

### Types of variable normalization formulas

1	Selection of objects and variables	data matrix $[x_{ij}]$		
2	Variable scale level	ratio	ratio	interval
	Selection of variable normalization formula	n6 – quotient transformation (x/sd) n7 – quotient transformation (x/range) n8 – quotient transformation (x/max) n9 – quotient transformation (x/mean) n10 – quotient transformation (x/sum) n11 – quotient transformation x/sqrt(SSQ)	n1 – standardization n2 – Weber standardization n3 – unitization n4 – unitization with zero minimum n5 – normalization in range [-1, 1]	n1 – standardization n2 – Weber standardization n3 – unitization n4 – unitization with zero minimum n5 – normalization in range [-1, 1]
	Transformed variable scale level	ratio	interval	interval

- (n1) 
$$z_{ij} = s_j^{-1} x_{ij} - \bar{x}_j s_j^{-1},$$
- (n2) 
$$z_{ij} = (x_{ij} - Me_j) / 1,4826 \cdot MAD_j,$$
- (n3) 
$$z_{ij} = r_j^{-1} x_{ij} - \bar{x}_j r_j^{-1},$$
- (n4) 
$$z_{ij} = \left[ x_{ij} - \min_i \{x_{ij}\} \right] / r_j$$
- (n5) 
$$z_{ij} = (x_{ij} - \bar{x}_j) / \max_i |x_{ij} - \bar{x}_j|$$
- (n6) 
$$x_{ij} / s_j$$
- (n7) 
$$x_{ij} / r_j$$
- n(8) 
$$x_{ij} / \max_i \{x_{ij}\}$$
- (n9) 
$$x_{ij} / \bar{x}_j$$
- (n10) 
$$x_{ij} / \sum_{i=1}^n x_{ij}$$
- (n11) 
$$x_{ij} / \sqrt{\sum_{i=1}^n x_{ij}^2}$$

where:  $x_{ij} (z_{ij})$  –  $i$ -th observation on  $j$ -th variable ( $i$ -th normalized observation on  $j$ -th variable),  
 $\bar{x}_j, s_j$  – mean and standard deviation for  $j$ -th variable,  
 $Me_j, MAD_j$  – median and median absolute deviation for  $j$ -th variable,  
 $r_j = \max_i \{x_{ij}\} - \min_i \{x_{ij}\}.$

### References

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