Analyses Using Achievement Levels Based on Plausible Values[1]

Qingshu Xie[2] and Ting Zhang[3]

Introduction

Achievement levels are important yardsticks in the National Assessment of Educational Progress (NAEP) program. They are intended to measure to what extent students' achievement in a certain subject matches the expected achievement defined in the assessment frameworks of various subjects adopted by the National Assessment Governing Board (NAGB). Authorized by the NAEP legislation and adopted by NAGB, the achievement levels in each main NAEP assessment are based on collective judgments about what students should know and can do in each subject-area assessment relative to a body of content reflected in the framework. Three levels—*Basic*, *Proficient*, and *Advanced*—are defined for each subject and each grade, with cut scores for each level determined through a standard-setting process.[4] Specific cut scores for each achievement level are set for each subject and by grade. The results using NAEP achievement levels are standard indicators of student performance reported in the *Nation's Report Card*. They also can be generated using the online NAEP Data Explorer (NDE)[5] at the National Center for Education Statistics (NCES).

As in the NDE, analyses using achievement levels can be conducted in three ways.

- **Discrete**: the percentage of students performing within each achievement level, counted separately from the other levels. These categories are the percentages of students scoring below *Basic*, at *Basic*, at *Proficient*, and at *Advanced*. The percentages at all mutually exclusive achievement levels add up to 100 percent.
- **Cumulative**: the percentage of students performing at or above each achievement level. These categories are percentages of students scoring below *Basic*, at or above *Basic*, at or above *Proficient*, and at *Advanced*. Except below *Basic* and at *Advanced*, the other two cumulative levels include students at the specific and all higher levels. Since they are not mutually exclusive, it is not meaningful to add all of these four percentages of cumulative achievement levels.
- **Achievement level as an independent variable**: of the students at each achievement level, discrete or cumulative, the percentage distribution of the students by selected

[1] This publication was prepared for NCES under Contract No. ED-IES-12-D-0002 with American Institutes for Research. Mention of trade names, commercial products, or organizations does not imply endorsement by the U.S. Government. The authors would like to thank Dr. Dan Sherman, Dr. Emmanuel Sikali, and Dr. Lauren Harrell for reviewing this document.
[2] MacroSys, 1901 N. Moore St., Ste. 602, Arlington, VA 22209
[3] American Institutes for Research, 1000 Thomas Jefferson St. NW, Washington, DC 20007
[4] For more information about NAEP achievement levels in each subject and their cut points, see https://nces.ed.gov/nationsreportcard/achievement.aspx.
[5] http://nces.ed.gov/nationsreportcard/naepdata/

characteristics can be computed. The percentages by the categories of the selected variable add up to 100 percent.

Methodologically, an analysis using NAEP achievement levels can be conducted with different approaches. For example, plausible values are used to compute the results by achievement levels, including discrete and cumulative levels, and by using achievement levels as independent variables. The online software NDE uses this approach. Another approach is built upon the marginal maximum likelihood (MML) composite regression to compute the estimates by achievement levels. The estimates are based on the posterior distributions from which plausible values are drawn. AM software,[6] a free statistical package customized for analyzing data from large-scale assessments such as NAEP, employs this technique. Currently, there are some restraints on these two approaches. First, the MML method is complicated in programming and not easily implemented with other statistical programs such as SAS, Stata, and R. In terms of the plausible value approach, there is not a complete set of well-documented methodologies for estimating achievement levels using plausible values. No formulae are provided for computing both discrete and cumulative achievement levels using plausible values at the NDE website; and the NDE formulae for computing the results using achievement levels as independent variables are not quite complete.[7]

In this document, we provide a detailed description of the plausible value method and demonstrate the method by replicating the analyses using SAS and comparing the replicated analysis results with those produced with NDE. It is intended to provide a complete document for NAEP data users to easily adapt the methodologies with other statistical software packages such as SAS, Stata, R, and so on. For the purpose of the demonstration, the 2011 and 2013 NAEP grade 8 mathematics assessment data sets are used. Since the two data sets contain different sets of plausible values (i.e., five sets of plausible values in the 2011 data set and 20 sets of plausible values in the 2013 data set), the method allows an opportunity to determine whether the increase in the number of plausible values affects how the results using achievement levels are computed.

In the following sections, the methodologies for the three types of analyses using achievement levels are described first. Second, analyses using achievement levels with selected variables are conducted using SAS; the results then are compared with those from the corresponding analyses with NDE to examine the accuracy of the results from the exercise. Finally, conclusions and recommendations are discussed at the end.

Methodology

When the achievement levels are based on plausible values, the analyses using NAEP achievement levels have to follow the standard methodology for NAEP data analysis in order to obtain appropriate estimates and variances.[8] The estimated percentage of students performing at a specific achievement level is computed by averaging the estimated percentages based on all plausible values; the variance of the estimated percentage is derived by combining sampling

[6] AM Statistical Software Beta Version: http://am.air.org/
[7] http://nces.ed.gov/nationsreportcard/NDEHelp/WebHelp/perfornance_measures_overview.htm
[8] For general descriptions, see https://nces.ed.gov/nationsreportcard/tdw/analysis/2004_2005/summary_proced_group.aspx.

variance and measurement variance among the estimated percentages based on all plausible values. Jackknife replication method is used in sampling variance estimation in this exercise.

All the results—including estimates, measurement variance, sampling variance, and standard errors—are to be presented in percentages.

Symbols and Notations

The following are the common major symbols and notations to be used in the following subsections. The meaning of a notation will change with its subscripts in reference to achievement level, plausible values, and the student and with its superscripts in reference to replicates. The meanings of specific notations are described respectively in corresponding subsections.

π: Percentage of students at a certain achievement level.

κ: Achievement level at level κ, with $k \in [1, \ldots, \mathrm{K}]$.

m: The mth set of plausible value, with $m \in [1, \ldots, \mathrm{M}]$.

i: The ith student in the sample, with $i \in [1, \ldots, \mathrm{N}]$.

r: The rth replicate, with $r \in [1, \ldots, \mathrm{R}]$, where R is 62 in the NAEP data.

w_i: The full sample weight of the ith student in the sample.

w_i^r: The rth replicate weight of the ith student in the sample.

A combination of these symbols in a notation has its special definition. For example, π_{km} means the percentage of students at achievement level *k* based on plausible value *m* in the full sample, π_{km}^r means the percentage of students at achievement level *k* based on plausible value *m* in replicate r, and π_k means the average percentage of students at achievement *k* based on all plausible values in the full sample.

Discrete achievement levels

Assume that variable $\pi_{km}$with $k \in [1, \ldots, \mathrm{K}], m \in [1, \ldots, \mathrm{M}]$ represents the percentage of students that demonstrate proficiency at one of *k* mutually exclusive levels based on plausible value *m* relative to all students. *K* indicates the total number of the achievement levels, and *M* indicates the total number of sets of plausible values used in the calculation.

$$\pi_{km} = \frac{\sum_{i=1}^{N} w_i I_{ikm}}{\sum_{i=1}^{N} w_i}. \qquad \text{(D-1)}$$

where I_{ikm} is a binary variable, which assumes value 1 if student *i* performs at level *k* based on plausible value *m* and assumes the value 0 otherwise. Also, *N* indicates all students in the sample, whereas w_i is the full sample weight for student *i*. Thus, the numerator is a weighted sum over students who demonstrate proficiency at that level *k*, whereas the denominator is a weighted sum over all students.

The percentage of students performed at *k* level is estimated as

$$\pi_k = \frac{1}{M}\sum_{m=1}^{M}\pi_{km} \,. \qquad \text{(D-2)}$$

The measurement variance of the estimated percentage is

$$V_{Measurement} = \frac{1}{M-1}\left(\sum_{m=1}^{M}(\pi_{km} - \pi_k)^2\right) . \qquad \text{(D-3)}$$

In order to obtain sampling variance of the estimated percentage, replicate weight w_i^r is used to estimate π_{km}^r, the percentage of students who demonstrate proficiency at level *k* based on plausible value *m* in replicate r, whereas $m \in [1,\ldots,\mathrm{M}], r \in [1,\ldots,\mathrm{R}]$. *M* is the total number of plausible values, and *R* is the total number of replicate weights.

$$\pi_{km}^r = \frac{\sum_{i=1}^{N} w_i^r I_{ikm}}{\sum_{i=1}^{N} w_i^r} \,. \qquad \text{(D-4)}$$

The average sampling variance of the estimated percentage over *M* sets of plausible values is

$$V_{Sampling} = \frac{1}{M}\left(\sum_{r=1}^{R}\sum_{m=1}^{M}(\pi_{km}^r - \pi_{km})^2\right) . \qquad \text{(D-5)}$$

The standard error of the percentage at achievement level *k* is the square root of the combination of two sources of variance: The sampling variance and the adjusted measurement variance is

$$Stderr_k = SQRT\left(V_{Sampling} + \frac{M+1}{M} V_{Measurement}\right) . \qquad \text{(D-6)}$$

To reduce computation burden, it is the convention to only use the first plausible value to compute the sampling variance of the estimate as defined in (D-7), which replaces the first component in (D-6). This is the method that has been employed by NAEP reports and NDE for analyses using achievement level.[9]

$$V_{Sampling} = \sum_{r=1}^{R}(\pi_{k1}^r - \pi_{k1})^2. \qquad \text{(D-7)}$$

Cumulative achievement levels

Percentages pertaining to different cumulative achievement levels are not mutually exclusive. To avoid any confusion, subscripts are not used for a cumulative level in the following notations. The following formulae are generalized and are applicable to any cumulative achievement level. Essentially, an analysis using a cumulative achievement level is to analyze the NAEP data with a set of binary variables defined by applying the cut score on all plausible values.

Suppose that variable $\pi_m$with $m \in [1,\ldots,\mathrm{M}]$ represents the percentage of students who demonstrate proficiency at or above a specific achievement level based on plausible value *m*

[9] Beaton, A.E., Rogers, A.M., Gonzalez, E., Hanly, M.B., Kolstad, A., Rust, K.F., Sikali, E., Stokes, L., and Jia, Y. (2011). *The NAEP Primer* (NCES 2011-463). U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office.

relative to all students. *M* indicates the total number of sets of plausible values used in the calculation.

$$\pi_m = \frac{\sum_{i=1}^{N} w_i I_{im}}{\sum_{i=1}^{N} w_i} \qquad \text{(C-1)}$$

where I_{im} is a binary variable, which assumes value 1 if student *i* performs at or above a specific achievement level based on plausible value *m* and assumes the value 0 otherwise. Also, *N* indicates all students in the sample. w_i is the full sample weight for student *i*. Thus, the numerator is a weighted sum over students who demonstrate proficiency at or above a specific achievement level, whereas the denominator is a weighted sum over all students.

The percentage of students who performed at or above the specific achievement level is estimated as

$$\pi = \frac{1}{M}\sum_{m=1}^{M} \pi_m. \qquad \text{(C-2)}$$

The measurement variance of the estimated percentage is

$$V_{Measurement} = \frac{1}{M-1}\left(\sum_{m=1}^{M}(\pi_m - \pi)^2\right). \qquad \text{(C-3)}$$

In order to obtain sampling variance of the estimated percentage, replicate weight w_i^r is used to estimate replicate percentage π_m^r, the percentage of students who demonstrate proficiency at or above the cumulative level based on plausible value *m* in replicate r, whereas $m \in [1, \ldots, \mathrm{M}], r \in [1, \ldots, \mathrm{R}]$. *M* is the total number of plausible values, and *R* is the total number of replicate weights.

$$\pi_m^r = \frac{\sum_{i=1}^{N} w_i^r I_{im}}{\sum_{i=1}^{N} w_i^r} \qquad \text{(C-4)}$$

The average sampling variance of the estimated percentage over the *M* sets of plausible values is

$$V_{Sampling} = \frac{1}{M}\left(\sum_{r=1}^{R}\sum_{m=1}^{M}(\pi_m^r - \pi_m)^2\right). \qquad \text{(C-5)}$$

The standard error of the percentage at or above the specific achievement level is

$$Stderr = SQRT\left(V_{Sampling} + \frac{M+1}{M} V_{Measurement}\right). \qquad \text{(C-6)}$$

To reduce computation burden, it is the convention to only use the first plausible value to compute the sampling variance of the estimate as defined in (C-7), which replaces the first component in (C-6). This is the method that has been employed by NAEP reports and NDE for analyses using achievement level.

$$V_{Sampling} = \sum_{r=1}^{R}(\pi_1^r - \pi_1)^2. \qquad \text{(C-7)}$$

Achievement levels as independent variables

When using achievement levels as independent variables, an example research question can be, "Of those students who are at each achievement level, what are the percentages of male students and female students, respectively?" Using achievement levels as independent variables facilitates analyses of student characteristics within a specific achievement level and helps to examine the change of the distribution of students by certain characteristics such as race/ethnicity and school location with the change of achievement level. Either a discrete or a cumulative achievement level can be used as an independent variable. This section describes the method of using discrete achievement levels as independent variables. To apply the following method with cumulative achievement levels, for each cumulative level, students are essentially divided into two mutually exclusive categories: one is below the specific cumulative level, and the other is at or above the specific level. Of our interest is the category at or above the specific level.

Let variable $\pi_{gkm}$with $g \in [1,\ldots,\mathrm{G}], k \in [1,\ldots,\mathrm{K}], m \in [1,\ldots,\mathrm{M}]$ represent the percentage of students in student group *g* that demonstrates proficiency at one of *k* mutually exclusive levels based on plausible value *m* relative to all students who demonstrate proficiency at that level *k*. *G* is the total number of student groups, *K* indicates the total number of the achievement levels, and *M* indicates the total number of plausible values used in the calculation.

$$\pi_{gkm} = \frac{\sum_{i=1}^{N_g} w_i I_{ikm}}{\sum_{i=1}^{N} w_i I_{ikm}} \qquad \text{(I-1)}$$

where I_{ikm} is a binary variable, which assumes value 1 if student *i* performs at level *k* based on plausible value *m* and assumes value 0 otherwise. Also, *N* indicates all students in the sample, whereas N_g only indicates those students in group *g*. $\mathrm{w_i}$ is the full sample weight for student *i*. Essentially, the numerator is a weighted sum over students who demonstrate proficiency at level *k* in group *g*, whereas the denominator is a weighted sum over all students who demonstrate proficiency at level *k*.

The percentage of students in group *g* who demonstrate proficiency at *k* level is estimated as

$$\pi_{gk} = \frac{1}{M}\sum_{m=1}^{M} \pi_{gkm}. \qquad \text{(I-2)}$$

The measurement variance of the estimated percentage is

$$V_{Measurement} = \frac{1}{M-1}\sum_{m=1}^{M}(\pi_{gkm} - \pi_{gk})^2. \qquad \text{(I-3)}$$

In order to obtain sampling variance of the estimated percentage, replicate weight w_i^r is used to estimate π_{gkm}^r, the percentage of students in student group *g* who demonstrate proficiency at level *k* based on plausible value *m* relative to all students who demonstrate proficiency at that level *k* in replicate r, whereas $m \in [1,\ldots,\mathrm{M}], r \in [1,\ldots,\mathrm{R}]$. *M* is the total number of plausible values, and *R* is the total number of replicate weights.

$$\pi_{gkm}^r = \frac{\sum_{i=1}^{N_g} w_i^r I_{ikm}}{\sum_{i=1}^{N} w_i^r I_{ikm}} \qquad \text{(I-4)}$$

The average sampling variance of the estimated percentage over the *M* sets of plausible values is

$$V_{Sampling} = \frac{1}{M}(\sum_{r=1}^{R}\sum_{m=1}^{M}(\pi_{gkm}^{r} - \pi_{gkm})^2). \quad \text{(I-5)}$$

The standard error of the estimated percentage is

$$Stderr = SQRT(V_{Sampling} + \frac{M+1}{M} V_{Measurement}). \quad \text{(I-6)}$$

To reduce computation burden, the convention is to only use the first plausible value to compute the sampling variance of the estimate as defined in (I-7), which replaces the first component in (I-6). This is the method currently employed by NDE for analysis using achievement level as an independent variable.

$$V_{Sampling} = \sum_{r=1}^{R}(\pi_{gk1}^{r} - \pi_{gk1})^2 \quad \text{(I-7)}$$

Research Design

We used SAS version 9.3 to run the analyses using achievement levels. The same analyses were conducted using NDE. The results from the analyses using SAS and NDE were compared. Data source and variables are presented as follows.

Data

- NAEP 2011 grade 8 mathematics assessment (5 sets of plausible values)
- NAEP 2013 grade 8 mathematics assessment (20 sets of plausible values)

Analyses using NDE

Select criteria

- Measure: Composite scale
- Jurisdiction: National

Variables

- All students
- Gender
- Race/ethnicity using 2011 guidelines, school-reported

Statistics reporting options

- Achievement levels—discrete
- Achievement levels—cumulative

In addition, we used the "Create New Report" function in NDE to conduct analyses using achievement level as an independent variable.

Analyses using SAS

We used the PROC SURVEYFREQ procedure to conduct the analyses. The first step for the analyses is to create categorical analysis variables by applying the cut scores onto all plausible

values. That means creating the same number of variables with each cut score as that of plausible values. The definitions of achievement levels are shown in table 1.

To conduct analyses using discrete achievement levels, a set of variables for achievement level is created with four categories according to the cut scores for grade 8 mathematics assessment: below *Basic*, at *Basic*, at *Proficient*, and at *Advanced*. These four categories are mutually exclusive. One way is to create four dummy variables for each of the four categories, and the other approach is to create a four-category variable with each category representing a discrete achievement level.

To conduct analyses using cumulative achievement levels, with each plausible value four binary variables are created for each cumulative achievement level, respectively, according to the cut scores for grade 8 mathematics assessment: below *Basic*, at or above *Basic*, at or above *Proficient*, and at *Advanced*.

Table 1. Definitions of achievement levels

Achievement level	Definition	Description
Discrete achievement level		
below *Basic*	[0, Basic)	Basic, Proficient and Advanced are cut points for *Basic*, *Proficient* and *Advanced* achievement in NAEP assessment. They vary by subject or by subject and assessment year. For grade 8 mathematics assessment, the cut scores for achievement levels are as follows: Basic = 262 Proficient = 299 Advanced = 333
at *Basic*	[Basic, Proficient)	
at *Proficient*	[Proficient, Advanced)	
at *Advanced*	[Advanced, maximum score]	
Cumulative achievement level		
below *Basic*	[0, Basic)	
at or above *Basic*	[Basic, maximum score]	
at or above *Proficient*	[Proficient, maximum score]	
at *Advanced*	[Advanced, maximum score]	

Results

The percentages by achievement level from NDE are with precision to 13 decimal places, and their standard errors are in 15 decimal places. The results by achievement level from SAS are all set to be in 14 decimal places. In table 2, percentages are presented in eight decimal places and standard errors in nine decimal places. As shown in table 2, both percentages and standard errors from SAS match those from NDE to at least seven decimal places for the national totals in 2011 and 2013. The results by sex and by race/ethnicity also match very well (see more results in tables A-1 to A-4 in the appendix).

Table 2. Percentage of students by achievement level for mathematics, grade 8: 2011 and 2013

Year/Achievement level	SAS estimate		NDE result		Comparison (SAS minus NDE)	
	Percentage	Standard error	Percentage	Standard error	Percentage	Standard error
2011						
Discrete level						
below *Basic*	26.62810861	0.245522309	26.62810860	0.245522298	0.00000000	0.000000011
Basic	38.63631387	0.290502417	38.63631386	0.290502424	0.00000000	-0.000000007
Proficient	26.47981687	0.196630163	26.47981687	0.196630157	0.00000000	0.000000006
Advanced	8.25576065	0.165213040	8.25576066	0.165213038	0.00000000	0.000000003
Cumulative level						
below *Basic*	26.62810861	0.245522309	26.62810860	0.245522298	0.00000000	0.000000011
at or above *Basic*	73.37189139	0.245522309	73.37189140	0.245522298	0.00000000	0.000000011
at or above *Proficient*	34.73557753	0.244085057	34.73557753	0.244085048	-0.00000001	0.000000009
at *Advanced*	8.25576065	0.165213040	8.25576066	0.165213038	0.00000000	0.000000003
2013						
Discrete level						
below *Basic*	26.04289256	0.271883147	26.04289256	0.271883144	0.00000000	0.000000002
Basic	38.46489023	0.295288807	38.46489023	0.295288802	-0.00000001	0.000000004
Proficient	26.68028903	0.239708178	26.68028903	0.239708177	0.00000000	0.000000001
Advanced	8.81192819	0.194737596	8.81192818	0.194737598	0.00000001	-0.000000002
Cumulative						
below *Basic*	26.04289256	0.271883147	26.04289256	0.271883144	0.00000000	0.000000002
at or above *Basic*	73.95710744	0.271883147	73.95710744	0.271883144	0.00000000	0.000000002
at or above *Proficient*	35.49221721	0.321072317	35.49221721	0.321072321	0.00000000	-0.000000005
at *Advanced*	8.81192819	0.194737596	8.81192818	0.194737598	0.00000001	-0.000000002

Unlike with the percentages of students by achievement levels, NDE uses a different level of precision for results using achievement level as an independent variable. Percentages are integers, and their standard errors contain only one decimal place. We generated corresponding results in the same level of precision. As shown in table 3, the results from SAS match those from NDE perfectly.

Table 3. Percentage of students at and at or above each achievement level for mathematics at grade 8, by gender: 2011 and 2013

Year/Achievement level	Sex	SAS estimate		NDE result		Comparison (SAS minus NDE)	
		Percentage	Standard error	Percentage	Standard error	Percentage	Standard error
2011							
Discrete level							
below *Basic*	Male	51	0.4	51	0.4	0	0.0
	Female	49	0.4	49	0.4	0	0.0
Basic	Male	49	0.3	49	0.3	0	0.0
	Female	51	0.3	51	0.3	0	0.0
Proficient	Male	51	0.5	51	0.5	0	0.0
	Female	49	0.5	49	0.5	0	0.0
Advanced	Male	57	0.7	57	0.7	0	0.0
	Female	43	0.7	43	0.7	0	0.0
Cumulative level			.				
below *Basic*	Male	51	0.4	51	0.4	0	0.0
	Female	49	0.4	49	0.4	0	0.0
at or above *Basic*	Male	51	0.2	51	0.2	0	0.0
	Female	49	0.2	49	0.2	0	0.0
at or above *Proficient*	Male	52	0.3	52	0.3	0	0.0
	Female	48	0.3	48	0.3	0	0.0
at *Advanced*	Male	57	0.7	57	0.7	0	0.0
	Female	43	0.7	43	0.7	0	0.0
2013							
Discrete level							
Below *Basic*	Male	52	0.4	52	0.4	0	0.0
	Female	48	0.4	48	0.4	0	0.0
Basic	Male	49	0.4	49	0.4	0	0.0
	Female	51	0.4	51	0.4	0	0.0
Proficient	Male	51	0.5	51	0.5	0	0.0
	Female	49	0.5	49	0.5	0	0.0
Advanced	Male	56	0.8	56	0.8	0	0.0
	Female	44	0.8	44	0.8	0	0.0
Cumulative level							
below *Basic*	Male	52	0.4	52	0.4	0	0.0
	Female	48	0.4	48	0.4	0	0.0
at or above *Basic*	Male	51	0.2	51	0.2	0	0.0
	Female	49	0.2	49	0.2	0	0.0
at or above *Proficient*	Male	52	0.4	52	0.4	0	0.0
	Female	48	0.4	48	0.4	0	0.0
at *Advanced*	Male	56	0.8	56	0.8	0	0.0
	Female	44	0.8	44	0.8	0	0.0

Conclusions and Discussions

The results from the replication exercise with SAS fully match those from NDE—the official NCES online NAEP data analysis tool. This indicates that the methods discussed in this document can be used as a guide to conduct analyses using achievement levels with other statistical software packages.

Currently, there are limitations to the analyses using achievement level with NDE. Only three types of analyses are available on the NDE, including analyses using discrete and cumulative achievement levels and analysis by using achievement levels as independent variables.

The analysis using achievement levels can be expanded to answer a broader range of research questions. For example, using achievement level as the dependent variable, a logistic regression analysis can be performed to determine the major factors that are associated with the students within a specific achievement level. Researchers who also are interested in NAEP achievement scores can perform the analysis of NAEP scores by achievement level or multiple regression analysis by achievement level. By analyzing NAEP scores by achievement level, the performance trend of the students at each performance level can be tracked, and the gap between the students at different performance levels over time can be measured. Using plausible values as the dependent variable, a multiple regression analysis by achievement level helps to determine the relationship between student performance and student and other characteristics at each performance level, which helps to reveal whether significant differences exist in the roles of such variables as sex, race/ethnicity, eligibility for the National School Lunch Program, parental education, urbanicity, and English language learner status among students across different performance levels.

In summary, the purpose of this technical document is to provide a guide for computing achievement level statistics using plausible values. We hope it can benefit more researchers who are interested in analyses using achievement levels.

Appendix. Analysis Results Using Achievement Level by Sex and Race/Ethnicity

Table A-1. Percentage of students at each achievement level for mathematics at grade 8, by sex and race/ethnicity: 2011

Characteristics/discrete level	SAS estimate		NDE result		Comparison (SAS minus NDE)	
	Percentage	Standard error	Percentage	Standard error	Percentage	Standard error
Sex						
Male						
below *Basic*	26.65720196	0.355921750	26.65720197	0.355921756	-0.00000001	-0.000000006
Basic	37.59574418	0.348081638	37.59574417	0.348081643	0.00000001	0.000000004
Proficient	26.53053533	0.293723096	26.53053533	0.293723095	0.00000000	0.000000001
Advanced	9.21651853	0.218790328	9.21651853	0.218790330	0.00000000	0.000000001
Female						
below *Basic*	26.59816011	0.245730532	26.59816010	0.245730523	0.00000001	0.000000009
Basic	39.70746910	0.358579837	39.70746910	0.358579849	0.00000000	-0.000000012
Proficient	26.42760764	0.322091758	26.42760765	0.322091753	-0.00000001	0.000000005
Advanced	7.26676315	0.190843055	7.26676316	0.190843050	-0.00000001	0.000000005
Race/ethnicity						
White						
below *Basic*	16.47768453	0.264905192	16.47768453	0.264905184	0.00000000	0.000000008
Basic	39.14687889	0.365123506	39.14687888	0.365123511	0.00000000	-0.000000004
Proficient	33.49224965	0.244141479	33.49224965	0.244141475	0.00000000	0.000000004
Advanced	10.88318693	0.213086572	10.88318694	0.213086571	-0.00000001	0.000000001
Black						
below *Basic*	49.08809860	0.615296127	49.08809858	0.615296143	0.00000002	-0.000000016
Basic	37.56331840	0.466261444	37.56331840	0.466261458	0.00000000	-0.000000014
Proficient	11.81594335	0.368475997	11.81594337	0.368475995	-0.00000002	0.000000002
Advanced	1.53263966	0.118512839	1.53263966	0.118512840	-0.00000001	-0.000000001
Hispanic						
below *Basic*	39.47302560	0.703137359	39.47302561	0.703137357	-0.00000001	0.000000002
Basic	40.26806711	0.731672836	40.26806710	0.731672831	0.00000001	0.000000005
Proficient	17.59677552	0.534540638	17.59677552	0.534540638	0.00000000	0.000000000
Advanced	2.66213177	0.183707435	2.66213177	0.183707430	0.00000000	0.000000005
Asian						
below *Basic*	12.15809358	0.901025901	12.15809359	0.901025869	-0.00000001	0.000000032
Basic	29.94075741	1.364638997	29.94075742	1.364639029	-0.00000001	-0.000000031
Proficient	34.22508541	1.216451371	34.22508540	1.216451382	0.00000001	-0.000000011
Advanced	23.67606360	1.049961996	23.67606359	1.049962011	0.00000001	-0.000000015
American Indian/Alaska Native						
below *Basic*	45.35522911	1.483335028	45.35522913	1.483335101	-0.00000002	-0.000000073
Basic	37.55950240	1.562835632	37.55950241	1.562835667	-0.00000001	-0.000000035
Proficient	13.64304105	1.306383023	13.64304103	1.306382990	0.00000002	0.000000033
Advanced	3.44222743	0.662544485	3.44222743	0.662544498	0.00000001	-0.000000013
Native Hawaiian/Pacific Islander						
below *Basic*	41.15587621	4.663518832	41.15587634	4.663518822	-0.00000013	0.000000011
Basic	37.02627300	4.177786827	37.02627291	4.177786753	0.00000009	0.000000073
Proficient	17.37894175	1.921452273	17.37894171	1.921452325	0.00000004	-0.000000052
Advanced	4.43890904	1.444267101	4.43890904	1.444267118	0.00000000	-0.000000017
Two or more races						
below *Basic*	22.48269603	1.070835707	22.48269597	1.070835769	0.00000006	-0.000000062
Basic	38.35142077	1.478429979	38.35142074	1.478430002	0.00000004	-0.000000023
Proficient	28.43234904	1.575668326	28.43234909	1.575668387	-0.00000006	-0.000000061
Advanced	10.73353416	0.948681008	10.73353420	0.948681036	-0.00000004	-0.000000028

Table A-2. Percentage of students at or above each achievement level for mathematics at grade 8, by sex and race/ethnicity: 2011

Characteristics/cumulative level	SAS estimate		NDE result		Comparison (SAS minus NDE)	
	Percentage	Standard error	Percentage	Standard error	Percentage	Standard error
Sex						
Male						
below *Basic*	26.65720196	0.355921750	26.65720197	0.35592176	-0.00000001	-0.00000001
at or above *Basic*	73.34279804	0.355921750	73.34279803	0.355921756	0.00000001	-0.00000001
at or above *Proficient*	35.74705386	0.333516351	35.74705386	0.333516351	0.00000000	0.00000000
at *Advanced*	9.21651853	0.218790328	9.21651853	0.218790330	0.00000000	0.00000000
Female						
below *Basic*	26.59816011	0.245730532	26.59816010	0.245730523	0.00000001	0.00000001
at or above *Basic*	73.40183989	0.245730532	73.40183990	0.245730523	-0.00000001	0.00000001
at or above *Proficient*	33.69437079	0.318658573	33.69437081	0.318658568	-0.00000001	0.00000001
at *Advanced*	7.26676315	0.190843055	7.26676316	0.190843050	-0.00000001	0.00000001
Race/ethnicity						
White						
below *Basic*	16.47768453	0.264905192	16.47768453	0.264905184	0.00000000	0.000000008
at or above *Basic*	83.52231547	0.264905192	83.52231547	0.264905184	0.00000000	0.000000008
at or above *Proficient*	44.37543658	0.319783505	44.37543659	0.319783495	-0.00000001	0.000000011
at *Advanced*	10.88318693	0.213086572	10.88318694	0.213086571	-0.00000001	0.000000001
Black						
below *Basic*	49.08809860	0.615296127	49.08809858	0.615296143	0.00000002	-0.000000016
at or above *Basic*	50.91190140	0.615296127	50.91190142	0.615296143	-0.00000002	-0.000000016
at or above *Proficient*	13.34858300	0.386483136	13.34858303	0.386483129	-0.00000002	0.000000007
at *Advanced*	1.53263966	0.118512839	1.53263966	0.118512840	-0.00000001	-0.000000001
Hispanic						
below *Basic*	39.47302560	0.703137359	39.47302561	0.703137357	-0.00000001	0.000000002
at or above *Basic*	60.52697440	0.703137359	60.52697439	0.703137357	0.00000001	0.000000002
at or above *Proficient*	20.25890729	0.554781159	20.25890729	0.554781165	0.00000000	-0.000000006
at *Advanced*	2.66213177	0.183707435	2.66213177	0.183707430	0.00000000	0.000000005
Asian						
below *Basic*	12.15809358	0.901025901	12.15809359	0.901025869	-0.00000001	0.000000032
at or above *Basic*	87.84190642	0.901025901	87.84190641	0.901025869	0.00000001	0.000000032
at or above *Proficient*	57.90114902	1.270775863	57.90114899	1.270775887	0.00000002	-0.000000024
at *Advanced*	23.67606360	1.049961996	23.67606359	1.049962011	0.00000001	-0.000000015
American Indian/Alaska Native						
below *Basic*	45.35522911	1.483335028	45.35522913	1.483335101	-0.00000002	-0.000000073
at or above *Basic*	54.64477089	1.483335028	54.64477087	1.483335101	0.00000002	-0.000000073
at or above *Proficient*	17.08526848	1.200892525	17.08526845	1.200892479	0.00000003	0.000000047
at *Advanced*	3.44222743	0.662544485	3.44222743	0.662544498	0.00000001	-0.000000013
Native Hawaiian/Pacific Islander						
below *Basic*	41.15587621	4.663518832	41.15587634	4.663518822	-0.00000013	0.000000011
at or above *Basic*	58.84412379	4.663518832	58.84412366	4.663518822	0.00000013	0.000000011
at or above *Proficient*	21.81785079	1.898069742	21.81785075	1.898069738	0.00000004	0.000000004
at *Advanced*	4.43890904	1.444267101	4.43890904	1.444267118	0.00000000	-0.000000017
Two or more races						
below *Basic*	22.48269603	1.070835707	22.48269597	1.070835769	0.00000006	-0.000000062
at or above *Basic*	77.51730397	1.070835707	77.51730403	1.070835769	-0.00000006	-0.000000062
at or above *Proficient*	39.16588319	1.721336002	39.16588329	1.721336135	-0.00000010	-0.000000133
at *Advanced*	10.73353416	0.948681008	10.73353420	0.948681036	-0.00000004	-0.000000028

Table A-3. Percentage of students at each achievement level for mathematics at grade 8, by sex and race/ethnicity: 2013

Characteristics/discrete level	SAS estimate		NDE result		Comparison (SAS minus NDE)	
	Percentage	Standard error	Percentage	Standard error	Percentage	Standard error
Sex						
Male						
below *Basic*	26.36478482	0.333978664	26.36478483	0.333978680	0.00000000	-0.000000016
Basic	37.37320633	0.413797459	37.37320634	0.413797459	-0.00000001	0.000000001
Proficient	26.57246985	0.323973612	26.57246985	0.323973624	0.00000000	-0.000000012
Advanced	9.68953900	0.273165842	9.68953899	0.273165845	0.00000001	-0.000000003
Female						
below *Basic*	25.70878398	0.359530753	25.70878398	0.359530739	0.00000001	0.000000014
Basic	39.59800524	0.372808409	39.59800524	0.372808407	0.00000000	0.000000003
Proficient	26.79220011	0.329729414	26.79220012	0.329729407	-0.00000001	0.000000007
Advanced	7.90101067	0.208544164	7.90101066	0.208544165	0.00000001	-0.000000001
Race/ethnicity						
White						
below *Basic*	16.24323783	0.243588636	16.24323784	0.243588644	0.00000000	-0.000000008
Basic	38.66571859	0.369603145	38.66571860	0.369603157	-0.00000001	-0.000000012
Proficient	33.49556813	0.369434941	33.49556813	0.369434942	0.00000000	-0.000000001
Advanced	11.59547544	0.235280642	11.59547543	0.235280633	0.00000001	0.000000009
Black						
below *Basic*	48.17699100	0.650425251	48.17699101	0.650425258	-0.00000002	-0.000000007
Basic	37.54771117	0.579847667	37.54771116	0.579847675	0.00000001	-0.000000008
Proficient	12.56712555	0.438663170	12.56712555	0.438663169	0.00000000	0.000000002
Advanced	1.70817228	0.167217635	1.70817227	0.167217628	0.00000001	0.000000007
Hispanic						
below *Basic*	37.69902207	0.628531186	37.69902205	0.628531143	0.00000002	0.000000043
Basic	41.15937356	0.722150554	41.15937357	0.722150549	-0.00000001	0.000000004
Proficient	18.08172263	0.486620902	18.08172265	0.486620911	-0.00000001	-0.000000009
Advanced	3.05988173	0.229435618	3.05988173	0.229435617	0.00000000	0.000000001
Asian						
below *Basic*	11.15832816	0.829553821	11.15832815	0.829553820	0.00000001	0.000000001
Basic	26.24725730	1.151481661	26.24725731	1.151481633	0.00000000	0.000000028
Proficient	35.76467474	1.340457804	35.76467477	1.340457868	-0.00000003	-0.000000064
Advanced	26.82973980	1.364408213	26.82973977	1.364408226	0.00000003	-0.000000013
American Indian/Alaska Native						
below *Basic*	41.04733171	1.662081135	41.04733167	1.662081105	0.00000004	0.000000030
Basic	37.89670937	1.965355137	37.89670940	1.965355107	-0.00000003	0.000000030
Proficient	17.66437515	1.460264537	17.66437515	1.460264499	0.00000000	0.000000039
Advanced	3.39158377	0.603732378	3.39158378	0.603732384	-0.00000001	-0.000000006
Native Hawaiian/Pacific Islander						
below *Basic*	32.79990323	3.490548597	32.79990318	3.490548525	0.00000005	0.000000072
Basic	43.30716101	3.495090746	43.30716107	3.495090818	-0.00000006	-0.000000072
Proficient	19.67405054	2.908713669	19.67405057	2.908713653	-0.00000003	0.000000016
Advanced	4.21888521	1.238519539	4.21888518	1.238519568	0.00000003	-0.000000029
Two or more races						
below *Basic*	23.56297792	1.181737426	23.56297796	1.181737353	-0.00000004	0.000000072
Basic	38.43131025	1.730071162	38.43131027	1.730071175	-0.00000002	-0.000000013
Proficient	27.27060003	1.382825664	27.27060003	1.382825676	0.00000000	-0.000000012
Advanced	10.73511180	1.160504569	10.73511175	1.160504567	0.00000005	0.000000003

Table A-4. Percentage of students at or above each achievement level for mathematics at grade 8, by sex and race/ethnicity: 2013

Characteristics/ cumulative level	SAS estimate		NDE result		Comparison (SAS minus NDE)	
	Percentage	Standard error	Percentage	Standard error	Percentage	Standard error
Sex						
Male						
below *Basic*	26.36478482	0.333978664	26.36478483	0.333978680	0.00000000	-0.000000016
at or above *Basic*	73.63521518	0.333978664	73.63521517	0.333978680	0.00000000	-0.000000016
at or above *Proficient*	36.26200885	0.399180514	36.26200884	0.399180540	0.00000001	-0.000000026
at *Advanced*	9.68953900	0.273165842	9.68953899	0.273165845	0.00000001	-0.000000003
Female						
below *Basic*	25.70878398	0.359530753	25.70878398	0.359530739	0.00000001	0.000000014
at or above *Basic*	74.29121602	0.359530753	74.29121602	0.359530739	-0.00000001	0.000000014
at or above *Proficient*	34.69321078	0.367749972	34.69321078	0.367749960	0.00000000	0.000000012
at *Advanced*	7.90101067	0.208544164	7.90101066	0.208544165	0.00000001	-0.000000001
Race/ethnicity						
White						
below *Basic*	16.24323783	0.243588636	16.24323784	0.243588644	0.00000000	-0.000000008
at or above *Basic*	83.75676217	0.243588636	83.75676216	0.243588644	0.00000000	-0.000000008
at or above *Proficient*	45.09104357	0.434186995	45.09104356	0.434187001	0.00000001	-0.000000006
at *Advanced*	11.59547544	0.235280642	11.59547543	0.235280633	0.00000001	0.000000009
Black						
below *Basic*	48.17699100	0.650425251	48.17699101	0.650425258	-0.00000002	-0.000000007
at or above *Basic*	51.82300900	0.650425251	51.82300899	0.650425258	0.00000002	-0.000000007
at or above *Proficient*	14.27529783	0.453793179	14.27529782	0.453793172	0.00000001	0.000000007
at *Advanced*	1.70817228	0.167217635	1.70817227	0.167217628	0.00000001	0.000000007
Hispanic						
below *Basic*	37.69902207	0.628531186	37.69902205	0.628531143	0.00000002	0.000000043
at or above *Basic*	62.30097793	0.628531186	62.30097795	0.628531143	-0.00000002	0.000000043
at or above *Proficient*	21.14160437	0.549945928	21.14160438	0.549945935	-0.00000001	-0.000000006
at *Advanced*	3.05988173	0.229435618	3.05988173	0.229435617	0.00000000	0.000000001
Asian						
below *Basic*	11.15832816	0.829553821	11.15832815	0.829553820	0.00000001	0.000000001
at or above *Basic*	88.84167184	0.829553821	88.84167185	0.829553820	-0.00000001	0.000000001
at or above *Proficient*	62.59441454	1.328647987	62.59441454	1.328647970	0.00000000	0.000000017
at *Advanced*	26.82973980	1.364408213	26.82973977	1.364408226	0.00000003	-0.000000013
American Indian / Alaska Native						
below *Basic*	41.04733171	1.662081135	41.04733167	1.662081105	0.00000004	0.000000030
at or above *Basic*	58.95266829	1.662081135	58.95266833	1.662081105	-0.00000004	0.000000030
at or above *Proficient*	21.05595892	1.466027940	21.05595893	1.466027912	-0.00000001	0.000000028
at *Advanced*	3.39158377	0.603732378	3.39158378	0.603732384	-0.00000001	-0.000000006
Native Hawaiian/Pacific Islander						
below *Basic*	32.79990323	3.490548597	32.79990318	3.490548525	0.00000005	0.000000072
at or above *Basic*	67.20009677	3.490548597	67.20009682	3.490548525	-0.00000005	0.000000072
at or above *Proficient*	23.89293575	2.932413071	23.89293575	2.932413048	0.00000001	0.000000023
at *Advanced*	4.21888521	1.238519539	4.21888518	1.238519568	0.00000003	-0.000000029
Two or more races						
below *Basic*	23.56297792	1.181737426	23.56297796	1.181737353	-0.00000004	0.000000072
at or above *Basic*	76.43702208	1.181737426	76.43702204	1.181737353	0.00000004	0.000000072
at or above *Proficient*	38.00571183	1.406656440	38.00571178	1.406656417	0.00000005	0.000000023
at *Advanced*	10.73511180	1.160504569	10.73511175	1.160504567	0.00000005	0.000000003