lmer for SAS PROC MIXED Users

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1 Introduction

The `lmer` function from the `Matrix` library for R is used to fit linear mixed-effects models. It is similar in scope to the SAS procedure `PROC MIXED` described in Littell et al. (1996).

A file on the SAS Institute web site (http://www.sas.com) contains all the data sets in the book and all the SAS programs used in Littell et al. (1996). We have converted the data sets from the tabular representation used for SAS to the `groupedData` objects used by `lmer`. To help users familiar with `SAS PROC MIXED` get up to speed with `lmer` more quickly, we provide transcripts of some `lmer` analyses paralleling the `SAS PROC MIXED` analyses in Littell et al. (1996).

In this paper we highlight some of the similarities and differences of `lmer` analysis and `SAS PROC MIXED` analysis.

2 Similarities between lmer and SAS PROC MIXED

Both `SAS PROC MIXED` and `lmer` can fit linear mixed-effects models expressed in the Laird-Ware formulation. For a single level of grouping Laird and Ware (1982) write the n_i-dimensional response vector $\boldsymbol{y}_i$ for the ith experimental

unit as

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{\epsilon}_i, \quad i = 1, \ldots, M \\ \boldsymbol{b}_i &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\epsilon}_i \sim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}) \end{aligned} \tag{1}$$

where $\boldsymbol{\beta}$ is the p-dimensional vector of *fixed effects*, $\boldsymbol{b}_i$ is the q-dimensional vector of *random effects*, $\boldsymbol{X}_i$ (of size $n_i \times p$) and $\boldsymbol{Z}_i$ (of size $n_i \times q$) are known fixed-effects and random-effects regressor matrices, and $\boldsymbol{\epsilon}_i$ is the n_i-dimensional *within-group error* vector with a spherical Gaussian distribution. The assumption $\mathrm{Var}(\boldsymbol{\epsilon}_i) = \sigma^2\boldsymbol{I}$ can be relaxed using additional arguments in the model fitting.

The basic specification of the model requires a linear model expression for the fixed effects and a linear model expression for the random effects. In `SAS PROC MIXED` the fixed-effects part is specified in the `model` statement and the random-effects part in the `random` statement. In `lmer` the arguments are called `fixed` and `random`.

Both `SAS PROC MIXED` and `lmer` allow a mixed-effects model to be fit by maximum likelihood (`method = ml` in SAS) or by maximum residual likelihood, sometimes also called restricted maximum likelihood or REML. This is the default criterion in `SAS PROC MIXED` and in `lmer`. To get ML estimates in `lmer`, set the optional argument `method="REML"`.

3 Important differences

The output from `PROC MIXED` typically includes values of the Akaike Information Criterion (AIC) and Schwartz's Bayesian Criterion (SBC). These are used to compare different models fit to the same data. The output of the `summary` function applied to the object created by `lmer` also produces values of AIC and BIC but the definitions used in `PROC MIXED` and in `lmer` are different. In `lmer` the definitions are such that "smaller is better". In `PROC MIXED` the definitions are such that "bigger is better".

When models are fit by REML, the values of AIC, SBC (or BIC) and the log-likelihood can only be compared between models with exactly the same fixed-effects structure. When models are fit by maximum likelihood these criteria can be compared between any models fit to the same data. That is, these quality-of-fit criteria can be used to evaluate different fixed-effects specifications or different random-effects specifications or different specifications of both fixed effects and random effects. The greater flexibility of model

comparisons when using maximum likelihood is the reason that this is the default criterion in `lmer`.

We encourage developing and testing the model using likelihood ratio tests or the AIC and BIC criteria. Once a form for both the random effects and the fixed effects has been determined, the model can be refit with `REML = TRUE` if the restricted estimates of the variance components are desired.

4 Data manipulation

Both `PROC MIXED` and `lmer` work with data in a tabular form with one row per observation. There are, however, important differences in the internal representations of variables in the data.

In SAS a qualitative factor can be stored either as numerical values or alphanumeric labels. When a factor stored as numerical values is used in `PROC MIXED` it is listed in the `class` statement to indicate that it is a factor. In S this information is stored with the data itself by converting the variable to a factor when it is first stored. If the factor represents an ordered set of levels, it should be converted to an `ordered` factor.

For example the SAS code

```
data animal;
 input trait animal y;
 datalines;
1 1 6
1 2 8
1 3 7
2 1 9
2 2 5
2 3 .
;
```

would require that the `trait` and `animal` variables be specified in a class statement in any model that is fit.

In S these data could be read from a file, say `animal.dat`, and converted to factors by

```
animal <- read.table("animal.dat", header = TRUE)
animal$trait <- as.factor(animal$trait)
animal$animal <- as.factor(animal$animal)
```

In general it is a good idea to check the types of variables in a data frame before working with it. One way of doing this is to apply the function data.class to each variable in turn using the sapply function.

```
> sapply(Animal, data.class)
        Sire          Dam AvgDailyGain
    "factor"     "factor"    "numeric"
> str(Animal)
'data.frame':        20 obs. of  3 variables:
 $ Sire        : Factor w/ 5 levels "1","2","3","4",..: 1 1 1 1 2 2 2 2 3 3 .
 $ Dam         : Factor w/ 2 levels "1","2": 1 1 2 2 1 1 2 2 1 1 ...
 $ AvgDailyGain: num  2.24 1.85 2.05 2.41 1.99 1.93 2.72 2.32 2.33 2.68 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 AvgDailyGain ~ 1 | Sire/Dam
  .. .. ..- attr(*, ".Environment")=length 0 <environment>
  ..$ order.groups:List of 2
  .. ..$ Sire: logi TRUE
  .. ..$ Dam : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 1
  .. ..$ AvgDailyGain: chr "Average Daily Weight Gain"
  ..$ units       : list()
```

To make specification of models in lmer easier and to make graphic presentations more informative, we recommend converting from a data.frame object to a groupedData object. This class of objects contains a formula specifying the response, the primary covariate (if there is one) and the grouping factor or factors. The data sets from Littell et al. (1996) have been converted to groupedData objects in this directory.

4.1 Unique levels of factors

Designs with nested grouping factors are indicated differently in the two languages. An example of such an experimental design is the semiconductor experiment described in section 2.2 of Littell et al. (1996) where twelve wafers are assigned to four experimental treatments with three wafers per treatment. The levels for the wafer factor are 1, 2, and 3 but the wafer factor is only meaningful within the same level of the treatment factor, et. There is nothing

associating wafer `1` of the third treatment group with wafer `1` of the first treatment group.

In `SAS` this nesting of factors is denoted by `wafer(et)`. In S the nesting is written with `ET/Wafer` and read "wafer within ET". If both levels of nested factors are to be associated with random effects then this is all you need to know. You would use an expression with a `"/"` in the grouping factor part of the formula for the `groupedData` object. Then the random effects could be specified as

```
random = list( ET = ~ 1, Wafer = ~ 1 )
```

or, equivalently

```
random = ~ 1 | ET/Wafer
```

In this case, however, there would not usually be any random effects associated with the "experimental treatment" or `ET` factor. The only random effects are at the `Wafer` level. It is necessary to create a factor that will have unique levels for each `Wafer` within each level of `ET`. One way to do this is to assign

```
> Semiconductor$Grp <- with(Semiconductor, ET:Wafer)
```

after which we could specify a random effects term of `(1 | Grp)`.

4.2 General approach

As a general approach to importing data into S for mixed-effects analysis you should:

- Create a `data.frame` with one row per observation and one column per variable.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- If necessary, use `getGroups` to create a factor with unique levels from inner nested factors.
- Specify the formula for the response, the primary covariate and the grouping structure to create a `groupedData` object from the data frame. Labels and units for the response and the primary covariate can also be specified at this time as can `outer` and `inner` factor expressions.

- Plot the data. Plot it several ways. The use of trellis graphics is closely integrated with the `nlme` library. The trellis plots can provide invaluable insight into the structure of the data. Use them.

5 Contrasts

When comparing estimates produced by `SAS PROC MIXED` and by `lmer` one must be careful to consider the contrasts that are used to define the effects of factors. In SAS a model with an intercept and a qualitative factor is defined in terms of the intercept and the indicator variables for all but the last level of the factor. The default behaviour in S is to use the Helmert contrasts for the factor. On a balanced factor these provide a set of orthogonal contrasts. In R the default is the "treatment" contrasts which are almost the same as the SAS parameterization except that they drop the indicator of the first level, not the last level.

When in doubt, check which contrasts are being used with the contrasts function.

To make comparisons easier, you may find it worthwhile to declare

```
> options(contrasts = c(factor = "contr.SAS", ordered = "contr.poly"))
```

at the beginning of your session.

References

Nan M. Laird and James H. Ware. Random-effects models for longitudinal data. *Biometrics*, 38:963–974, 1982.

Ramon C. Littell, George A. Milliken, Walter W. Stroup, and Russell D. Wolfinger. *SAS System for Mixed Models*. SAS Institute, Inc., 1996.

A AvgDailyGain

```
> print(xyplot(adg ~ Treatment | Block, AvgDailyGain, type = c("g",
+     "p", "r"), xlab = "Treatment (amount of feed additive)",
+     ylab = "Average daily weight gain (lb.)", aspect = "xy",
+     index.cond = function(x, y) coef(lm(y ~ x))[1]))
```

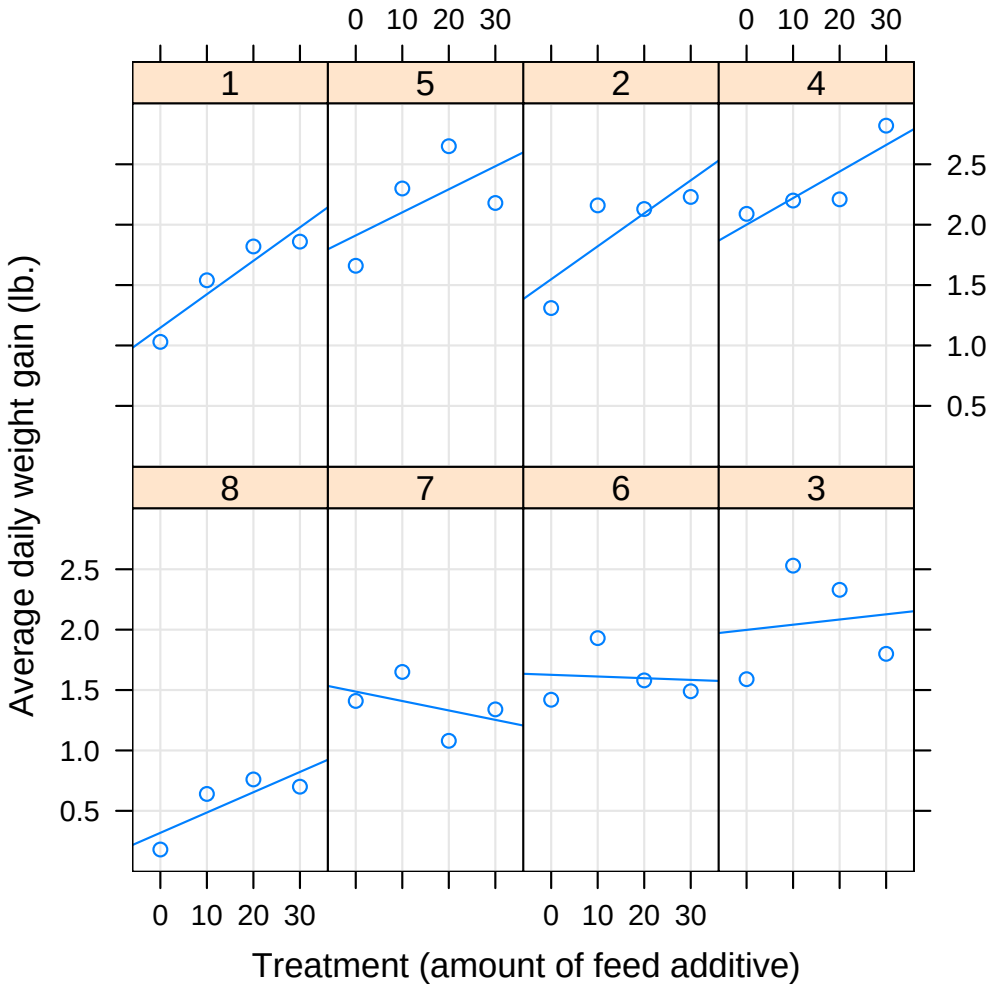

Figure 1: Average daily weight gain

```
> (fm1Adg <- lmer(adg ~ (Treatment - 1) * InitWt + (1 | Block),
+     AvgDailyGain))
Linear mixed-effects model fit by REML
Formula: adg ~ (Treatment - 1) * InitWt + (1 | Block)
   Data: AvgDailyGain
   AIC   BIC logLik MLdeviance REMLdeviance
 83.33 96.52 -32.66      10.10        65.33
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.25930  0.50922
 Residual             0.04943  0.22233
number of obs: 32, groups: Block, 8

Fixed effects:
                    Estimate Std. Error t value
Treatment0          0.439128   0.711092  0.6175
Treatment10         1.426113   0.637549  2.2369
Treatment20         0.479621   0.548889  0.8738
Treatment30         0.200115   0.775204  0.2581
InitWt              0.004448   0.002082  2.1368
Treatment0:InitWt  -0.002154   0.002786 -0.7732
Treatment10:InitWt -0.003365   0.002515 -1.3381
Treatment20:InitWt -0.001082   0.002488 -0.4351

Correlation of Fixed Effects:
            Trtmn0 Trtm10 Trtm20 Trtm30 InitWt Tr0:IW T10:IW
Treatment10  0.039
Treatment20  0.080  0.334
Treatment30  0.011  0.097  0.043
InitWt       0.050 -0.032  0.035 -0.967
Trtmnt0:InW -0.640  0.046 -0.024  0.754 -0.780
Trtmnt10:IW -0.021 -0.535 -0.178  0.781 -0.808  0.617
Trtmnt20:IW -0.040 -0.106 -0.512  0.828 -0.856  0.666  0.775
> anova(fm1Adg)
Analysis of Variance Table
                Df Sum Sq Mean Sq
Treatment        4 5.7250  1.4313
InitWt           1 0.5495  0.5495
Treatment:InitWt  3 0.1381  0.0460
> (fm2Adg <- lmer(adg ~ InitWt + Treatment + (1 | Block), AvgDailyGain))
```

```
Linear mixed-effects model fit by REML
Formula: adg ~ InitWt + Treatment + (1 | Block)
   Data: AvgDailyGain
   AIC   BIC logLik MLdeviance REMLdeviance
 48.34 57.13 -18.17      13.62        36.34
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.24084  0.49076
 Residual             0.05008  0.22379
number of obs: 32, groups: Block, 8

Fixed effects:
             Estimate Std. Error t value
(Intercept)  0.8011075  0.3556610   2.252
InitWt       0.0027797  0.0008334   3.336
Treatment0  -0.5520737  0.1148132  -4.808
Treatment10 -0.0685662  0.1189691  -0.576
Treatment20 -0.0881292  0.1162879  -0.758

Correlation of Fixed Effects:
            (Intr) InitWt Trtmn0 Trtm10
InitWt      -0.844
Treatment0   0.036 -0.224
Treatment10  0.139 -0.340  0.534
Treatment20  0.079 -0.272  0.530  0.545
> anova(fm2Adg)
Analysis of Variance Table
          Df  Sum Sq Mean Sq
InitWt     1 0.51456 0.51456
Treatment  3 1.52670 0.50890
> (fm3Adg <- lmer(adg ~ InitWt + Treatment - 1 + (1 | Block),
+     AvgDailyGain))
Linear mixed-effects model fit by REML
Formula: adg ~ InitWt + Treatment - 1 + (1 | Block)
   Data: AvgDailyGain
   AIC   BIC logLik MLdeviance REMLdeviance
 48.34 57.13 -18.17      13.62        36.34
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.24084  0.49076
```

```
 Residual             0.05008  0.22379
number of obs: 32, groups: Block, 8

Fixed effects:
             Estimate Std. Error t value
InitWt      0.0027797  0.0008334   3.336
Treatment0  0.2490338  0.3776318   0.659
Treatment10 0.7325413  0.3903798   1.876
Treatment20 0.7129784  0.3827685   1.863
Treatment30 0.8011075  0.3556610   2.252

Correlation of Fixed Effects:
            InitWt Trtmn0 Trtm10 Trtm20
Treatment0  -0.863
Treatment10 -0.873  0.957
Treatment20 -0.867  0.957  0.958
Treatment30 -0.844  0.953  0.953  0.953
```

B BIB

```
> print(xyplot(y ~ x | Block, BIB, groups = Treatment, type = c("g",
+     "p"), aspect = "xy", auto.key = list(points = TRUE, space = "right",
+     lines = FALSE)))

> (fm1BIB <- lmer(y ~ Treatment * x + (1 | Block), BIB))
Linear mixed-effects model fit by REML
Formula: y ~ Treatment * x + (1 | Block)
   Data: BIB
   AIC   BIC logLik MLdeviance REMLdeviance
 122.9 133.5 -52.45       93.5        104.9
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 18.238   4.2706
 Residual              1.201   1.0959
number of obs: 24, groups: Block, 8

Fixed effects:
             Estimate Std. Error t value
(Intercept)  22.36814    3.10213   7.211
Treatment1    4.42936    3.36581   1.316
```

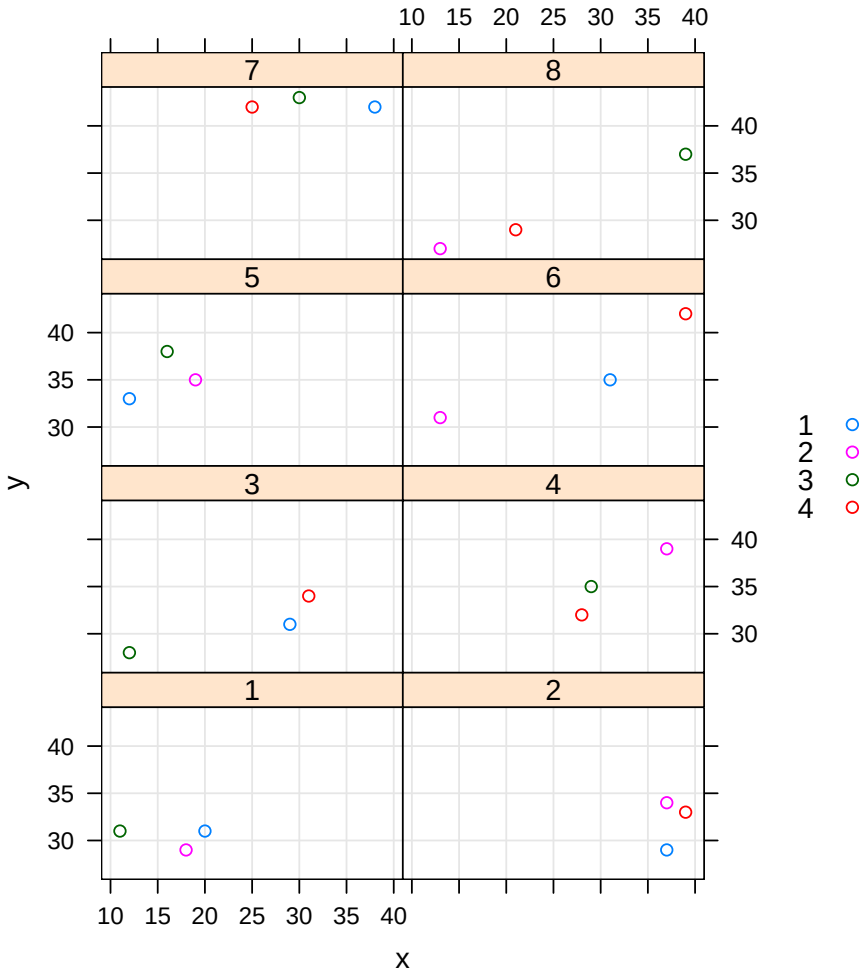

Figure 2: Balanced incomplete block design

```
Treatment2   -0.43747    2.93388  -0.149
Treatment3    6.27836    3.28278   1.913
x             0.44254    0.08708   5.082
Treatment1:x -0.22376    0.10611  -2.109
Treatment2:x  0.05339    0.09716   0.549
Treatment3:x -0.17917    0.11574  -1.548

Correlation of Fixed Effects:
            (Intr) Trtmn1 Trtmn2 Trtmn3 x      Trtm1: Trtm2:
Treatment1  -0.729
Treatment2  -0.778  0.797
Treatment3  -0.797  0.827  0.826
x           -0.859  0.797  0.865  0.886
Treatmnt1:x  0.709 -0.979 -0.774 -0.797 -0.799
Treatmnt2:x  0.722 -0.731 -0.965 -0.763 -0.829  0.729
Treatmnt3:x  0.769 -0.789 -0.790 -0.976 -0.879  0.777  0.748
> anova(fm1BIB)
Analysis of Variance Table
            Df  Sum Sq Mean Sq
Treatment    3  23.448   7.816
x            1 136.808 136.808
Treatment:x  3  18.426   6.142
> (fm2BIB <- lmer(y ~ Treatment + x:Grp + (1 | Block), BIB))
Linear mixed-effects model fit by REML
Formula: y ~ Treatment + x:Grp + (1 | Block)
   Data: BIB
   AIC   BIC logLik MLdeviance REMLdeviance
 113.2 121.4 -49.59      94.09        99.18
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 18.5214  4.3036
 Residual              1.0380  1.0188
number of obs: 24, groups: Block, 8

Fixed effects:
            Estimate Std. Error t value
(Intercept) 20.94523    2.06223  10.157
Treatment1   5.34139    1.97584   2.703
Treatment2   1.13555    0.71404   1.590
Treatment3   8.18098    1.77022   4.621
```

```
x:Grp13      0.23952    0.04297   5.575
x:Grp24      0.48923    0.04412  11.087

Correlation of Fixed Effects:
           (Intr) Trtmn1 Trtmn2 Trtmn3 x:Gr13
Treatment1 -0.501
Treatment2 -0.431  0.559
Treatment3 -0.527  0.942  0.581
x:Grp13     0.027 -0.663 -0.165 -0.605
x:Grp24    -0.639  0.651  0.452  0.688  0.042
> anova(fm2BIB)
Analysis of Variance Table
          Df  Sum Sq Mean Sq
Treatment  3  23.424   7.808
x:Grp      2 154.733  77.366
```

C Bond

```
> (fm1Bond <- lmer(pressure ~ Metal + (1 | Ingot), Bond))
Linear mixed-effects model fit by REML
Formula: pressure ~ Metal + (1 | Ingot)
   Data: Bond
   AIC   BIC logLik MLdeviance REMLdeviance
 115.8 120.0  -53.9      115.7        107.8
Random effects:
 Groups   Name        Variance Std.Dev.
 Ingot    (Intercept) 11.452   3.3841
 Residual             10.370   3.2203
number of obs: 21, groups: Ingot, 7

Fixed effects:
            Estimate Std. Error t value
(Intercept)  71.1000     1.7656   40.27
Metalc       -0.9143     1.7213   -0.53
Metali        4.8000     1.7213    2.79

Correlation of Fixed Effects:
       (Intr) Metalc
Metalc -0.487
Metali -0.487  0.500
```

```
> anova(fm1Bond)
Analysis of Variance Table
      Df Sum Sq Mean Sq
Metal  2 131.90   65.95
```

D Cultivation

```
> str(Cultivation)
'data.frame':        24 obs. of  4 variables:
 $ Block: Factor w/ 4 levels "1","2","3","4": 1 1 1 1 1 1 2 2 2 2 ...
 $ Cult : Factor w/ 2 levels "a","b": 1 1 1 2 2 2 1 1 1 2 ...
 $ Inoc : Factor w/ 3 levels "con","dea","liv": 1 2 3 1 2 3 1 2 3 1 ...
 $ drywt: num  27.4 29.7 34.5 29.4 32.5 34.4 28.9 28.7 33.4 28.7 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 drywt ~ 1 | Block/Cult
  .. .. ..- attr(*, ".Environment")=length 7 <environment>
  ..$ order.groups:List of 2
  .. ..$ Block: logi TRUE
  .. ..$ Cult : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Cult:Class 'formula' length 2 ~Inoc
  .. .. .. ..- attr(*, ".Environment")=length 7 <environment>
  ..$ labels      :List of 1
  .. ..$ drywt: chr "Yield"
  ..$ units       : list()
> xtabs(~Block + Cult, Cultivation)
     Cult
Block a b
    1 3 3
    2 3 3
    3 3 3
    4 3 3
> (fm1Cult <- lmer(drywt ~ Inoc * Cult + (1 | Block) + (1 |
+     Cult), Cultivation))
Linear mixed-effects model fit by REML
Formula: drywt ~ Inoc * Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
   AIC   BIC logLik MLdeviance REMLdeviance
```

```
 84.49 93.91 -34.24      74.94        68.49
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.20737  1.0988
 Cult     (Intercept) 0.26585  0.5156
 Residual             1.19632  1.0938
number of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
             Estimate Std. Error t value
(Intercept)    33.5250     0.9310   36.01
Inoccon        -5.5000     0.7734   -7.11
Inocdea        -2.8750     0.7734   -3.72
Culta          -0.3750     1.0629   -0.35
Inoccon:Culta   0.2500     1.0938    0.23
Inocdea:Culta  -1.0250     1.0938   -0.94

Correlation of Fixed Effects:
            (Intr) Inoccn Inocde Culta  Incc:C
Inoccon     -0.415
Inocdea     -0.415  0.500
Culta       -0.571  0.364  0.364
Inoccon:Clt  0.294 -0.707 -0.354 -0.514
Inocdea:Clt  0.294 -0.354 -0.707 -0.514  0.500
> anova(fm1Cult)
Analysis of Variance Table
          Df  Sum Sq Mean Sq
Inoc       2 118.176  59.088
Cult       1   0.656   0.656
Inoc:Cult  2   1.826   0.913
> (fm2Cult <- lmer(drywt ~ Inoc + Cult + (1 | Block) + (1 |
+     Cult), Cultivation))
Linear mixed-effects model fit by REML
Formula: drywt ~ Inoc + Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
   AIC   BIC logLik MLdeviance REMLdeviance
 85.75 92.82 -36.88       76.9        73.75
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.21290  1.10132
```

```
 Cult     (Intercept) 0.25844  0.50837
 Residual             1.16298  1.07842
number of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
            Estimate Std. Error t value
(Intercept)  33.6542     0.8692   38.72
Inoccon      -5.3750     0.5392   -9.97
Inocdea      -3.3875     0.5392   -6.28
Culta        -0.6333     0.8430   -0.75

Correlation of Fixed Effects:
        (Intr) Inoccn Inocde
Inoccon -0.310
Inocdea -0.310  0.500
Culta   -0.485  0.000  0.000
> anova(fm2Cult)
Analysis of Variance Table
     Df  Sum Sq Mean Sq
Inoc  2 118.176  59.088
Cult  1   0.656   0.656
> (fm3Cult <- lmer(drywt ~ Inoc + (1 | Block) + (1 | Cult),
+     Cultivation))
Linear mixed-effects model fit by REML
Formula: drywt ~ Inoc + (1 | Block) + (1 | Cult)
   Data: Cultivation
   AIC   BIC logLik MLdeviance REMLdeviance
 85.68 91.57 -37.84      77.32        75.68
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.21031  1.10014
 Cult     (Intercept) 0.10386  0.32228
 Residual             1.16327  1.07855
number of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
            Estimate Std. Error t value
(Intercept)  33.3375     0.7070   47.15
Inoccon      -5.3750     0.5393   -9.97
Inocdea      -3.3875     0.5393   -6.28
```

```
Correlation of Fixed Effects:
        (Intr) Inoccn
Inoccon -0.381
Inocdea -0.381  0.500
> anova(fm3Cult)
Analysis of Variance Table
     Df  Sum Sq Mean Sq
Inoc  2 118.176  59.088
```

E Demand

```
> (fm1Demand <- lmer(log(d) ~ log(y) + log(rd) + log(rt) +
+     log(rs) + (1 | State) + (1 | Year), Demand))
Linear mixed-effects model fit by REML
Formula: log(d) ~ log(y) + log(rd) + log(rt) + log(rs) + (1 | State) +      (
   Data: Demand
    AIC    BIC logLik MLdeviance REMLdeviance
 -226.2 -209.8  120.1     -260.5       -240.2
Random effects:
 Groups   Name        Variance   Std.Dev.
 Year     (Intercept) 0.00026462 0.016267
 State    (Intercept) 0.02944077 0.171583
 Residual             0.00111723 0.033425
number of obs: 77, groups: Year, 11; State, 7

Fixed effects:
            Estimate Std. Error t value
(Intercept) -1.28470    0.72339  -1.776
log(y)       1.06989    0.10392  10.295
log(rd)     -0.29539    0.05246  -5.631
log(rt)      0.03989    0.02789   1.430
log(rs)     -0.32675    0.11439  -2.856

Correlation of Fixed Effects:
        (Intr) log(y) lg(rd) lg(rt)
log(y)  -0.976
log(rd)  0.383 -0.227
log(rt)  0.077 -0.062 -0.337
log(rs)  0.444 -0.600 -0.270 -0.323
```

F HR

```
> (fm1HR <- lmer(HR ~ Time * Drug + baseHR + (Time | Patient),
+     HR))
Linear mixed-effects model fit by REML
Formula: HR ~ Time * Drug + baseHR + (Time | Patient)
   Data: HR
   AIC   BIC logLik MLdeviance REMLdeviance
 787.6 815.5 -383.8      788.1        767.6
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 60.590   7.7839
          Time        37.760   6.1449   -0.563
 Residual             24.371   4.9367
number of obs: 120, groups: Patient, 24

Fixed effects:
            Estimate Std. Error t value
(Intercept)  33.9863    10.2761   3.307
Time         -3.1970     3.0848  -1.036
Druga         3.5988     4.2302   0.851
Drugb         7.0911     4.2082   1.685
baseHR        0.5433     0.1161   4.681
Time:Druga   -7.5013     4.3625  -1.719
Time:Drugb   -3.9894     4.3625  -0.914

Correlation of Fixed Effects:
           (Intr) Time   Druga  Drugb  baseHR Tim:Drg
Time       -0.162
Druga      -0.308  0.394
Drugb      -0.244  0.396  0.501
baseHR     -0.957  0.000  0.110  0.041
Time:Druga  0.115 -0.707 -0.557 -0.280  0.000
Time:Drugb  0.115 -0.707 -0.279 -0.560  0.000  0.500
> anova(fm1HR)
Analysis of Variance Table
          Df Sum Sq Mean Sq
Time       1 379.43  379.43
Drug       2  93.08   46.54
baseHR     1 534.05  534.05
Time:Drug  2  72.15   36.08
```

```
> (fm3HR <- lmer(HR ~ Time + Drug + baseHR + (Time | Patient),
+     HR))
Linear mixed-effects model fit by REML
Formula: HR ~ Time + Drug + baseHR + (Time | Patient)
   Data: HR
   AIC   BIC logLik MLdeviance REMLdeviance
 795.8 818.1 -389.9      791.2        779.8
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 61.490   7.8415
          Time        41.057   6.4076   -0.572
 Residual             24.370   4.9366
number of obs: 120, groups: Patient, 24

Fixed effects:
            Estimate Std. Error t value
(Intercept)  36.0596    10.1799   3.542
Time         -7.0273     1.8191  -3.863
Druga        -0.4567     3.5094  -0.130
Drugb         4.9343     3.4830   1.417
baseHR        0.5433     0.1160   4.684

Correlation of Fixed Effects:
       (Intr) Time   Druga  Drugb
Time   -0.097
Druga  -0.297  0.000
Drugb  -0.219  0.000  0.502
baseHR -0.966  0.000  0.132  0.050
> anova(fm3HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq
Time    1 363.67  363.67
Drug    2  93.23   46.61
baseHR  1 534.77  534.77
> (fm4HR <- lmer(HR ~ Time + baseHR + (Time | Patient), HR))
Linear mixed-effects model fit by REML
Formula: HR ~ Time + baseHR + (Time | Patient)
   Data: HR
   AIC   BIC logLik MLdeviance REMLdeviance
 803.1 819.9 -395.6      794.3        791.1
```

```
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 62.578   7.9106
          Time        40.614   6.3729   -0.553
 Residual             24.429   4.9426
number of obs: 120, groups: Patient, 24

Fixed effects:
            Estimate Std. Error t value
(Intercept)  36.9198     9.8722   3.740
Time         -7.0273     1.8151  -3.872
baseHR        0.5509     0.1172   4.701

Correlation of Fixed Effects:
       (Intr) Time
Time   -0.098
baseHR -0.984  0.000
> anova(fm4HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq
Time    1 366.16  366.16
baseHR  1 539.83  539.83
```

G Mississippi

```
> (fm1Miss <- lmer(y ~ 1 + (1 | influent), Mississippi))
Linear mixed-effects model fit by REML
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
   AIC   BIC logLik MLdeviance REMLdeviance
 256.4 259.6 -126.2      256.6        252.4
Random effects:
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 63.324   7.9576
 Residual             42.658   6.5313
number of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)   21.223      3.429   6.189
```

```
> (fm1MLMiss <- lmer(y ~ 1 + (1 | influent), Mississippi, method = "ML"))
Linear mixed-effects model fit by maximum likelihood
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
   AIC   BIC logLik MLdeviance REMLdeviance
 260.6 263.8 -128.3      256.6        252.4
Random effects:
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 51.255   7.1592
 Residual             42.697   6.5343
number of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)   21.217      3.122   6.796
> ranef(fm1MLMiss)
An object of class "ranef.lmer"
[[1]]
  (Intercept)
1   0.3097833
2  -6.5772271
3  -3.7862742
4   2.8826708
5  -5.8435201
6  13.0145672
> ranef(fm1Miss)
An object of class "ranef.lmer"
[[1]]
  (Intercept)
1    0.309286
2   -6.719335
3   -3.897948
4    2.946106
5   -6.012988
6   13.374879
> VarCorr(fm1Miss)
$influent
1 x 1 Matrix of class "dpoMatrix"
            (Intercept)
(Intercept)    63.32364
```

```
attr(,"sc")
   scale
6.531315
> (fm2Miss <- lmer(y ~ Type + (1 | influent), Mississippi))
Linear mixed-effects model fit by REML
Formula: y ~ Type + (1 | influent)
   Data: Mississippi
   AIC   BIC logLik MLdeviance REMLdeviance
 242.5 249.0 -117.3      247.5        234.5
Random effects:
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 15.031   3.8770
 Residual             42.502   6.5194
number of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)   36.400      4.851   7.504
Type1        -20.800      5.941  -3.501
Type2        -16.462      5.524  -2.980

Correlation of Fixed Effects:
      (Intr) Type1
Type1 -0.816
Type2 -0.878  0.717
> anova(fm2Miss)
Analysis of Variance Table
     Df Sum Sq Mean Sq
Type  2 540.26  270.13
```

H Multilocation

```
> str(Multilocation)
'data.frame':       108 obs. of  7 variables:
 $ obs     : num  3 4 6 7 9 10 12 16 19 20 ...
 $ Location: Factor w/ 9 levels "A","B","C","D",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ Block   : Factor w/ 3 levels "1","2","3": 1 1 1 1 2 2 2 2 3 3 ...
 $ Trt     : Factor w/ 4 levels "1","2","3","4": 3 4 2 1 2 1 3 4 1 2 ...
 $ Adj     : num  3.16 3.12 3.16 3.25 2.71 ...
```

```
 $ Fe      : num  7.10 6.68 6.83 6.53 8.25 ...
 $ Grp     : Factor w/ 27 levels "A/1","A/2","A/3",..: 1 1 1 1 2 2 2 2 3 3 ..
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 Adj ~ 1 | Location/Block
  .. .. ..- attr(*, ".Environment")=length 17 <environment>
  ..$ order.groups:List of 2
  .. ..$ Location: logi TRUE
  .. ..$ Block   : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Block:Class 'formula' length 2 ~Trt
  .. .. .. ..- attr(*, ".Environment")=length 17 <environment>
  ..$ labels      :List of 1
  .. ..$ Adj: chr "Adjusted yield"
  ..$ units       : list()
> Multilocation$Grp <- with(Multilocation, Block:Location)
> (fm1Mult <- lmer(Adj ~ Location * Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location * Trt + (1 | Grp)
   Data: Multilocation
   AIC   BIC logLik MLdeviance REMLdeviance
 84.65 183.9 -5.323     -87.15        10.65
Random effects:
 Groups   Name        Variance Std.Dev.
 Grp      (Intercept) 0.005624 0.074993
 Residual             0.034576 0.185945
number of obs: 108, groups: Grp, 27

Fixed effects:
               Estimate Std. Error t value
(Intercept)     2.35923    0.11576  20.381
LocationA       0.64930    0.16371   3.966
LocationB       0.06643    0.16371   0.406
LocationC       0.54533    0.16371   3.331
LocationD       0.37413    0.16371   2.285
LocationE       0.55000    0.16371   3.360
LocationF       0.99810    0.16371   6.097
LocationG       0.36057    0.16371   2.203
LocationH       1.01403    0.16371   6.194
```

```
Trt1            0.22720    0.15182   1.496
Trt2           -0.00140    0.15182  -0.009
Trt3            0.42323    0.15182   2.788
LocationA:Trt1 -0.18853    0.21471  -0.878
LocationB:Trt1 -0.27523    0.21471  -1.282
LocationC:Trt1 -0.04000    0.21471  -0.186
LocationD:Trt1 -0.53513    0.21471  -2.492
LocationE:Trt1 -0.26297    0.21471  -1.225
LocationF:Trt1 -0.27153    0.21471  -1.265
LocationG:Trt1  0.20323    0.21471   0.947
LocationH:Trt1 -0.14953    0.21471  -0.696
LocationA:Trt2 -0.09347    0.21471  -0.435
LocationB:Trt2 -0.32273    0.21471  -1.503
LocationC:Trt2  0.08960    0.21471   0.417
LocationD:Trt2 -0.29693    0.21471  -1.383
LocationE:Trt2 -0.30693    0.21471  -1.430
LocationF:Trt2 -0.30993    0.21471  -1.443
LocationG:Trt2 -0.10860    0.21471  -0.506
LocationH:Trt2 -0.33060    0.21471  -1.540
LocationA:Trt3 -0.40247    0.21471  -1.874
LocationB:Trt3 -0.56550    0.21471  -2.634
LocationC:Trt3 -0.12247    0.21471  -0.570
LocationD:Trt3 -0.54840    0.21471  -2.554
LocationE:Trt3 -0.32863    0.21471  -1.531
LocationF:Trt3 -0.46257    0.21471  -2.154
LocationG:Trt3 -0.25297    0.21471  -1.178
LocationH:Trt3 -0.37203    0.21471  -1.733

Correlation of Fixed Effects:
            (Intr) LoctnA LoctnB LoctnC LoctnD LoctnE LoctnF LoctnG LoctnH
LocationA   -0.707
LocationB   -0.707  0.500
LocationC   -0.707  0.500  0.500
LocationD   -0.707  0.500  0.500  0.500
LocationE   -0.707  0.500  0.500  0.500  0.500
LocationF   -0.707  0.500  0.500  0.500  0.500  0.500
LocationG   -0.707  0.500  0.500  0.500  0.500  0.500  0.500
LocationH   -0.707  0.500  0.500  0.500  0.500  0.500  0.500  0.500
Trt1        -0.656  0.464  0.464  0.464  0.464  0.464  0.464  0.464  0.464
Trt2        -0.656  0.464  0.464  0.464  0.464  0.464  0.464  0.464  0.464
```

```
Trt3        -0.656  0.464  0.464  0.464  0.464  0.464  0.464  0.464  0.464
LoctnA:Trt1  0.464 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnB:Trt1  0.464 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnC:Trt1  0.464 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnD:Trt1  0.464 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328
LoctnE:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328
LoctnF:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328
LoctnG:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328
LoctnH:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656
LoctnA:Trt2  0.464 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnB:Trt2  0.464 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnC:Trt2  0.464 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnD:Trt2  0.464 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328
LoctnE:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328
LoctnF:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328
LoctnG:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328
LoctnH:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656
LoctnA:Trt3  0.464 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnB:Trt3  0.464 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnC:Trt3  0.464 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnD:Trt3  0.464 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328
LoctnE:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328
LoctnF:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328
LoctnG:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328
LoctnH:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656
            Trt1   Trt2   Trt3   LcA:T1 LcB:T1 LcC:T1 LcD:T1 LcE:T1 LcF:T1
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
Trt1
Trt2         0.500
Trt3         0.500  0.500
LoctnA:Trt1 -0.707 -0.354 -0.354
LoctnB:Trt1 -0.707 -0.354 -0.354  0.500
LoctnC:Trt1 -0.707 -0.354 -0.354  0.500  0.500
```

```
LoctnD:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500
LoctnE:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500
LoctnF:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500  0.500
LoctnG:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500  0.500  0.500
LoctnH:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500  0.500  0.500
LoctnA:Trt2 -0.354 -0.707 -0.354  0.500  0.250  0.250  0.250  0.250  0.250
LoctnB:Trt2 -0.354 -0.707 -0.354  0.250  0.500  0.250  0.250  0.250  0.250
LoctnC:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.500  0.250  0.250  0.250
LoctnD:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.500  0.250  0.250
LoctnE:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.500  0.250
LoctnF:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.250  0.500
LoctnG:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.250  0.250
LoctnH:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.250  0.250
LoctnA:Trt3 -0.354 -0.354 -0.707  0.500  0.250  0.250  0.250  0.250  0.250
LoctnB:Trt3 -0.354 -0.354 -0.707  0.250  0.500  0.250  0.250  0.250  0.250
LoctnC:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.500  0.250  0.250  0.250
LoctnD:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.500  0.250  0.250
LoctnE:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.500  0.250
LoctnF:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.250  0.500
LoctnG:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.250  0.250
LoctnH:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.250  0.250
            LcG:T1 LcH:T1 LcA:T2 LcB:T2 LcC:T2 LcD:T2 LcE:T2 LcF:T2 LcG:T2
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
Trt1
Trt2
Trt3
LoctnA:Trt1
LoctnB:Trt1
LoctnC:Trt1
LoctnD:Trt1
LoctnE:Trt1
LoctnF:Trt1
LoctnG:Trt1
```

LoctnH:Trt1	0.500								
LoctnA:Trt2	0.250	0.250							
LoctnB:Trt2	0.250	0.250	0.500						
LoctnC:Trt2	0.250	0.250	0.500	0.500					
LoctnD:Trt2	0.250	0.250	0.500	0.500	0.500				
LoctnE:Trt2	0.250	0.250	0.500	0.500	0.500	0.500			
LoctnF:Trt2	0.250	0.250	0.500	0.500	0.500	0.500	0.500		
LoctnG:Trt2	0.500	0.250	0.500	0.500	0.500	0.500	0.500	0.500	
LoctnH:Trt2	0.250	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
LoctnA:Trt3	0.250	0.250	0.500	0.250	0.250	0.250	0.250	0.250	0.250
LoctnB:Trt3	0.250	0.250	0.250	0.500	0.250	0.250	0.250	0.250	0.250
LoctnC:Trt3	0.250	0.250	0.250	0.250	0.500	0.250	0.250	0.250	0.250
LoctnD:Trt3	0.250	0.250	0.250	0.250	0.250	0.500	0.250	0.250	0.250
LoctnE:Trt3	0.250	0.250	0.250	0.250	0.250	0.250	0.500	0.250	0.250
LoctnF:Trt3	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.500	0.250
LoctnG:Trt3	0.500	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.500
LoctnH:Trt3	0.250	0.500	0.250	0.250	0.250	0.250	0.250	0.250	0.250
	LcH:T2	LcA:T3	LcB:T3	LcC:T3	LcD:T3	LcE:T3	LcF:T3	LcG:T3	
LocationA									
LocationB									
LocationC									
LocationD									
LocationE									
LocationF									
LocationG									
LocationH									
Trt1									
Trt2									
Trt3									
LoctnA:Trt1									
LoctnB:Trt1									
LoctnC:Trt1									
LoctnD:Trt1									
LoctnE:Trt1									
LoctnF:Trt1									
LoctnG:Trt1									
LoctnH:Trt1									
LoctnA:Trt2									
LoctnB:Trt2									
LoctnC:Trt2									

```
LoctnD:Trt2
LoctnE:Trt2
LoctnF:Trt2
LoctnG:Trt2
LoctnH:Trt2
LoctnA:Trt3  0.250
LoctnB:Trt3  0.250  0.500
LoctnC:Trt3  0.250  0.500  0.500
LoctnD:Trt3  0.250  0.500  0.500  0.500
LoctnE:Trt3  0.250  0.500  0.500  0.500  0.500
LoctnF:Trt3  0.250  0.500  0.500  0.500  0.500  0.500
LoctnG:Trt3  0.250  0.500  0.500  0.500  0.500  0.500  0.500
LoctnH:Trt3  0.500  0.500  0.500  0.500  0.500  0.500  0.500  0.500
> anova(fm1Mult)
Analysis of Variance Table
            Df Sum Sq Mean Sq
Location     8 6.9450  0.8681
Trt          3 1.2217  0.4072
Location:Trt 24 0.9966  0.0415
> (fm2Mult <- lmer(Adj ~ Location + Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location + Trt + (1 | Grp)
   Data: Multilocation
 AIC   BIC logLik MLdeviance REMLdeviance
  20 54.87  3.001     -51.22       -6.001
Random effects:
 Groups   Name        Variance  Std.Dev.
 Grp      (Intercept) 0.0050913 0.071354
 Residual             0.0367122 0.191604
number of obs: 108, groups: Grp, 27

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.53296    0.07600   33.33
LocationA    0.47818    0.09753    4.90
LocationB   -0.22443    0.09753   -2.30
LocationC    0.52712    0.09753    5.40
LocationD    0.02902    0.09753    0.30
LocationE    0.32537    0.09753    3.34
LocationF    0.73709    0.09753    7.56
```

```
LocationG    0.32098    0.09753    3.29
LocationH    0.80099    0.09753    8.21
Trt1         0.05834    0.05215    1.12
Trt2        -0.18802    0.05215   -3.61
Trt3         0.08379    0.05215    1.61

Correlation of Fixed Effects:
          (Intr) LoctnA LoctnB LoctnC LoctnD LoctnE LoctnF LoctnG LoctnH
LocationA -0.642
LocationB -0.642  0.500
LocationC -0.642  0.500  0.500
LocationD -0.642  0.500  0.500  0.500
LocationE -0.642  0.500  0.500  0.500  0.500
LocationF -0.642  0.500  0.500  0.500  0.500  0.500
LocationG -0.642  0.500  0.500  0.500  0.500  0.500  0.500
LocationH -0.642  0.500  0.500  0.500  0.500  0.500  0.500  0.500
Trt1      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
Trt2      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
Trt3      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
          Trt1   Trt2
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
Trt1
Trt2       0.500
Trt3       0.500  0.500
> (fm3Mult <- lmer(Adj ~ Location + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location + (1 | Grp)
   Data: Multilocation
   AIC   BIC logLik MLdeviance REMLdeviance
 29.82 56.64  -4.91     -22.17         9.82
Random effects:
 Groups   Name        Variance  Std.Dev.
 Grp      (Intercept) 0.0016547 0.040678
```

```
 Residual             0.0504386 0.224585
number of obs: 108, groups: Grp, 27

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.52149    0.06895   36.57
LocationA    0.47818    0.09752    4.90
LocationB   -0.22443    0.09752   -2.30
LocationC    0.52712    0.09752    5.41
LocationD    0.02902    0.09752    0.30
LocationE    0.32537    0.09752    3.34
LocationF    0.73709    0.09752    7.56
LocationG    0.32098    0.09752    3.29
LocationH    0.80099    0.09752    8.21

Correlation of Fixed Effects:
          (Intr) LoctnA LoctnB LoctnC LoctnD LoctnE LoctnF LoctnG
LocationA -0.707
LocationB -0.707  0.500
LocationC -0.707  0.500  0.500
LocationD -0.707  0.500  0.500  0.500
LocationE -0.707  0.500  0.500  0.500  0.500
LocationF -0.707  0.500  0.500  0.500  0.500  0.500
LocationG -0.707  0.500  0.500  0.500  0.500  0.500  0.500
LocationH -0.707  0.500  0.500  0.500  0.500  0.500  0.500  0.500
> (fm4Mult <- lmer(Adj ~ Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Trt + (1 | Grp)
   Data: Multilocation
   AIC   BIC logLik MLdeviance REMLdeviance
 41.51 54.92 -15.75      14.95        31.51
Random effects:
 Groups   Name        Variance Std.Dev.
 Grp      (Intercept) 0.110923 0.33305
 Residual             0.036715 0.19161
number of obs: 108, groups: Grp, 27

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.86567    0.07395   38.75
```

```
Trt1         0.05834    0.05215    1.12
Trt2        -0.18802    0.05215   -3.61
Trt3         0.08379    0.05215    1.61

Correlation of Fixed Effects:
     (Intr) Trt1   Trt2
Trt1 -0.353
Trt2 -0.353  0.500
Trt3 -0.353  0.500  0.500
> (fm5Mult <- lmer(Adj ~ 1 + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ 1 + (1 | Grp)
   Data: Multilocation
   AIC   BIC logLik MLdeviance REMLdeviance
 51.33 56.69 -23.66      43.75        47.33
Random effects:
 Groups   Name        Variance Std.Dev.
 Grp      (Intercept) 0.107491 0.32786
 Residual             0.050439 0.22459
number of obs: 108, groups: Grp, 27

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.85419    0.06669   42.79
> anova(fm2Mult)
Analysis of Variance Table
         Df Sum Sq Mean Sq
Location  8 7.3733  0.9217
Trt       3 1.2217  0.4072
> (fm2MultR <- lmer(Adj ~ Trt + (Trt - 1 | Location) + (1 |
+     Block), Multilocation, control = list(msV = 1, niterEM = 200)))
  0      58.1948: 0.888889 0.888889 0.888889 0.888889  0.00000  0.00000  0.00
  1      24.0707:  1.03246 0.969626 0.994051 0.990267 -0.406757 -0.437449 -0.
  2      15.9679: 0.884835 0.815369  1.10073  1.41171 -0.341162 -0.401693 -0.
  3      15.4463: 0.977758 0.604674 0.969809  1.58337 0.0458003 -0.0495717 -1
  4      15.4463: 0.977758 0.604674 0.969809  1.58337 0.0458003 -0.0495717 -1
  5      15.4463: 0.977758 0.604674 0.969809  1.58337 0.0458003 -0.0495717 -1
  6      15.4463: 0.977758 0.604674 0.969809  1.58337 0.0458003 -0.0495717 -1
Linear mixed-effects model fit by REML
Formula: Adj ~ Trt + (Trt - 1 | Location) + (1 | Block)
```

```
   Data: Multilocation
   AIC   BIC logLik MLdeviance REMLdeviance
 45.45 85.68 -7.723      2.553        15.45
Random effects:
 Groups   Name        Variance   Std.Dev.   Corr
 Location Trt1        1.4555e-01 3.8151e-01
          Trt2        7.6954e-02 2.7741e-01 0.716
          Trt3        1.0610e-01 3.2574e-01 0.719 0.653
          Trt4        5.8897e-02 2.4269e-01 0.866 0.840 0.812
 Block    (Intercept) 1.8599e-11 4.3126e-06
 Residual             3.7197e-02 1.9287e-01
number of obs: 108, groups: Location, 9; Block, 3

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.86567    0.08900   32.20
Trt1         0.05834    0.08754    0.67
Trt2        -0.18802    0.07271   -2.59
Trt3         0.08379    0.08257    1.01

Correlation of Fixed Effects:
     (Intr) Trt1   Trt2
Trt1  0.126
Trt2 -0.253  0.181
Trt3 -0.107  0.249  0.177
```

I PBIB

```
> str(PBIB)
'data.frame':        60 obs. of  3 variables:
 $ response : num  2.4 2.5 2.6 2 2.7 2.8 2.4 2.7 2.6 2.8 ...
 $ Treatment: Factor w/ 15 levels "1","10","11",..: 7 15 1 5 11 13 14 1 2 1 .
 $ Block    : Factor w/ 15 levels "1","10","11",..: 1 1 1 1 8 8 8 8 9 9 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 response ~ Treatment | Block
  .. .. ..- attr(*, ".Environment")=length 24 <environment>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
```

```
  ..$ labels      : list()
  ..$ units       : list()
> (fm1PBIB <- lmer(response ~ Treatment + (1 | Block), PBIB))
Linear mixed-effects model fit by REML
Formula: response ~ Treatment + (1 | Block)
   Data: PBIB
   AIC   BIC logLik MLdeviance REMLdeviance
 83.98 117.5 -25.99      22.83        51.98
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.046579 0.21582
 Residual             0.085533 0.29246
number of obs: 60, groups: Block, 15

Fixed effects:
             Estimate Std. Error t value
(Intercept)  2.891351   0.166407  17.375
Treatment1  -0.073802   0.222036  -0.332
Treatment10 -0.400266   0.222036  -1.803
Treatment11  0.007305   0.222036   0.033
Treatment12  0.161431   0.222036   0.727
Treatment13 -0.273531   0.222036  -1.232
Treatment14 -0.400000   0.227177  -1.761
Treatment15 -0.032128   0.222036  -0.145
Treatment2  -0.486030   0.222036  -2.189
Treatment3  -0.436409   0.222036  -1.965
Treatment4  -0.107616   0.227177  -0.474
Treatment5  -0.086451   0.222036  -0.389
Treatment6   0.019336   0.222036   0.087
Treatment7  -0.102384   0.222036  -0.461
Treatment8  -0.109716   0.222036  -0.494

Correlation of Fixed Effects:
            (Intr) Trtmn1 Trtm10 Trtm11 Trtm12 Trtm13 Trtm14 Trtm15 Trtmn2
Treatment1  -0.667
Treatment10 -0.667  0.500
Treatment11 -0.667  0.477  0.500
Treatment12 -0.667  0.500  0.500  0.500
Treatment13 -0.667  0.500  0.500  0.500  0.500
Treatment14 -0.683  0.512  0.512  0.512  0.512  0.512
```

```
Treatment15 -0.667  0.500  0.477  0.500  0.500  0.500  0.512
Treatment2  -0.667  0.500  0.500  0.500  0.477  0.500  0.512  0.500
Treatment3  -0.667  0.500  0.500  0.500  0.500  0.477  0.512  0.500  0.500
Treatment4  -0.683  0.512  0.512  0.512  0.512  0.512  0.500  0.512  0.512
Treatment5  -0.667  0.500  0.477  0.500  0.500  0.500  0.512  0.477  0.500
Treatment6  -0.667  0.477  0.500  0.477  0.500  0.500  0.512  0.500  0.500
Treatment7  -0.667  0.500  0.500  0.500  0.477  0.500  0.512  0.500  0.477
Treatment8  -0.667  0.500  0.500  0.500  0.500  0.477  0.512  0.500  0.500
            Trtmn3 Trtmn4 Trtmn5 Trtmn6 Trtmn7
Treatment1
Treatment10
Treatment11
Treatment12
Treatment13
Treatment14
Treatment15
Treatment2
Treatment3
Treatment4   0.512
Treatment5   0.500  0.512
Treatment6   0.500  0.512  0.500
Treatment7   0.500  0.512  0.500  0.500
Treatment8   0.477  0.512  0.500  0.500  0.500
```

J SIMS

```
> str(SIMS)
'data.frame':        3691 obs. of  3 variables:
 $ Pretot: num  29 38 31 31 29 23 23 33 30 32 ...
 $ Gain  : num  2 0 6 6 5 9 7 2 1 3 ...
 $ Class : Factor w/ 190 levels "1","10","100",..: 1 1 1 1 1 1 1 1 1 1 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 Gain ~ Pretot | Class
  .. .. ..- attr(*, ".Environment")=length 25 <environment>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 2
  .. ..$ Pretot: chr "Sum of pre-test core item scores"
```

```
  .. ..$ Gain  : chr "Gain in mathematics achievement score"
  ..$ units       : list()
> (fm1SIMS <- lmer(Gain ~ Pretot + (Pretot | Class), SIMS))
Linear mixed-effects model fit by REML
Formula: Gain ~ Pretot + (Pretot | Class)
   Data: SIMS
   AIC   BIC logLik MLdeviance REMLdeviance
 22391 22422 -11190      22373        22381
Random effects:
 Groups   Name        Variance  Std.Dev. Corr
 Class    (Intercept) 14.489218 3.806471
          Pretot       0.009204 0.095937 -0.641
 Residual             22.235736 4.715478
number of obs: 3691, groups: Class, 190

Fixed effects:
            Estimate Std. Error t value
(Intercept)   7.0596     0.3659   19.29
Pretot       -0.1860     0.0161  -11.56

Correlation of Fixed Effects:
       (Intr)
Pretot -0.760
```