# Vignette of the JoSAE package

Johannes Breidenbach*

6 October 2011: JoSAE 0.2

## 1 Introduction

The aim in the analysis of sample surveys is frequently to derive estimates of subpopulation characteristics. This task is denoted small area estimation (SAE) (Rao, 2003). Often, the sample available for the subpopulation is, however, too small to allow a reliable estimate. Frequently, auxiliary variables exists that are correlated with the variable of interest. Several estimators can make use of auxiliary information which may reduce the variance of the estimate (Rao, 2003). Another term for *small area* is *domain*. These two terms will be used interchangeable in the following.

The `JoSAE` package implements the *generalized regression* (GREG) (Särndal , 1984) and unit level *empirical best linear unbiased prediction* EBLUP (Battese et al., 1988) estimators and their variances. The *synthetic regression* and the *simple random sample* (SRS) estimates are also calculated. The purpose of the `JoSAE` package is to document the functions used in the publication of (Breidenbach and Astrup , 2011). The data used in that study are also provided.

If `R` is running, the `JoSAE` package can be installed by typing

```
> install.packages("JoSAE")
```

into the console[1].

The command

```
> library(JoSAE)
```

loads the package into the current workspace. We can get an overview of the packages' contents by typing

```
> `?`(JoSAE)
```

## 2 Using the provided functions - small area estimates

For our small area estimates, we need

- sample data which contain the variable of interest and the auxiliary variables of all sampled population elements and
- domain data which contain the mean of the auxiliary variables of all population elements within each domain of interest.

Both data sets need to have a corresponding domain ID.

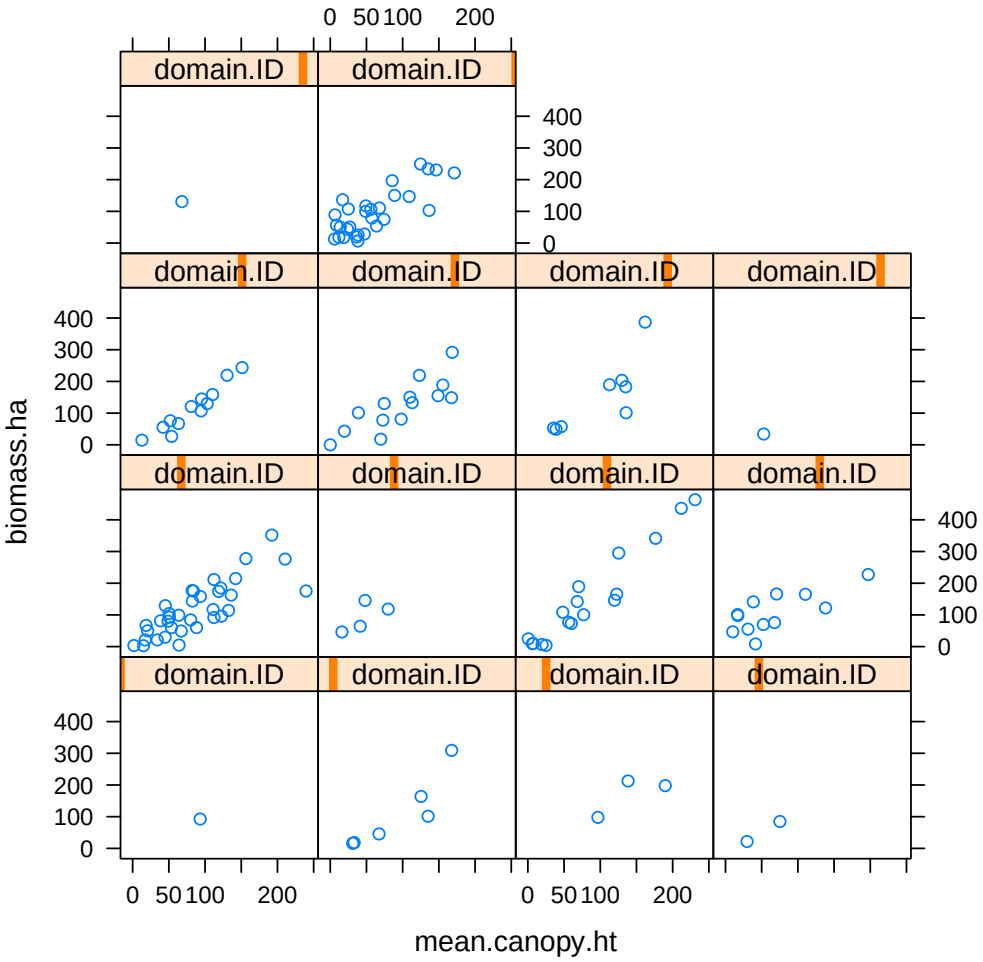
*Norwegian Forest and Landscape Institute, 1431 Ås, Norway, job@skogoglandskap.no, Tel.: +47 64 94 89 81

[1]The character ">" is not part of the command. A working Internet connection is required.

## 2.1 Mean forest biomass within Norwegian municipalities

To load and plot the data used by (Breidenbach and Astrup , 2011) we write:

```
> data(JoSAE.domain.data)
> data(JoSAE.sample.data)
> library(lattice)
> print(xyplot(biomass.ha ~ mean.canopy.ht | domain.ID, data = JoSAE.sample.data))
```




The data set `JoSAE.sample.data` contains the above-ground forest biomass (the variable of interest) observed on sample plots of the Norwegian National Forest Inventory (NNFI) and the mean canopy height derived from overlapping digital aerial images (the auxiliary variable). The domain ID indicates in which of 14 municipalities (i.e., our small areas) the sample plot was located.

The data set `JoSAE.domain.data` contains the mean canopy height derived from overlapping digital aerial images within the forest of a municipality. All population elements (i.e., not only those elements where field data from the NNFI were available) were used to derive this mean.

In order to make use of the auxiliary variables, a statistical model needs to be fit that links the variable of interest to the auxiliary variables. We fit a linear mixed-effects model (Pinheiro et al., 2011) with a random intercept on the municipality level to our data:

```
> summary(fit.lme <- lme(biomass.ha ~ mean.canopy.ht, data = JoSAE.sample.data,
+     random = ~1 | domain.ID))

Linear mixed-effects model fit by REML
 Data: JoSAE.sample.data
       AIC      BIC    logLik
  1553.764 1565.616 -772.8822
```

```
Random effects:
 Formula: ~1 | domain.ID
        (Intercept) Residual
StdDev:    10.30361 49.85829

Fixed effects: biomass.ha ~ mean.canopy.ht
                   Value Std.Error  DF   t-value p-value
(Intercept)    6.694678  8.334032 130  0.803294  0.4233
mean.canopy.ht 1.375782  0.077531 130 17.744832  0.0000
 Correlation:
               (Intr)
mean.canopy.ht -0.754

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-3.12149463 -0.56323615 -0.05238025  0.55696863  3.11427777

Number of Observations: 145
Number of Groups: 14
```

In combination with the domain-level data, the functions provided in the `JoSAE` package can now be used to calculate domain level EBLUP estimates and their variances. Since the functions expect variable names in the domain data and the sample data to be the same, we first have to do some renaming:

```
> d.data <- JoSAE.domain.data
> names(d.data)[3] <- "mean.canopy.ht"
```

Then we can use the `eblup.mse.f.wrap` function, which does all the work. This function is a wrapper function that calls several other `JoSAE` functions. All attributes the function needs are the domain data and the fitted model (an lme object).

```
> result <- eblup.mse.f.wrap(domain.data = d.data, lme.obj = fit.lme)
```

Besides the EBLUP estimate and its variance, the function calculates the GREG and SRS estimate as well as a synthetic regression estimate based on a linear model fitted with the fixed-part of the lme formula. Many other domain characteristics are calculated by the `eblup.mse.f.wrap` function. The help page lists the details. Let's print some of the most interesting results in Tables 1 and 2.

| domain.ID | N.i.domain | n.i.sample | sample.mean | GREG | EBLUP | Synth |
|---|---|---|---|---|---|---|
| 1 | 105267 | 1 | 92.73 | 112.97 | 153.76 | 155.73 |
| 2 | 202513 | 6 | 109.06 | 87.43 | 107.82 | 113.81 |
| 3 | 134156 | 3 | 169.54 | 105.08 | 132.74 | 136.82 |
| 4 | 193807 | 2 | 53.29 | 99.76 | 123.88 | 126.45 |
| 5 | 1379945 | 35 | 118.39 | 115.20 | 118.49 | 124.05 |
| 6 | 176731 | 4 | 93.63 | 136.18 | 116.91 | 114.23 |
| 7 | 474615 | 17 | 152.52 | 135.54 | 117.73 | 105.72 |
| 8 | 442280 | 12 | 106.40 | 105.79 | 99.86 | 97.69 |
| 9 | 495568 | 12 | 113.70 | 112.59 | 116.84 | 119.66 |
| 10 | 520141 | 14 | 124.14 | 100.89 | 110.76 | 117.47 |
| 11 | 230756 | 8 | 152.95 | 142.97 | 135.89 | 133.98 |
| 12 | 83441 | 1 | 34.11 | 74.37 | 118.19 | 120.66 |
| 13 | 57858 | 1 | 130.78 | 124.36 | 95.01 | 94.67 |
| 14 | 905387 | 29 | 97.77 | 106.32 | 102.46 | 98.42 |

Table 1: Number of population and sampled elements as well as simple random sample, synthetic, GREG and EBLUP estimates of the mean above-ground forest biomass within 14 Norwegian municipalities.

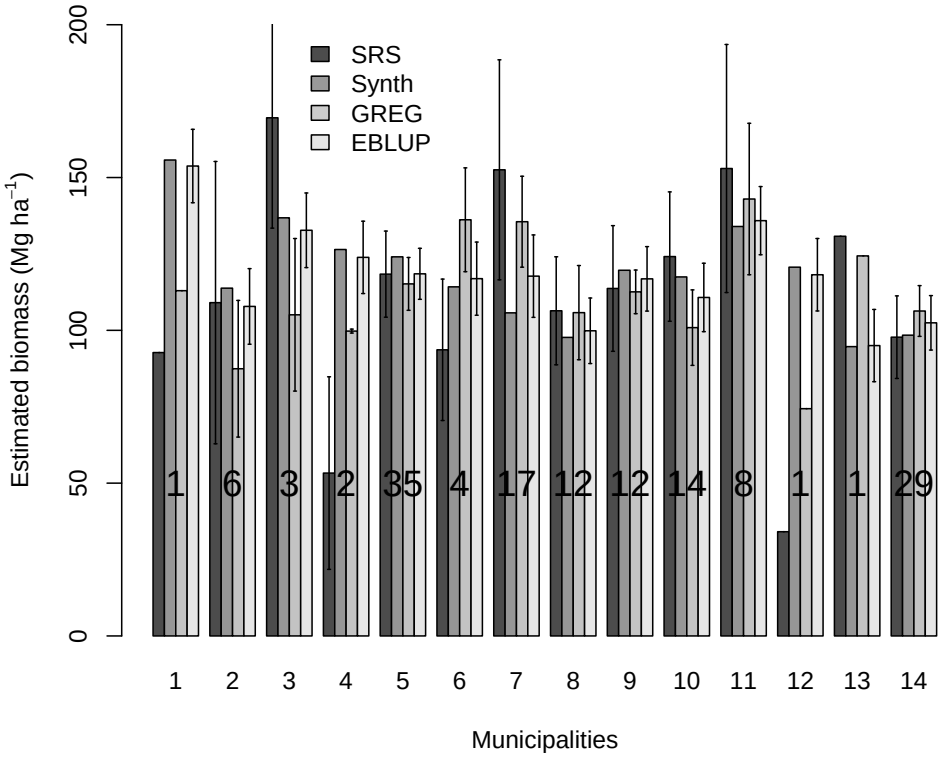
| domain.ID | n.i.sample | sample.se | GREG.se | EBLUP.se.1 | EBLUP.se.2 |
|---|---|---|---|---|---|
| 1 | 1 | | | 11.93 | 12.01 |
| 2 | 6 | 46.19 | 22.36 | 12.62 | 12.37 |
| 3 | 3 | 36.10 | 24.96 | 12.26 | 12.21 |
| 4 | 2 | 31.51 | 0.65 | 11.59 | 11.84 |
| 5 | 35 | 14.09 | 8.64 | 7.74 | 8.34 |
| 6 | 4 | 23.14 | 16.98 | 11.77 | 11.97 |
| 7 | 17 | 35.99 | 14.88 | 15.63 | 13.48 |
| 8 | 12 | 17.67 | 15.41 | 9.83 | 10.72 |
| 9 | 12 | 20.56 | 7.14 | 9.48 | 10.54 |
| 10 | 14 | 21.16 | 12.35 | 11.11 | 11.20 |
| 11 | 8 | 40.58 | 24.78 | 10.29 | 11.16 |
| 12 | 1 | | | 11.80 | 11.86 |
| 13 | 1 | | | 11.65 | 11.82 |
| 14 | 29 | 13.51 | 8.30 | 8.28 | 8.91 |

Table 2: Number of population and sampled elements as well as standard errors of the simple random sample, GREG and EBLUP estimates of the mean above-ground forest biomass within 14 Norwegian municipalities.

The `eblup.mse.f.wrap` function does not return a standard error for the synthetic regression estimate, since no estimators exist that consider its model bias. In Table 2, it needs to be noted that variances for the SRS and GREG estimates are unstable for small sample sizes within domains (say <6 observations). A variance estimate is technically impossible for domains with just one observation. The EBLUP variances are frequently smaller than the GREG variances and stable even for domains with just one observation. However, the EBLUP variance is model-based and thus relies on the correctness of the fitted model. Rao (2003) suggests two different EBLUP variances estimates. Both are returned by the `eblup.mse.f.wrap` function (Table 2).

The data can be visualized by:

```
> tmp <- result[, c("biomass.ha.sample.mean", "Synth", "GREG",
+     "EBLUP")]
> tmp1 <- barplot(t(as.matrix(tmp)), beside = T, names.arg = result$domain.ID,
+     xlab = "Municipalities", ylab = expression(paste("Estimated biomass (Mg ",
+         ha^{
+             -1
+         }, ")")), ylim = c(0, 200))
> text(tmp1[2, ] + 0.5, y = 50, labels = result$n.i.sample, cex = 1.5)
> tmp2 <- result[, c("sample.se", "sample.se", "GREG.se", "EBLUP.se.2")]
> tmp2[is.na(tmp2)] <- 0
> arrows(x0 = tmp1[1, ], y0 = tmp[, 1] + tmp2[, 1], x1 = tmp1[1,
+     ], y1 = tmp[, 1] - tmp2[, 1], length = 0.01, angle = 90,
+     code = 3)
> arrows(x0 = tmp1[3, ], y0 = tmp[, 3] + tmp2[, 3], x1 = tmp1[3,
+     ], y1 = tmp[, 3] - tmp2[, 3], length = 0.01, angle = 90,
+     code = 3)
> arrows(x0 = tmp1[4, ], y0 = tmp[, 4] + tmp2[, 4], x1 = tmp1[4,
+     ], y1 = tmp[, 4] - tmp2[, 4], length = 0.01, angle = 90,
+     code = 3)
> legend(13, 200, fill = grey(c(0.3, 0.6, 0.8, 0.9)), legend = c("SRS",
+     "Synth", "GREG", "EBLUP"), bty = "n")
```




## 2.2 County crop areas in Iowa

Battese et al. (1988) the EBLUP estimator and demonstrated its application using Landsat data to estimate the mean hectares of corn and soybeans within counties (small areas) in north-central Iowa. Thanks to Schoch (2011) the Landsat data are available in R. The functions in the `JoSAE` package should give approximately similar results as those presented by Battese et al. (1988) and Rao (2003, Table 7.3,p.144).

Let's get the data, split the data sets into a domain-specific and sample specific data frame and add a numeric domain ID to both. We will also exclude an "outlying" domain[2] in row 33 as was suggested by Battese et al. (1988):

```
> library(rsae)
> data(landsat)
> landsat.domains <- unique(landsat[-33, c(1, 7:8, 10)])
> landsat.domains$domain.ID <- 1:nrow(landsat.domains)
> names(landsat.domains)[2:3] <- c("PixelsCorn", "PixelsSoybeans")
> tmp <- landsat[-33, c(2:6, 10)]
> landsat.sample <- merge(landsat.domains[4:5], tmp, by = "CountyName")
```

Now we can fit a linear mixed-effects model and obtain our small area estimates:

```
> summary(landsat.lme <- lme(HACorn ~ PixelsCorn + PixelsSoybeans,
+     data = landsat.sample, random = ~1 | domain.ID))

Linear mixed-effects model fit by REML
 Data: landsat.sample
       AIC      BIC    logLik
  308.3666 315.8492 -149.1833
```

[2]The `rsae` package was specifically developed for robust estimation where outliers do not need to be excluded.

```
Random effects:
 Formula: ~1 | domain.ID
        (Intercept) Residual
StdDev:    11.83317 12.13543

Fixed effects: HACorn ~ PixelsCorn + PixelsSoybeans
                  Value Std.Error DF   t-value p-value
(Intercept)    51.07040 24.409705 22  2.092217  0.0482
PixelsCorn      0.32872  0.049876 22  6.590780  0.0000
PixelsSoybeans -0.13457  0.055194 22 -2.438092  0.0233
 Correlation:
               (Intr) PxlsCr
PixelsCorn     -0.935
PixelsSoybeans -0.892  0.723

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-1.87576686 -0.70964548 -0.08543768  0.72472023  1.65660575

Number of Observations: 36
Number of Groups: 12
```

```
> result <- eblup.mse.f.wrap(domain.data = landsat.domains, lme.obj = landsat.lme)
```

| County.name | n_i | EBLUP | EBLUP.se.1 | EBLUP.se.2 | GREG.se |
|---|---|---|---|---|---|
| Cerro Gordo | 1 | 122.20 | 9.04 | 9.52 | |
| Hamilton | 1 | 126.22 | 9.04 | 9.46 | |
| Worth | 1 | 106.70 | 10.66 | 10.19 | |
| Humboldt | 2 | 108.44 | 8.11 | 8.18 | 19.88 |
| Franklin | 3 | 144.28 | 7.10 | 6.89 | 6.86 |
| Pocahontas | 3 | 112.14 | 6.68 | 6.70 | 6.65 |
| Winnebago | 3 | 112.80 | 6.62 | 6.66 | 9.13 |
| Wright | 3 | 122.00 | 6.29 | 6.55 | 8.73 |
| Webster | 4 | 115.33 | 5.95 | 5.92 | 4.09 |
| Hancock | 5 | 124.42 | 5.13 | 5.28 | 4.22 |
| Kossuth | 5 | 106.90 | 5.62 | 5.48 | 3.18 |
| Hardin | 5 | 143.01 | 5.57 | 5.63 | 5.06 |

Table 3: EBLUP estimates of county means of hectares under corn and estimated standard errors of the EBLUP and GREG estimates.

Comparing Table 3 with the reference (Battese et al., 1988; Rao, 2003) suggests that the results are quite similar but not exactly the same. The EBLUP estimates for the county means are slightly different because Battese et al. (1988) adjusted the estimates to sum up to the approximately unbiased Survey-Regression estimate for the total area. The standard errors are slightly different since Battese et al. (1988) used the method of *fitting of constants* to estimate the model parameters but REML was used here. Finally, Battese et al. (1988) obtained standard errors also for Survey-Regression estimates within domains with just one observation. Unfortunately, no details were elaborated. Given that the Survey-Regression estimator should be the same as the GREG (Rao, 2003, p. 20), it is unclear to me how this was done (any hints would be appreciated).

All in all, it looks like the functions in the `JoSAE` package are correctly implemented.

## 3 Acknowledgments

I would like to thank Tobias Schoch, the author of the `rsae` package (Schoch , 2011) for the provision of the Landsat data set.

## References

Battese, G.E., R.M. Harter, and W.A. Fuller (1988): *An Error-Components Model for Prediction of County Crop Areas Using Survey and Satellite Data*, Journal of the American Statistical Association 83, pp. 28-36.

Breidenbach, J. and R. Astrup (2011): *Small area estimation of forest attributes in the Norwegian National Forest Inventory*. European Journal of Forest Research, under review.

Pinheiro, J., D. Bates, S. DebRoy, D. Sarkar and the R Development Core Team (2011): *nlme: Linear and Nonlinear Mixed Effects Models*. R package version 3.1-101.

Rao, J.N.K. (2003): *Small Area Estimation*, New Work: John Wiley and Sons.

Särndal, C. (1984): *Design-consistent versus model-dependent estimation for small domains*. Journal of the American Statistical Association, JSTOR, 624-631

Schoch, T. (2011): *rsae: Robust Small Area Estimation*, R package version 0.1-3.