

## Von Bertalanffy's Individual Growth Model

The “individual growth model” published by von Bertalanffy in 1934 is widely used in biological models. Although it has a number of variations, we focus here on its simplest form:

$$L_t = L_\infty \left[ 1 - e^{-K(t-t_0)} \right],$$

a formula that describes the length  $L_t$  of an individual at age  $t$ . The model involves three parameters:

$L_\infty$  = horizontal asymptote of the curve, corresponding to the ultimate length,

$K$  = growth rate toward the asymptotic length,

$t_0$  = theoretical age at which an individual has length 0.

Motivation for this model comes from the differential equation

$$\frac{dL_t}{dt} = K(L_\infty - L_t),$$

which states that the growth rate is proportional to the difference between the asymptotic length  $L_\infty$  and the current length  $L_t$ .

Schnute (1981) published a variation of this growth model that is also illustrated in *PBS Modelling*. See the example “SGM” in the GUI generated by `runExamples()`.

### References

Quinn II, T.J. and Deriso, R.B. (1999) *Quantitative Fish Dynamics*. Oxford University Press, New York NY, 542 pp.

Schnute, J. (1981) A versatile growth model with statistically stable parameters. *Canadian Journal of Fisheries and Aquatic Sciences* **38**, 1128-1140.

von Bertalanffy, L. (1934) Untersuchungen über die Gesetzmäßigkeit des Wachstums. I. Allgemeine Grundlagen der Theorie; mathematische und physiologische Gesetzmäßigkeiten des Wachstums bei Wassertieren. *Arch. Entwicklungsmech.* **131**, 613-652.

Wikipedia: [http://en.wikipedia.org/wiki/Von\\_Bertalanffy](http://en.wikipedia.org/wiki/Von_Bertalanffy)