

# Tutorial on the package `hmmm`

**Manuela Cazzaro**  
Università di Milano-Bicocca

**Roberto Colombi**  
Università di Bergamo

**Sabrina Giordano**  
Università della Calabria

---

## Abstract

In this tutorial we show how complete hierarchical multinomial marginal (hmm) models for categorical variables can be defined, estimated and tested using the `hmmm` package.

*Keywords:* marginal models, generalized interactions, chi-bar-square distribution.

---

## 1. Introduction

Marginal models are defined for categorical variables by imposing restrictions on marginal distributions of contingency tables, (Agresti (2012), Ch. 12). A complete hierarchical multinomial marginal model (hmm) is specified by an ordered set of marginal distributions and a set of interactions (contrasts of logarithms of probabilities) defined within different marginal distributions according to the rules of hierarchy and completeness (Bergsma and Rudas (2002), Bartolucci, Colombi, and Forcina (2007)). In particular, in hmm models every interaction is defined in one and only one marginal distribution (completeness) and within the first marginal set which contains it (hierarchy). By imposing equality and inequality constraints on marginal interactions, interesting hypotheses (i.e. independence in sub-tables, where some categories are collapsed, association in marginal tables, conditional independence or additive effects of covariates in marginal tables, marginal homogeneity, monotone dependence, positive association, among others) can be tested in hmm models.

In this tutorial we show how to define and estimate hmm models with interactions restricted under equality and inequality constraints or influenced by the effects of covariates using the `hmmm` package.

## 2. How to define and estimate marginal models

The starting point for the marginal modeling of categorical data is a multidimensional table representing the joint distribution of two or more unordered and/or (partially) ordered categorical variables.

In the `hmmm` package the input data must be a vector of cell frequencies arranged in antilexicographic order. To start with, we show how to get a named vector of frequencies from the `accident` data frame, regarding accidents occurred to workers, whose columns contain the variables: var. 1 *Type* of the injury (with 3 levels), var. 2 *Time* to recover (with 4 levels), var. 3 *Age* of the worker (with 3 levels) and var. 4 solar *Hour* (with 2 levels) along with the counts for each configuration of the variables (last column). Note that variables are denoted

by integers, the lower the number identifying the variable, the faster its category subscript changes in the vectorized contingency table.

```
> library("hmmm")
> data(accident, package="hmmm")
> y<-getnames(accident,st=9)
```

The length of the row names is controlled by the `st` argument. Row names identify the cells of the contingency table and are used in the outputs displaying estimated cell probabilities. Only the first twelve rows are printed to give an example.

	cell names	counts
[1,]	uncertain 0  -- 7 <=25 morning	21
[2,]	avoidable 0  -- 7 <=25 morning	9
[3,]	not-avoid 0  -- 7 <=25 morning	0
[4,]	uncertain 7  -- 21 <=25 morning	10
[5,]	avoidable 7  -- 21 <=25 morning	9
[6,]	not-avoid 7  -- 21 <=25 morning	0
[7,]	uncertain 21  -- 60 <=25 morning	5
[8,]	avoidable 21  -- 60 <=25 morning	1
[9,]	not-avoid 21  -- 60 <=25 morning	1
[10,]	uncertain >= 60 <=25 morning	2
[11,]	avoidable >= 60 <=25 morning	0
[12,]	not-avoid >= 60 <=25 morning	1

Let us start by defining a saturated hmm model, i.e. a model without any restrictions on the interactions. First, the list of the marginal sets has to be declared and the command `marg.list` serves the need. Here, with respect to `accident` data, it defines the bivariate distribution of the variables 3, 4; the two joint distributions of the variables 1, 3, 4 and 2, 3, 4; and the joint distribution. The symbol “*b*” states that all the log-linear interactions in every marginal set are of baseline type (sections 3 and 4 are devoted to illustrate the use of more general types of interactions). The second statement uses the function `hmmm.model` to define the hmmm model. In the input arguments, as well as the marginal sets, information on the number of categories and on the names of the variables involved are specified. The output illustrates how the interactions are allocated according to the principles of hierarchy and completeness.

```
> margin<-marg.list(c("marg-marg-b-b", "b-marg-b-b", "marg-b-b-b", "b-b-b-b"))
> m<-hmmm.model(marg=margin, lev=c(3,4,3,2),
+ names=c("Type", "Time", "Age", "Hour"))
> m
```

	inter.	inter.names	marg.	marg.names	type	npar	start	end
[1,]	3	Age	34	Age,Hour	b	2	1	2
[2,]	4	Hour	34	Age,Hour	b	1	3	3
[3,]	34	Age.Hour	34	Age,Hour	bb	2	4	5
[4,]	1	Type	134	Type,Age,Hour	b	2	6	7

[5,]	13	Type.Age	134	Type, Age, Hour	bb	4	8	11
[6,]	14	Type.Hour	134	Type, Age, Hour	bb	2	12	13
[7,]	134	Type.Age.Hour	134	Type, Age, Hour	bbb	4	14	17
[8,]	2	Time	234	Time, Age, Hour	b	3	18	20
[9,]	23	Time.Age	234	Time, Age, Hour	bb	6	21	26
[10,]	24	Time.Hour	234	Time, Age, Hour	bb	3	27	29
[11,]	234	Time.Age.Hour	234	Time, Age, Hour	bbb	6	30	35
[12,]	12	Type.Time	1234	Type, Time, Age, Hour	bb	6	36	41
[13,]	123	Type.Time.Age	1234	Type, Time, Age, Hour	bbb	12	42	53
[14,]	124	Type.Time.Hour	1234	Type, Time, Age, Hour	bbb	6	54	59
[15,]	1234	Type.Time.Age.Hour	1234	Type, Time, Age, Hour	bbbb	12	60	71

A non-saturated model can be defined by imposing equality constraints on certain interactions. For example, we can set to zero the interactions that occupy the positions 12 : 13, 14 : 17 in the vector of the parameters in order to state that the conditional independence  $1 \perp\!\!\!\perp 4 \mid 3$  holds for the variables in `accident` data. This can be achieved by specifying the argument `sel` of the `hmmm.model` function

```
> modelB<-hmmm.model(marg=margin,lev=c(3,4,3,2),
+ names=c("Type","Time","Age","Hour"),
+ sel=c(12:13,14:17))
> modB<-hmmm.mlfit(y,modelB)
```

The model is then estimated by the command `hmmm.mlfit`

```
> modB
```

SUMMARY of MODEL:

OVERALL GOODNESS OF FIT:

Likelihood Ratio Stat (df= 6 ): Gsq = 6.02965 (p = 0.41988 )

Further, estimated parameters can be printed by the following statement

```
> print(modB,aname="model B",printflag=TRUE)
```

A much more detailed output with estimated standard errors and estimated cell probabilities is given by

```
> summary(modB)
```

When the constrained interactions are log-linear parameters defined in the joint distribution (Agresti (2012)) it is convenient to use the argument `formula` of the `hmmm.model` function for specifying the log-linear model without the interactions we impose to be zero. For example, if in addition to the previous constraints, we would verify also if the odds ratios of the distribution of *Type* and *Time* do not depend on the levels of *Age* and *Hour*, we must set to zero the interactions of second and third order arranged in the positions from 42 to 71. These log-linear interactions are defined in the joint distribution and we can use the statements

```
> modelA<-hmmm.model(marg=margin,lev=c(3,4,3,2),
+ names=c("Type","Time","Age","Hour"),sel=c(12:13,14:17),
+ formula=~Type*Age*Hour+Time*Age*Hour+Type:Time)
> modA<-hmmm.mlfit(y,modelA)
```

Thus, modelA is nested in modelB. The likelihood ratio test to compare the two nested models is obtained by the function `anova`

```
> anova(modA,modB)
```

	statistics	value	df	pvalue
model A	34.589455	36	0.5356700	
model B	6.029646	6	0.4198800	
LR test	28.559810	30	0.5407972	

Note that the previous model A is not log-linear because some constrained interactions are defined in marginal distributions. A log-linear model can be defined and estimated by the following statements

```
> modellog<-loglin.model(lev=c(3,4,3,2),
+ formula=~Type*Age*Hour+Time*Age*Hour+Type:Time,
+ names=c("Type","Time","Age","Hour"))
> modlog<-hmmm.mlfit(y,modellog)
```

### 3. Generalized marginal interactions

In the previous section all the interactions defined within the marginal distributions were of log-linear type. [Bartolucci \*et al.\* \(2007\)](#) have shown that more general types of interactions can be used to parameterize marginal models. This possibility is particularly useful as, in presence of ordered categorical variables, the univariate marginal distributions are parameterized more appropriately using non standard logits such as the global and continuation ones for example, or bivariate distributions are parameterized by non standard odds ratios such as the global, global-continuation and the continuation ones. This extension is also important since several hypotheses of restrictive association and monotone dependence can be expressed by inequality constraints on these generalized interactions (see section 5 on testing inequality constraints in hmm models). [Bartolucci \*et al.\* \(2007\)](#), in particular, showed that the generalized marginal interactions within a marginal set can be defined by assigning a logit type to every variable of the marginal distribution. For example, if we use global logits for both variables in a bivariate distribution then this distribution is parameterized by global logits and global log odds ratios. If we assign continuation logits to one variable and global logits to the other one then we have a parametrization in terms of continuation logits, global logits and continuation-global log odds ratios. For details on generalized interactions and their interpretation refer to [Bartolucci \*et al.\* \(2007\)](#) and to section 5 where their usefulness for testing hypotheses of stochastic orderings is clarified.

The `marg.list` command is used to make clear the logit type assigned to every variable in a marginal distribution as every generalized interaction depends on it. The types of logit

allowed in **hmmm** are baseline (b), local (l), continuation (c) and reverse continuation (rc). A different type of logits is discussed in the next section.

For example, we consider the data **Madsen** with variables: *Influence* (var. 1 with 3 levels), *Satisfaction* (var. 2 with 3 levels), *Contact* (var. 3 with 2 levels), *Housing* (var. 4 with 4 levels). Remind that variables are denoted by integers, the lower the number identifying the variable, the faster its category subscript changes in the vectorized contingency table.

For the **Madsen** data, consider the statements

```
> margin<-marg.list(c("marg-marg-1-1","g-marg-1-1","marg-g-1-1","g-g-1-1"))
> model<-hmmm.model(marg=margin,lev=c(3,3,2,4),names=c("In","Sa","Co","Ho"))
> model
```

	inter.	inter.names	marg.	marg.names	type	npar	start	end
[1,]	3	Co	34	Co,Ho	l	1	1	1
[2,]	4	Ho	34	Co,Ho	l	3	2	4
[3,]	34	Co.Ho	34	Co,Ho	ll	3	5	7
[4,]	1	In	134	In,Co,Ho	g	2	8	9
[5,]	13	In.Co	134	In,Co,Ho	gl	2	10	11
[6,]	14	In.Ho	134	In,Co,Ho	gl	6	12	17
[7,]	134	In.Co.Ho	134	In,Co,Ho	gll	6	18	23
[8,]	2	Sa	234	Sa,Co,Ho	g	2	24	25
[9,]	23	Sa.Co	234	Sa,Co,Ho	gl	2	26	27
[10,]	24	Sa.Ho	234	Sa,Co,Ho	gl	6	28	33
[11,]	234	Sa.Co.Ho	234	Sa,Co,Ho	gll	6	34	39
[12,]	12	In.Sa	1234	In,Sa,Co,Ho	gg	4	40	43
[13,]	123	In.Sa.Co	1234	In,Sa,Co,Ho	ggl	4	44	47
[14,]	124	In.Sa.Ho	1234	In,Sa,Co,Ho	ggl	12	48	59
[15,]	1234	In.Sa.Co.Ho	1234	In,Sa,Co,Ho	ggll	12	60	71

This means that in the bivariate distribution of variables 3, 4 all the interactions are of local type, while in the joint distribution of 1, 3, 4 the interactions 1 are global logits, the interactions 13 and 14 are global-local log-odds ratios. In this marginal distribution, the interactions 134 are differences between the logarithms of two global-local odds ratios. A similar comment holds for the joint distribution of the variables 2, 3, 4.

To test if there is an additive effect of variables 3 and 4 on the global logits of variables 1 and 2 in the marginal distributions 134 and 234 we can use the following statements

```
> modelad1<-hmmm.model(marg=margin,lev=c(3,3,2,4),
+ names=c("In","Sa","Co","Ho"),sel=c(18:23,34:39))

> data(madsen,package="hmmm")
> y<-getnames(madsen,st=6)
> modadd1<-hmmm.mlfit(y,modelad1)
> modadd1
```

SUMMARY of MODEL:

OVERALL GOODNESS OF FIT:

Likelihood Ratio Stat (df= 12 ): Gsq = 14.76183 (p = 0.25472 )

Moreover, to add the hypothesis that the global odds ratios of the variables 1 and 2 do not depend on the levels of the other two variables, the `ggl` and `ggll` interactions that occupy the positions 44 : 71 in the vector of parameters have to be constrained to zero

```
> modelad2<-hmmm.model(marg=margin,lev=c(3,3,2,4),
+ names=c("In","Sa","Co","Ho"),sel=c(18:23,34:39,44:71))
> modadd2<-hmmm.mlfit(y,modelad2)
> modadd2
```

SUMMARY of MODEL:

OVERALL GOODNESS OF FIT:

Likelihood Ratio Stat (df= 40 ): Gsq = 45.61355 (p = 0.25008 )

For an alternative way of specifying other similar hypotheses see section 6 where the effect of covariates on interactions is taken into account.

## 4. Recursive marginal interactions

Cazzaro and Colombi (2008) extended the class of generalized interactions by introducing a new type of logits: the recursive (or nested) logits. In the simplest case these logits are defined in correspondence of a partition of the categories of a variable. As an example we consider the data `relpol` with variable *Religion* with levels: *Pr* ‘Protestant’, *Ca* ‘Catholic’, *No* ‘None’ and variable *Politics* with levels: *El* ‘Extremely liberal’, *Li* ‘Liberal’, *Sl* ‘Slightly liberal’, *Mo* ‘Moderate’, *Sc* ‘Slightly conservative’, *Co* ‘Conservative’, *Ec* ‘Extremely conservative’. For *Religion* we consider the partition with sets  $R=\{Pr, Ca\}$ ,  $N=\{No\}$  and for *Politics* the partition in the sets:  $L=\{El, Li, Sl\}$ ,  $M=\{Mo\}$  and  $C=\{Sc, Co, Ec\}$ .

A first sets of logits are the baseline logits which are defined within every set of the partition (the reference category can be chosen arbitrarily in every set). For example, this kind of recursive logits for *Religion* and *Politics* are:  $\log[pr(Ca)/pr(Pr)]$  and  $\log[pr(El)/pr(Li)]$ ,  $\log[pr(Sl)/pr(Li)]$ ,  $\log[p(Sc)/p(Co)]$ ,  $\log[pr(Ec)/pr(Co)]$ , respectively.

A second sets of logits are the baseline logits defined on the probabilities of the sets of the partition (the reference set can be chosen arbitrarily). Considering the `relpol` data, the recursive logits of the variables *Religion* and *Politics* in this case are:  $\log[pr(N)/pr(R)]$  and  $\log[pr(C)/pr(L)]$ ,  $\log[pr(M)/pr(L)]$ , respectively.

The number of recursive logits is always equal to the number of categories minus one.

For a more general definition, and for several examples, see Cazzaro and Colombi (2008) and Cazzaro and Colombi (2013).

The use of interactions based on recursive logits is requested in `marg.list` by the use of “*r*” instead of “*l*”, “*b*”, “*c*”, “*rc*”, “*g*”. The recursive logits are specified by the function `recursive` that must have an argument for every variable. The argument is 0 for every variable to which a recursive logit is not assigned otherwise it is a matrix. The rows of this matrix specify the categories whose probabilities appear in the numerator and denominator of every recursive logit. In a row a value 1 (-1) corresponds to the categories whose probability is cumulated in the numerator (denominator), 0 if the category is not involved. Finally the output of `recursive` must be assigned to the argument `cocacontr` of `hmmm.model`.

With the reference to the `relpol` data the necessary statements are

```

> marginals<-marg.list(c("r-marg","marg-r","r-r"))
> R1<-matrix(c(-1,-1,1,
+             -1,1,0),2,3,byrow=T)
> R2<-matrix(c(-1,-1,-1, 0, 1, 1, 1,
+             -1,-1,-1, 1, 0, 0, 0,
+             1,-1, 0, 0, 0, 0, 0,
+             0,-1, 1, 0, 0, 0, 0,
+             0, 0, 0, 0, 1,-1, 0,
+             0, 0, 0, 0, 0,-1, 1),6,7,byrow=T)
> rec<-recursive(R1,R2)
> model<-hmmm.model(marg=marginals,lev=c(3,7),names=c("Rel","Pol"),
+ cocacontr=rec)
> model

```

	inter.	inter.names	marg.	marg.names	type	npar	start	end
[1,]	1	Rel	1	Rel	r	2	1	2
[2,]	2	Pol	2	Pol	r	6	3	8
[3,]	12	Rel.Pol	12	Rel,Pol	rr	12	9	20

Here we report only the results for the saturated model, see [Cazzaro and Colombi \(2013\)](#) for examples on non-saturated models.

```

> data(relpol, package="hmmm")
> y<-getnames(relpol,st=4)
> modr<-hmmm.mlfit(y,model)
> print(modr,printflag=T)

```

SUMMARY of MODEL:

OVERALL GOODNESS OF FIT:

Likelihood Ratio Stat (df= 0 ): Gsq = 0

	inter.	marg.	type	STRATA_1
link1	Rel	Rel	r	2.198445
link2	Rel	Rel	r	1.085646
link3	Pol	Pol	r	-0.368723
link4	Pol	Pol	r	-0.404021
link5	Pol	Pol	r	1.699386
link6	Pol	Pol	r	-0.263191
link7	Pol	Pol	r	-0.045611
link8	Pol	Pol	r	1.714798
link9	Rel.Pol	Rel,Pol	rr	-1.665426
link10	Rel.Pol	Rel,Pol	rr	-0.257114
link11	Rel.Pol	Rel,Pol	rr	-0.96283
link12	Rel.Pol	Rel,Pol	rr	0.276553
link13	Rel.Pol	Rel,Pol	rr	-0.065654

```

link14 Rel.Pol Rel,Pol rr -0.85745
link15 Rel.Pol Rel,Pol rr -0.567533
link16 Rel.Pol Rel,Pol rr -0.386656
link17 Rel.Pol Rel,Pol rr 0.797325
link18 Rel.Pol Rel,Pol rr 0.348307
link19 Rel.Pol Rel,Pol rr 1.071584
link20 Rel.Pol Rel,Pol rr -0.95906

```

## 5. Inequality constraints on interactions

Hypotheses of monotone dependence and positive/negative association between ordered categorical variables can be ascertained by testing marginal models with inequality constraints on certain interactions. We illustrate how to define, fit and test models with parameters constrained by inequalities using the dataset `polbirth`.

In the dataset `polbirth` involving data on political orientation and opinion on teenage birth control, variable 1 is *Politics* with categories: *Extremely liberal*, *Liberal*, *Slightly liberal*, *Moderate*, *Slightly conservative*, *Conservative*, *Extremely conservative* and variable 2 is *Birth* with categories *Strongly agree*, *Agree*, *Disagree*, *Strongly disagree*.

With these variables, for example, we can test the hypothesis that the distributions of *Politics* given the levels of *Birth* are ordered according to the simple dominance criterion coherently with the strength of the opinion on *Birth* control. This hypothesis is equivalent to require that all the global-local log-odds ratios are non-negative. Continuation-local or local log-odds ratios can be constrained to consider successively stronger notions of monotone dependence (uniform and likelihood ratio stochastic orderings), see [Dardanoni and Forcina \(1998\)](#) and [Shaked and Shanthikumar \(1994\)](#).

We test the simple monotone dependence of *Politics* on *Birth*.

The marginal sets, the logit types and the labels of the variables are declared below

```

> data(polbirth,package="hmmm")
> y<-getnames(polbirth)
> marginals<-marg.list(c("g-marg","marg-1","g-1"))
> names<-c("Politics","Birth")

```

The interactions subject to inequality constraints, the marginal set where they are defined and the types of logit used for each variable are listed as follows, so that in this case the log-odds ratios of global-local types are the interactions to be constrained

```

> ineq<-list(marg=c(1,2),int=list(c(1,2)),types=c("g","1"))

```

The marginal model with inequalities on global-local interactions is defined using the function `hmmm.model` where `ineq` is assigned to the argument `dismarg`

```

> model<-hmmm.model(marg=marginals,dismarg=ineq,lev=c(7,4),names=names)

```

More than one list, like that specified in `ineq`, can compose `dismarg` if interactions defined in different marginal distributions have to be constrained (see details in the help of the `hmmm.model` function).

The model with non-negative global-local log-odds ratios (Simple monotone dependence model) is estimated with the function `hmmm.mlfit` where the input `noineq` is declared `FALSE`

```
> mlr<-hmmm.mlfit(y,model,noineq=FALSE)
```

If the previous inequality constraints are turned into equality, all the global-local log-odds ratios are null and the corresponding model is the stochastic independence model

```
> model0<-hmmm.model(marg=marginals,lev=c(7,4),sel=c(10:27),names=names)
> mnull<-hmmm.mlfit(y,model0)
```

The model estimated without any inequality constraints on parameters is, in this case, the saturated model

```
> msat<-hmmm.mlfit(y,model)
```

The fitted models are compared through the function `hmmm.chibar`. The arguments of `hmmm.chibar` are the estimated models with inequality constraints turned into equalities (`nullfit`), with inequality constraints (`disfit`) and without inequality constraints on parameters (`satfit`).

```
> test<-hmmm.chibar(nullfit=mnull,disfit=mlr,satfit=msat)
```

Function `hmmm.chibar` tests problems of type A and B (Silvapulle and Sen (2005) pg. 61): the test of type A compares the model `nullfit` under  $H_0$  against the model `disfit` under  $H_1$ ; while the type B problem means testing  $H_0$ : model `disfit` against  $H_1$ : model `satfit`. The main difference between type A and type B problems is that inequalities are present in the alternative hypothesis of type A and the null hypothesis of type B problems.

The null distribution of the likelihood ratio statistic  $G^2$  for or against inequality constraints turns out to be chi-bar-square, that is a mixture of chi-square distributions. Its tail probabilities are computed by simulation (the method *Simulation 2* described in Silvapulle and Sen (2005) pg. 79 is implemented).

The output of `hmmm.chibar` provides the values of the likelihood ratio statistics  $G^2$  and their simulated pvalues for both tests of type A and B.

```
> test
```

```
chibar simulated pvalues

      test      pvalue
testA 64.457490 3.80154e-09
testB  2.033941 9.35751e-01
```

A much detailed output is printed by `summary`.

## 6. Covariates effects on the response variables

Different models can be estimated taking into account the effects of covariates on the response variables (as in [Marchetti and Lupparelli \(2011\)](#) and [Glonck and McCullagh \(1995\)](#)).

We consider the data `accident` but note that, now, var. 1 *Type* of the injury (3 levels), var. 2 *Time* to recover (4 levels) are considered as response variables and var. 3 *Age* of the worker (3 levels) and var. 4 solar *Hour* (2 levels) as covariates. Remind that the lower the variable number is the faster the variable sub-script changes in the vectorized table. Furthermore, the categories of the covariates determine the strata and the data must be arranged in such a way that the subscript of the response variable changes faster than the subscripts of the covariates.

In order to estimate different models taking into account the covariate effects on the response variables, first, the list of the marginal sets of the response variables has to be specified (see section 2 for details about `marg.list`). With respect to the vector of counts from `accident` data, the necessary statement is

```
> marginals<-marg.list(c("b-marg", "marg-g", "b-g"))
```

It is stated that in the marginal distribution of *Type* the interactions are baseline logits, in the marginal distribution of *Time* the interactions are global logits and, in the bivariate distribution of *Type* and *Time*, the interactions are baseline-global odds ratios.

Successively, a list of components, each for every interaction specified above, defining the effects of the covariates on such interactions, is needed. The following statements account for additive effect of the covariates *Age* and *Hour* on the marginal logits of the response variables *Type* and *Time* and on the association (log-odds ratios) between the responses *Type* and *Time*.

```
> al<-list(
+ Type=~Type*(Age+Hour),
+ Time=~Time*(Age+Hour),
+ Type.Time=~Type.Time*(Age+Hour)
+ )
```

It is worthwhile to note that each component of the list has the name of the interaction and contains the model formula of the covariate effects on such interaction.

The model that takes into account the covariate effects on the response variables is then specified through the function `hmmm.model.X`. Several arguments are included in `hmmm.model.X`: the marginal sets and the names of the response variables (`names`) defined in the previous statements and their number of categories, the names of the covariate variables (`fnames`) and their number of categories (`strata`) but, in particular, the main argument is `Formula` to which a list as `al` must be assigned.

```
> model<-hmmm.model.X(marg=marginals,lev=c(3,4),names=c("Type","Time"),
+ Formula=al,strata=c(3,2),fnames=c("Age","Hour"))
```

The model is then estimated by the command `hmmm.mlfit`.

```
> data(accident,package="hmmm")
> y<-getnames(accident,st=9,sep=";")
> mod1<-hmmm.mlfit(y,model,y.eps=0.1)
> mod1
```

SUMMARY of MODEL:

OVERALL GOODNESS OF FIT:

Likelihood Ratio Stat (df= 22 ): Gsq = 16.47375 (p = 0.7917 )

More detailed outputs (the estimated effects and the estimated standard errors, among others) are given by the following statement. Note that the covariate effects preceded by the main general effect (`Intercept`) are listed for every interaction.

```
> summary(mod1)
```

The necessary list of model formulas to test another interesting hypothesis in which there is the covariates *Age*, *Hour* additive effect on the marginal logits of the responses and the stochastic independence between *Type* and *Time* in each sub-table identified by the levels of *Age* and *Hour*, is

```
> alind<-list(
+ Type=~Type*Age+Type*Hour,
+ Time=~Time*Age+Time*Hour,
+ Type.Time="zero"
+ )
```

Note that we use "zero" to constrain to zero all the interactions of a given type, in this case the log-odds ratios between *Type* and *Time*.

If we want to test the so-called 'Parallel log-odds model', that is if the effect of the covariates *Age* and *Hour* is identical for each of the logits and the odds ratios of the responses *Type* and *Time*, we need the following statement

```
> alpar<-list(
+ Type=~Type+Age+Hour,
+ Time=~Time+Age+Hour,
+ Type.Time=~Type.Time+Age+Hour
+ )
```

Note that the model tested in this section are Glonek-McCullagh *Multivariate Logistic Models* with categorical covariate variables.

## 7. Further remarks

The complete hierarchical marginal models, that can be analyzed with the `hmmm` package, are a generalization of several models proposed in the literature of categorical data analysis.

For example, *Log-linear Models* are hmm models where all the interactions are defined within the joint distribution (section 2). The Bergsma and Rudas (2002) *Marginal Models* are hmm models where the interactions of log-linear type are defined in different marginal distributions (section 2). The models described in the other sections are extensions of the Bergsma-Rudas models involving more general type of interactions. Finally, Glonek and McCullagh (1995) *Multivariate Logistic Models* (see the examples of section 6) are hmm models which use all the marginal distributions and the parameters are the highest order interactions that can be defined within every marginal distribution.

Note that the **hmmm** package can estimate the parameters of all the previous models under inequality constraints.

Furthermore, there are other topics that this tutorial does not cover: (i) *Hidden Markov models* with several observed categorical variables whose distributions conditioned by the latent states are defined as hmm models; (ii) Lang (2004) *Multinomial Poisson homogeneous models* that can be estimated with the **hmmm** package also under inequality constraints.

## References

- Agresti A (2012). *Categorical data analysis - third edition*. John Wiley and Sons, New Jersey.
- Bartolucci F, Colombi R, Forcina A (2007). “An extended class of marginal link functions for modelling contingency tables by equality and inequality constraints.” *Statistica Sinica*, (17), 691–711.
- Bergsma WP, Rudas T (2002). “Marginal models for categorical data.” *The Annals of Statistics*, (30), 140–159.
- Cazzaro M, Colombi R (2008). “Modelling two way contingency tables with recursive logits and odds ratios.” *Statistical Methods and Applications*, (17), 435–453.
- Cazzaro M, Colombi R (2013). “Marginal nested interactions for contingency tables.” *Communications in Statistics - Theory and Methods*, p. To appear.
- Dardanoni V, Forcina A (1998). “A unified approach to likelihood inference on stochastic orderings in a nonparametric context.” *The Journal of the American Statistical Association*, (93), 1112–1123.
- Glonek GFV, McCullagh P (1995). “Multivariate logistic models.” *The Journal of the Royal Statistical Society - B*, (57), 533–546.
- Lang JB (2004). “Multinomial Poisson homogeneous models for contingency tables.” *The Annals of Statistics*, (32), 340–383.
- Marchetti GM, Lupporelli M (2011). “Chain graph models of multivariate regression type for categorical data.” *Bernoulli*, (17), 827–844.
- Shaked M, Shanthikumar JG (1994). *Stochastic orders and their applications*. Academic, San Diego.

Silvapulle MJ, Sen PK (2005). *Constrained statistical inference*. John Wiley and Sons, New Jersey.

**Affiliation:**

Manuela Cazzaro  
Dipartimento di Statistica e Metodi Quantitativi  
Scuola di Economia e Statistica  
Università di Milano-Bicocca  
20126 Milano, Italia  
E-mail: [manuela.cazzaro@unimib.it](mailto:manuela.cazzaro@unimib.it)

Roberto Colombi  
Dipartimento di Ingegneria dell'Informazione e Metodi Matematici  
Facoltà di Ingegneria  
Università di Bergamo  
24044 Dalmine (Bergamo), Italia  
E-mail: [roberto.colombi@unibg.it](mailto:roberto.colombi@unibg.it)

Sabrina Giordano  
Dipartimento di Scienze Economiche, Statistiche e Finanziarie  
Facoltà di Economia  
Università della Calabria  
87036 Arcavacata di Rende (Cosenza), Italia  
E-mail: [sabrina.giordano@unical.it](mailto:sabrina.giordano@unical.it)