual expected value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_{ij} = 0$.

- If `"log"` link is used, expected values are computed as above and then exponentiated, while predicted values are draws from the log-normal distribution whose logarithm has mean and variance equal to μ_{ij} and σ^2, respectively.

Output Values

The output of each Zelig command contains useful information which you may view. You may examine the available information in `z.out` by using `slotNames(z.out)`, see the fixed effect coefficients by using `summary(z.out)@coefs`, and a default summary of information through `summary(z.out)`. Other elements available through the operator are listed below.

- From the `zelig()` output stored in `summary(z.out)`, you may extract:
  - `fixef`: numeric vector containing the conditional estimates of the fixed effects.
  - `ranef`: numeric vector containing the conditional modes of the random effects.
  - `frame`: the model frame for the model.
- From the `sim()` output stored in `s.out`, you may extract quantities of interest stored in a data frame:
  - `qi$pr`: the simulated predicted values drawn from the distributions defined by the expected values.
  - `qi$ev`: the simulated expected values for the specified values of x.
  - `qi$fd`: the simulated first differences in the expected values for the values specified in x and x1.
  - `qi$ate.pr`: the simulated average predicted treatment effect for the treated from conditional prediction models.
  - `qi$ate.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.

How to Cite

To cite the *ls.mixed* Zelig model:

Delia Bailey and Ferdinand Alimadhi. 2007. ”ls.mixed: Mixed effects linear model” in Kosuke Imai, Gary King, and Olivia Lau, ”Zelig: Everyone’s Statistical Software,”`http://gking.harvard.edu/zelig`

To cite Zelig as a whole, please reference these two sources:

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Imai, Kosuke, Gary King, and Olivia Lau. (2008). "Toward A Common Framework for Statistical Analysis and Development." Journal of Computational and Graphical Statistics, Vol. 17, No. 4 (December), pp. 892-913.

See also

Mixed effects linear regression is part of `lme4` package by Douglas M. Bates (Bates 2007). For a detailed discussion of mixed-effects models, please see **?**

Bibliography

Bates, D. (2007), *lme4: Fit linear and generalized linear mixed-effects models.*