ClustVarLV:
A package for the clustering of variables around latent variables

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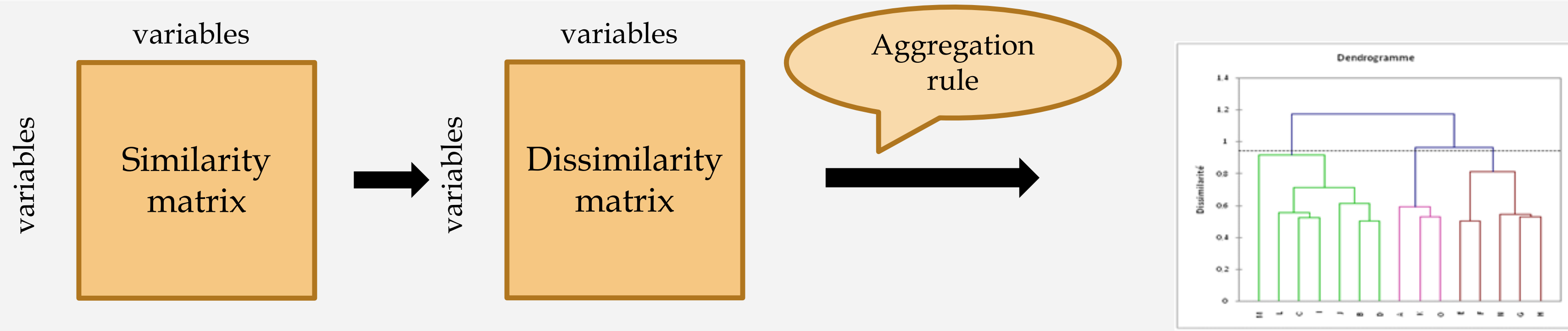
Outline

- Context : the clustering of variables
- CLV method: data structure / types of groups
- Algorithms et main functions in « ClustVarLV »
- *Illustration 1:* psychological scales
- *Illustration* 2: preference mapping
- ClustVarLV et ClustOfVar
- Conclusion and perspectives

The clustering of variables



The clustering of variables

variables

variables

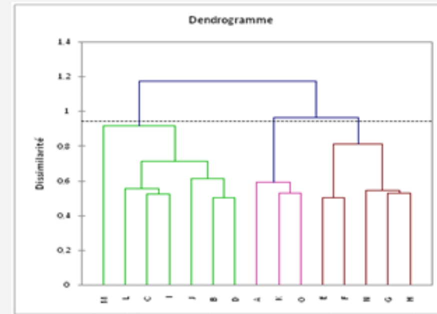
Similarity matrix

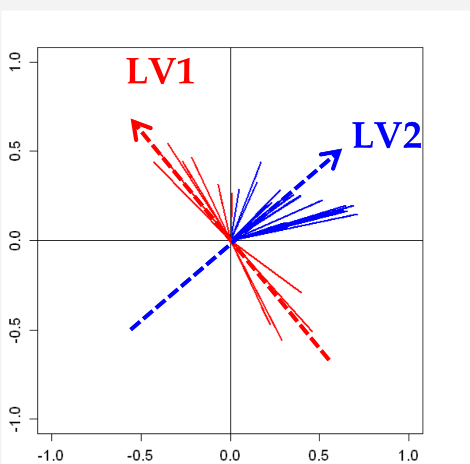
variables

variables

Dissimilarity matrix

Aggregation rule





Factor analysis / exploratory approaches :
Identifying groups of variables
defined around Latent Variables (LV)

CLV (Clustering of variables around Latent Variables)
available on R

VARCLUS : procédure SAS/STAT

Highlighting the inter-correlations structure between the variables

- **Principal Components Analysis (PCA)**

  ⇨ analysis of the linear relationships between the variables and dimensionality reduction using the first Principal Components (PC).

- **Principal Components with rotation** (RC)

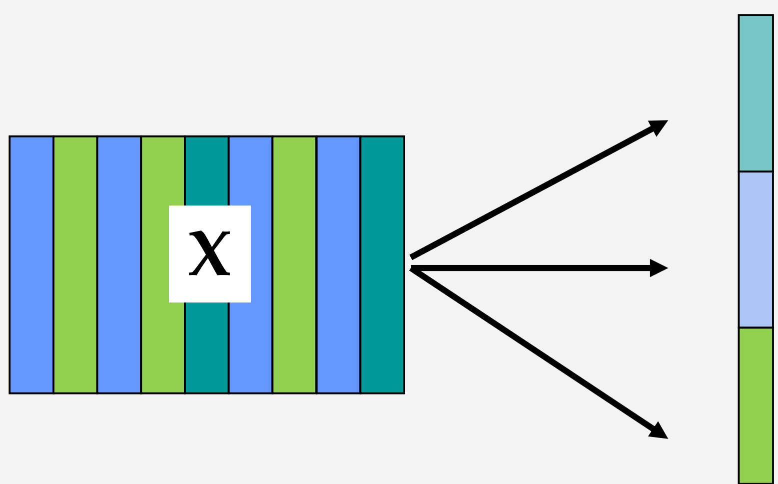
  ⇨ Linear combinations of the initial variables more easy to interpret than the PC.

- **CLV approach**

  ⇨ dimensionality reduction (K latent variables (LV) associated with groups of variables).

  ⇨ easier interpretation (each LV is a linear combinayion of the variables belonging to the associated group).

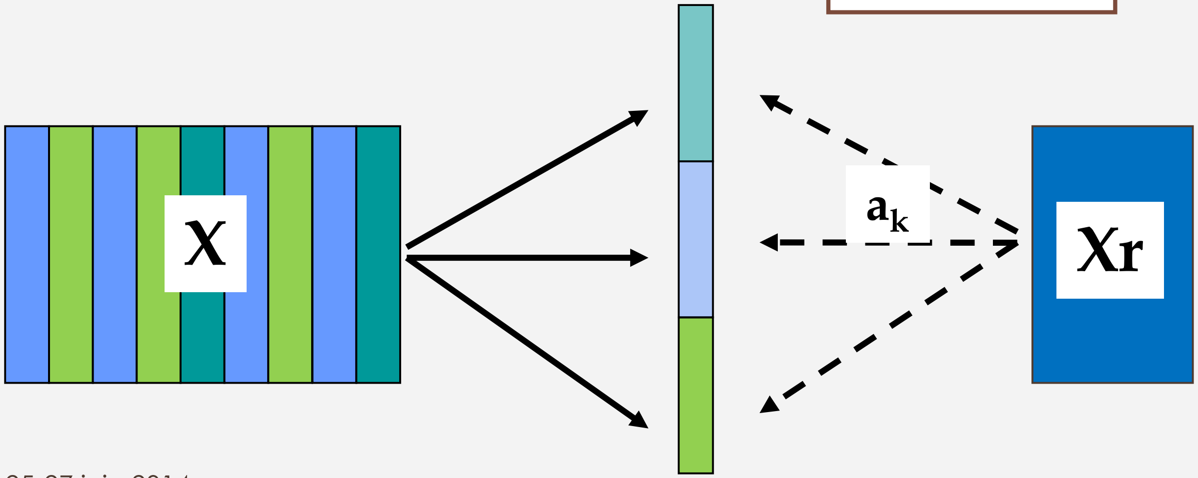
CLV method for various data structures



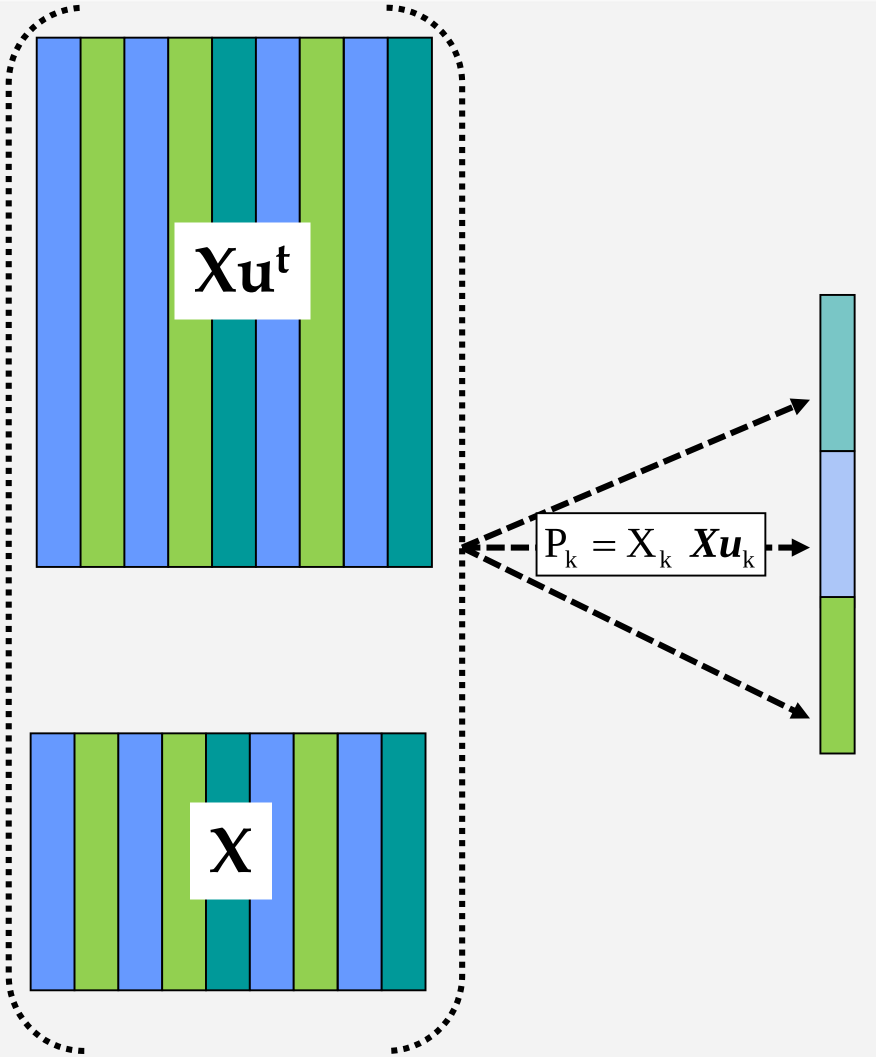
$\boldsymbol{c}_k$: latent variable of group G_k

CLV method for various data structures

$$\boxed{t_k = \boldsymbol{Xr}\, a_k}$$

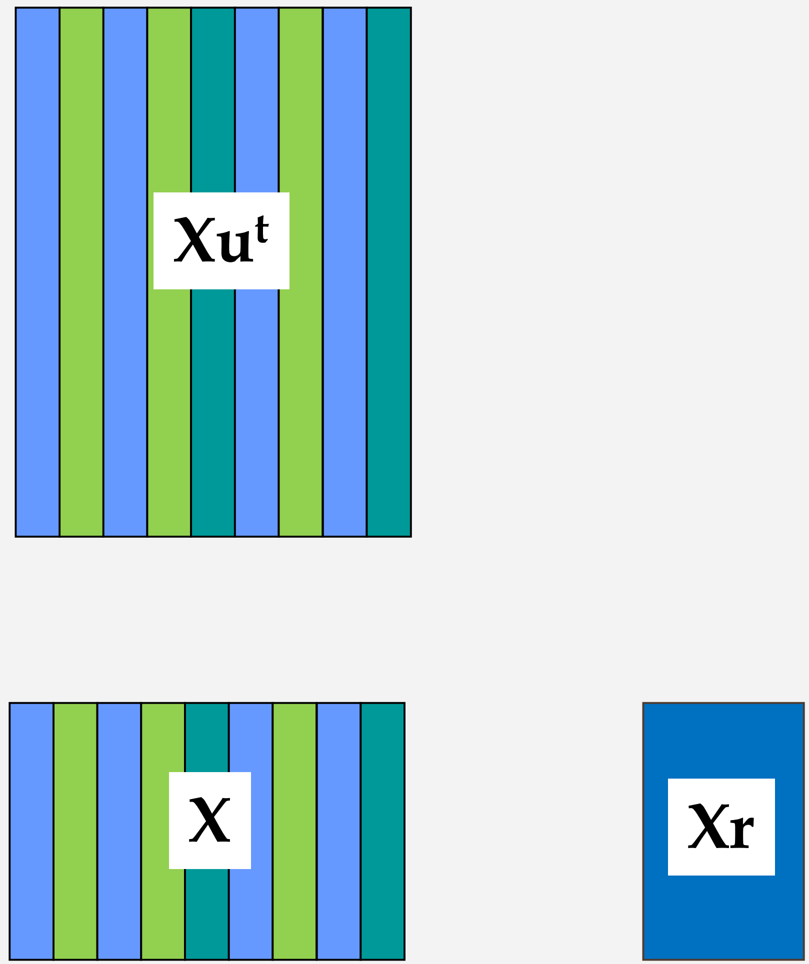


CLV method for various data structures



$$\tau_k = P_k \, u_k$$

CLV method for various data structures (L-shaped data)



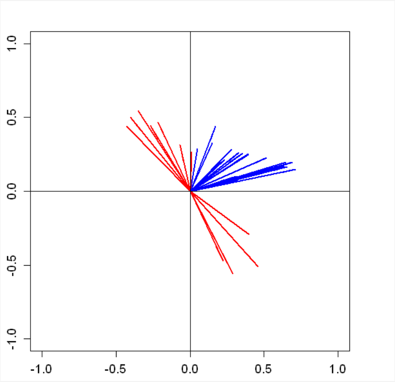
$$\tau_k = P_k \, u_k$$

$$t_k = Xr \, a_k$$

CLV method: two types of groups

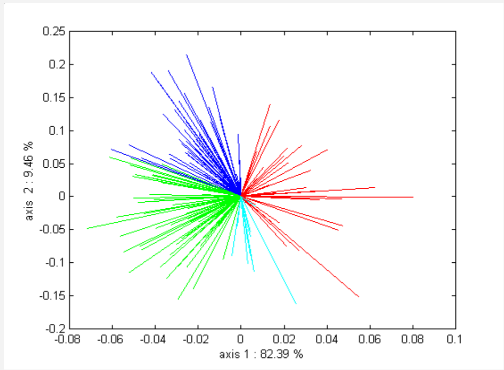
Directional groups

High positive or negative correlations ⇨ agreement



Local groups

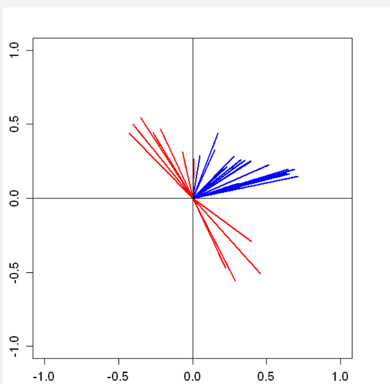
High positive correlations ⇨ agreement
High negative correlations ⇨ disagrrement



CLV method: two types of groups

<u>Directional groups</u>

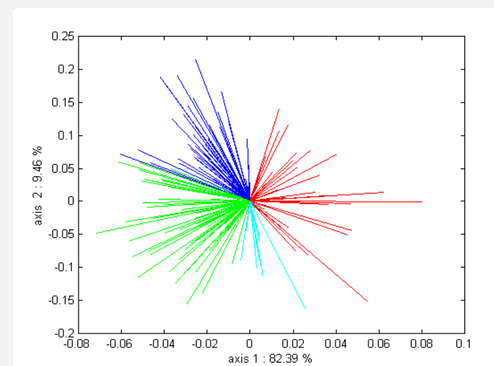
High positive or negative correlations ⇨ agreement



method = 1
or method="directional"

<u>Local groups</u>

High positive correlations ⇨ agreement
High negative correlations ⇨ disagrrement



method = 2
or method= "local"

```
> CLV(X, method= …)
```

CLV method: two types of groups

<u>Directionnal groups</u>
`method = 1`

<u>Local groups</u>
`method = 2`

Maximization of

nb of groups

$$T = n \sum_{k=1}^{K} \sum_{j=1}^{p} \delta_{kj} \operatorname{cov}^2(\boldsymbol{x}_j, \boldsymbol{c}_k)$$

Latent Variable

Group's membership indicator

$$S = \sqrt{n} \sum_{k=1}^{K} \sum_{j=1}^{p} \delta_{kj} \operatorname{cov}(\boldsymbol{x}_j, \boldsymbol{c}_k)$$

avec $\boldsymbol{c}_k{}' \boldsymbol{c}_k = 1$

Algorithm (1)

Partitioning algorithm

⓿ **Initialization :** user's choice (…) or
at random (`nstart`)

❶ **Estimation of the LV**

`method=1`, matrix **X** : $\mathbf{c}_k$ (k=1,…K) is the first standardized principal component of X_k

`method=2`, matrix **X** : $\mathbf{c}_k$ (k=1,…K) is proportional to the averaged variable $\bar{x}_k$

❷ **Etape d'affectation des variables**

cas `method=1`, matrix **X** : $\delta_{kj} = 1 \text{ if } \max_{l=1,\dots,K}\left\{\operatorname{cov}^2(x_j, c_l)\right\} = \operatorname{cov}^2(x_j, c_k)$

cas `method=2`, matrix **X** : $\delta_{kj} = 1 \text{ if } \max_{l=1,\dots,K}\left\{\operatorname{cov}(x_j, c_l)\right\} = \operatorname{cov}(x_j, c_k)$

until convergence

Function (1)

Partitioning algorithm

```
>  CLV_kmeans(X, method=1 , sX=TRUE, clust= K,nstart=100)
```

data matrix (n x p)

type of groups :
-method=1 or « directional »
-method=2 or « local »

sX=TRUE / FALSE
standardization of the variables, or not

nb of repetitions of the algorithm. (`nstart=1` if initialization by a partition given by the user)

- if `clust` is a scalar, say K : nb de groups in the partition
- if `clust` is a vector of p integers $\in\{1,\ldots,K\}$: initial partition

<u>Outputs</u> :

⇨ partition into K groups (if `nstart>1`, optimal partition among the `nstart` solutions is given)

⇨ Latent variables for each group of variables (not standardized)

+ value of the criterion at convergence, nb of iterations before convergence, summary for the `nstart` solutions

Algorithm (2)

Ascendant hierarchical algorithm

- At the beginning (step 1) : each variable is a group by itself (K=p)
- At the end (step p) : all the variables are in the same group (K=1)

illustration for
method=
"directional"

- At step j — value of the criterion T_j — partition : {A, B, ….}

⬇ ⬇

- At step j+1 — value of the criterion $T_{j+1} < T_j$ — partition : {A $\cup$ B, …}

aggregation criterion: $\Delta T_j = (T_j - T_{j+1}) \quad > 0$

Rule : at each step, j, the two groups, A et B, for which ΔT_j is minimized are merged together (loss of within-group coherence as small as possible)

Advantages :

- Initialization of the partitioning algorithm
- Help for choosing the number of groups, K, on the basis of the variations of ΔT_j

Function (2)

Ascendant hierarchical algorithm with consolidation by the *k-means* algorithm

```
> resclv<-CLV(X,method= "directional", sX=TRUE, nmax= 20)
```

Maximal size of the partition for which a *k-means* consolidation is performed (20, by default).

Outputs :

⇨ partitions into 1, 2, 3, …, `nmax` groups before consolidation (by cutting the dendrogram) **and** after consolidation (*k-means*).

⇨ Latent variables for each group associated to each partition.

⇨ detailed results of the hierarchy.

Functions (3)

The same functions are used
with or without external variables

Example (available with the package) :

```
> data(apples_sh)
# local groups with external variables Xr
> resclvYX <- CLV_kmeans (X = apples_sh$pref,
                   Xr = apples_sh$senso, method = "local ",
                   sX = FALSE, sXr = TRUE, graph = TRUE)
```

Illustration 1 : exploratory analysis for psychological scales

- **AUPALESENS project** (France, 2010-2014)
  "Making eating more enjoyable for seniors to promote healthy aging and prevent malnutrition"

- n=559 subjects (>65 ans)

- Pluridisciplinary questionnaire … only considered here
  scales used for assessing psychological behaviour (5-points Likert scale)

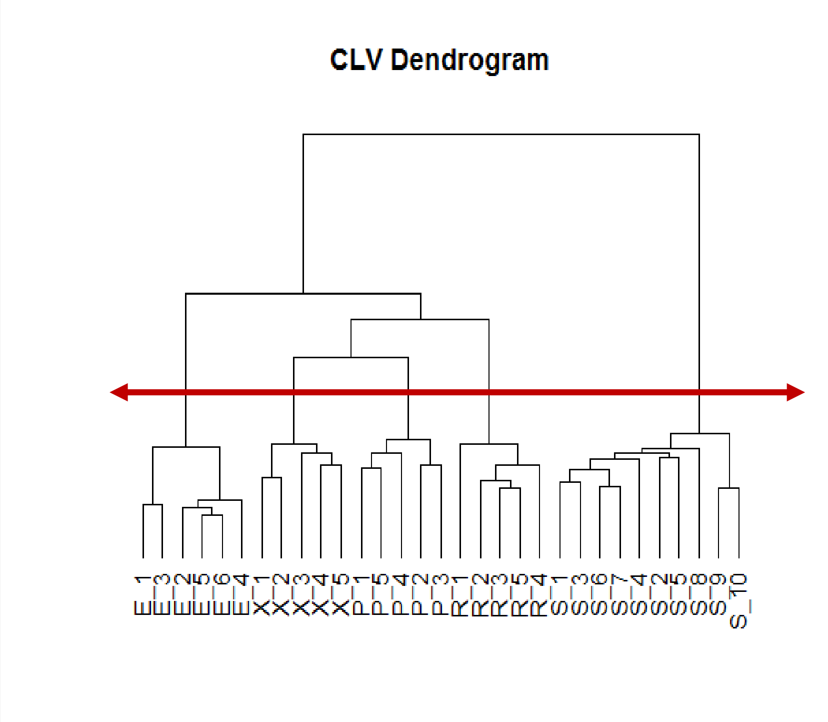
**Bailly, Maitre, Amand, Hervé, Alaphilippe (2012). Appetite, 59(853-858)*

Eating behaviour
(based on DEBQ)

- « Emotional eating » (E) : 6 items
- « eXternal eating » (X) : 5
- « Restrained eating» (R) : 5 items
- « Food enjoyment» (P) : 5 items
- « Self estime» (S) : 10 items

Illustration 1

```
> load("AUPA_psycho.rda")
> X<-AUPA_psycho
> dim(X)
[1] 559  31
> res.clv<-CLV(X,method= "directional",sX=TRUE)
> plot(res.clv, type= " dendrogram " )
> plot(res.clv, type= " delta " )
```



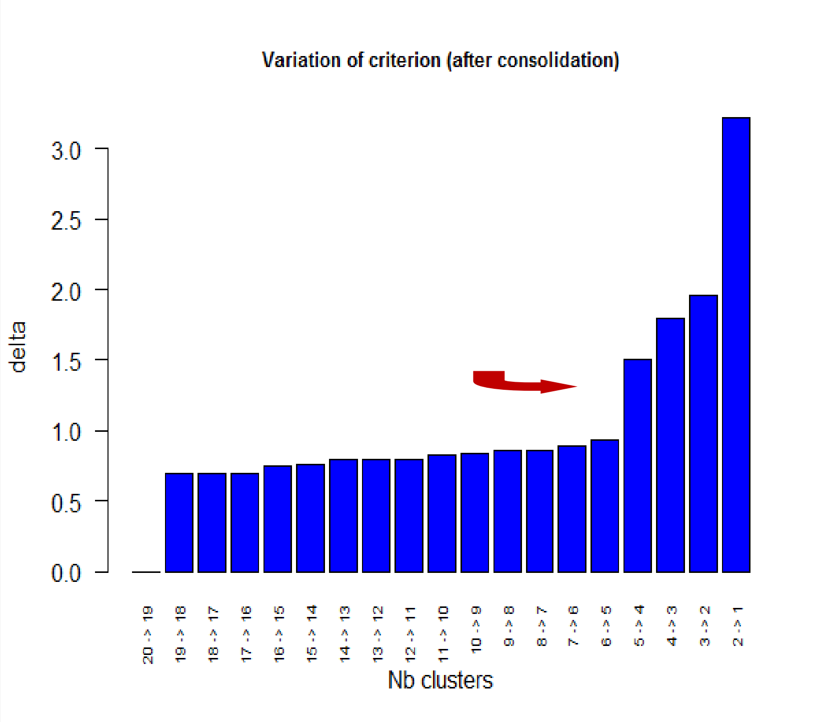


Illustration 1

```
> summary_clv(res.clv,K=5)

$number  1  2  3  4  5
         6  5  5  5 10

$prop_within
 Group.1 Group.2 Group.3 Group.4 Group.5
 0.6036  0.4077  0.4653   0.388  0.3614

$prop_tot    0.4368

$cormatrix
      Comp1 Comp2 Comp3 Comp4 Comp5
Comp1  1.00  0.36  0.27  0.08  0.20
Comp2  0.36  1.00  0.23  0.23  0.11
Comp3  0.27  0.23  1.00  0.14  0.05
Comp4  0.08  0.23  0.14  1.00 -0.16
Comp5  0.20  0.11  0.05 -0.16  1.00
```

Within-group variability explained by the Latent Variable of the group

Total variability explained by the 5 Latent Variables

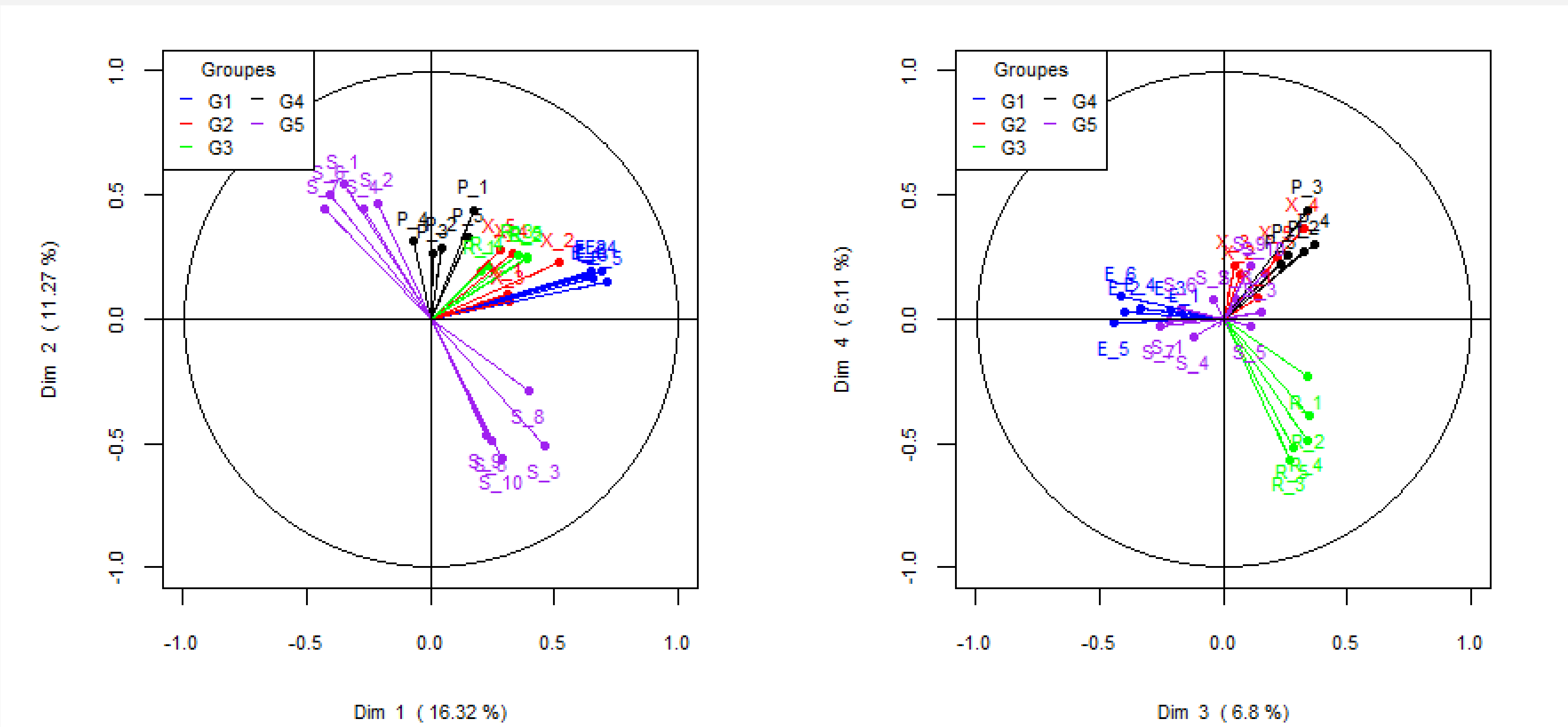
Correlation matrix betwwen the Latent Variables

Illustration 1

```
> Summary_clv(res.clv,K=5)

$groups[[1]]
    cor in group  |cor|next group
E_5         0.85             0.25
E_4         0.80             0.34
E_6         0.80             0.25
E_2         0.79             0.25
E_3         0.73             0.31
E_1         0.68             0.29

$groups[[2]]
    cor in group  |cor|next group
X_2         0.76             0.38
X_4         0.67             0.30
X_5         0.65             0.19
X_1         0.58             0.17
X_3         0.51             0.22

$groups[[3]]
    cor in group  |cor|next group
R_5         0.77             0.25
R_3         0.76             0.21
R_2         0.71             0.23
R_4         0.66             0.11
R_1         0.47             0.14

$groups[[4]]
    cor in group   cor|next group
P_1         0.72             0.18
P_3         0.63             0.14
P_2         0.61             0.10
P_4         0.58            -0.14
P_5         0.57             0.19

$groups[[5]]
     cor in group  |cor|next group
S_3          0.70             0.21
S_1         -0.68            -0.10
S_6         -0.66             0.17
S_7         -0.65            -0.17
S_10         0.65             0.07
S_5          0.55            -0.12
S_4         -0.53             0.10
S_9          0.53            -0.10
S_2         -0.51             0.14
S_8          0.49             0.23
```

Illustration 1: exploratory analysis of the scales

```
> plot_var(res.clv,K=5,axeh=1,axev=2,label=TRUE)
> plot_var(res.clv,K=5,axeh=3,axev=4,label=TRUE)
```



The groups of variables perfectly coincide with the underlying psychological scales

Illustration 2: preference mapping of apple using L-CLV

Xu^t

Consumers questionnaire

- Frequency of consumption,
- Apple cultivars known
- Important sensory attributes,
- Modalities of consumption (peeled/during meal/ …)
- Purchase criteria
- ….
- Age, gender, professional activity….

Vigneau, Charles, Chen (2013). *Food Quality and Preference*, 22(4), 83-92

produits

X

hedonic test

224 regular apple consumers
31 apples varieties

Liking scores on a 9-points scale

Xr

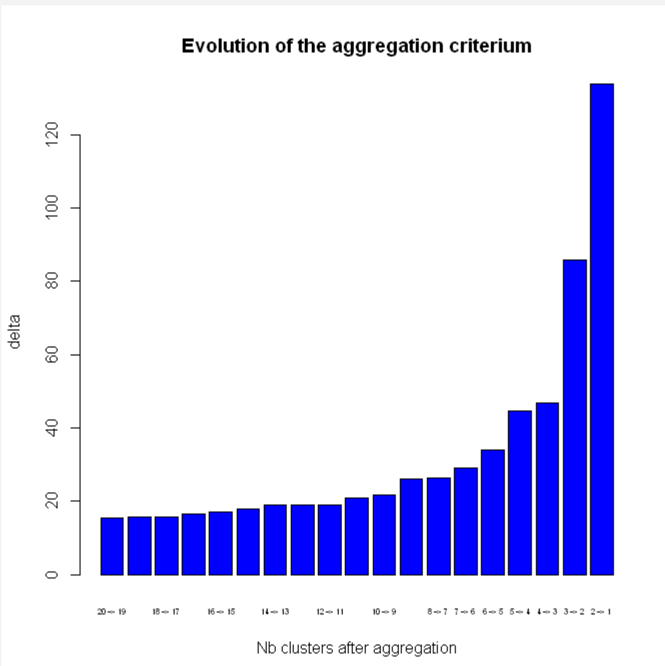
Sensory descriptive analysis

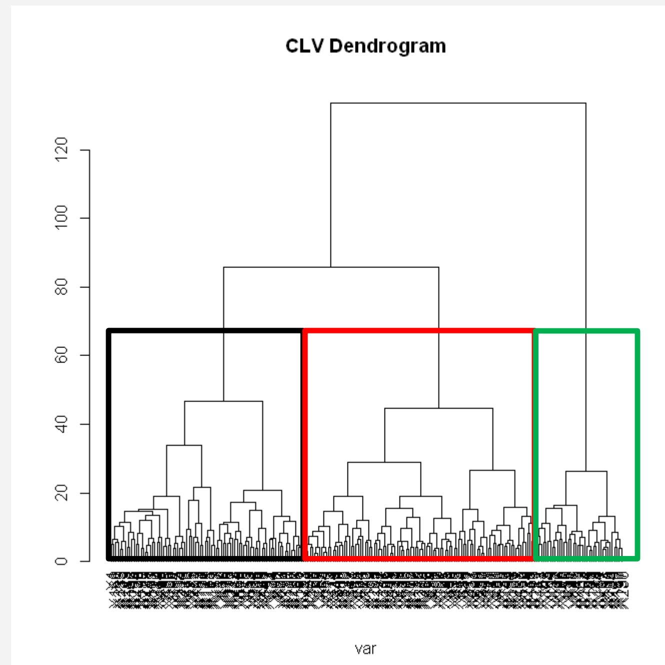
15 assessors, 15 attributes

Crunchy	A_Pineapple/Banana
Juicy	A_Sweet/Rose
Fondant	A_Woody/Earthy
	A_Rustic
Sweet	A_Lemon
Acid	A_White flowers
	A_Ripe fruit
Odour intensity	A_Green
Aroma intensity	

Illustration 2

```
> resL<-LCLV(X=pref, Xr=senso, Xu=questions, sX=TRUE,
                                          _ sXr=TRUE, sXu=FALSE)
> plot(res.clv, type= " dendrogram " )
> plot(res.clv, type= " delta " )
```



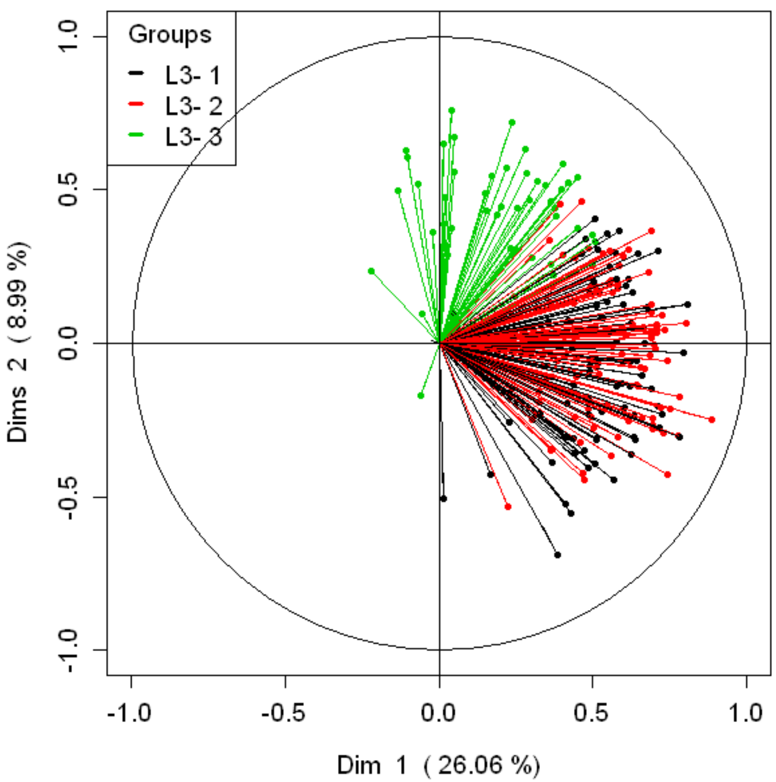



```
> get_partition(resL,K=3)
```

Segment L3-1	82 consumers	(37%)
Segment L3-2	96 consumers	(43%)
Segment L3-3	46 consumers	(20%)

Illustration 2

```
> plot_var(resL,K=3,axeh=1,axev=2,label=FALSE)
> plot_var(resL,K=3,axeh=2,axev=3,label=FALSE)
```



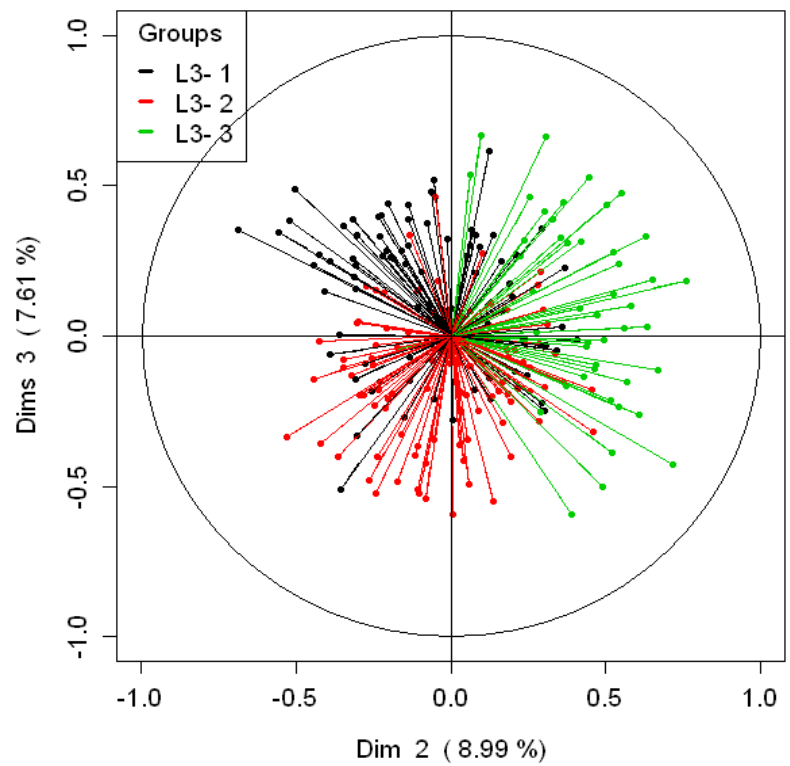


Illustration 2

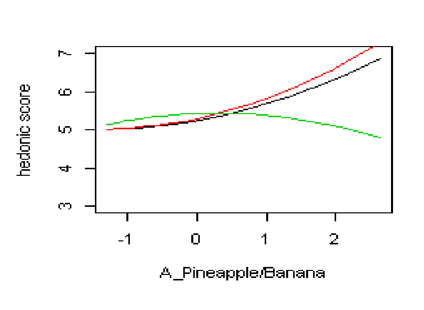
Interpretation of the segmentation of consumers panel

```
> get_load(resL,K=3)
```

❖ According to the sensory *drivers*

loadings (a_k) associated with the variables in Xr

- Consumers in the segments **1** and **2** appreciated the juicy and sweet varieties of apple, with « ananas/banana » aroma.
- Consumers in the segment **3** appreciated more fondant apples, with « rustic » and « ripe fruit » aroma. They dislike acifity and « green » aroma in apples.



❖ According to the Usage & Attitude items and the socio-demographic characteristics of the consumers

loadings (u_k) associated with the variables in Xu

- Segment **1** : mainly, the youngest in the panel
- Segment **2** et **3** : in majority, > 40 years old

….

are attentive to appearance, color, packaging cultivar, origin.

ClustVarLV et ClustOfVar

Both based on the CLV approach
Similar algorithms (hierarchical and k-means)

Type of groups	
directional or local	directional, only
Standardization	
choice	quantitative variables are standardized
Categorical variables	
data coding with dummy variables, clustering of the modalities	*integrated* clustering criterion updated
Variables externes	
integrated, associated with the obs. and/or the variables	-

Conclusion et perspectives

ClustVarLV : clustering of variables

… but not only that:

- data dimensionality reduction (latent variables)
- CLV components easier to understand

Many different areas of application: sensory analysis and consumer's preference analysis, chemometry (IR, RMN spectroscopy), omic- data, psychometry, satisfaction questionnaires …

Developpments in progress

- « discarding » the atypical variables / the variables which are not well associated with the group's structure in the dataset.
- Supervised clustering of variables (by taking into account of a response variable)