Validation of a virtual population

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Contents

List of Figures

List of Tables

1 Introduction

A virtual population is a population generated by simulation. The population is valid if it is sufficiently close to the real population used as a reference. The validity of the simulation depends on the model, the data and the assumptions. The validity of the simulation is assessed by comparing demographic indicators are of the virtual population with similar indicators of the reference population.

The paper reports on the validity of the virtual population produced by the VirtualPop package Version 2.0 (https://cran.r-project.org/web/packages/VirtualPop/index.html) from mortality and fertility rates of the United States. Mortality rates are age- and sex-specific, while fertility rates age- and parity-specific. Two types of virtual populations are considered in this paper. The first is generated from period mortality and fertility rates, i.e. rates experienced by a population in a given calendar year. The second is generated by cohort rates, i.e. rates experienced by people born in a given calendar year. The period and cohort rates used in the paper are published by the Human Mortality Database (HMD) (https://www.mortality.org) and the Human Fertility Database (HFD) (https://www.humanfertility.org). The virtual population consists of several generations and is stratified by age and sex. Women are distinguished by parity and newborns by birth order. Children are born to a couple, irrespective of their legal status or living arrangement.

Validity tests require that four conditions are satisfied:

a. The virtual population is sufficiently large to limit the effect of Monte Carlo variation caused by sampling probability distributions. In this paper, the virtual population consists of an initial cohort of 10,000 individuals and their offspring.
b. The virtual and reference populations should be followed during the same period of time or age interval. In other words, the same observation window should apply to the two populations. Differences in observation window are important reasons for differences in summary indicators. Differences can be attribtuted to differences in life course segments covered and in contextual factors during the observation period.
c. The demographic indicators used for comparison should be computed or estimated using the same method. If that is not feasible, the methods should be as close as possible.
d. The metric used to assess the validity of the virtual population should be informative and easy to interpret. In this paper, virtual and real populations are compared using distributions rather than single indices summarizing the differences.

In the paper, three tests are used to assess the validity of a virtual (simulated) population. The first (Section 2) is a comparison of age at death distributions in the virtual population and in the period life table included in the Human Mortality Database. Life expectancies are also compared with indicators published by the U.S. National Center for Health Statistics. The second test (Section 3) is a comparison of the distribution of women by number of children ever born (CEB) in the virtual population and the distribution reported in (a) the HFD fertility table and (b) the Current Population Survey (CPS) of June 2018. The comparison covers both the total CEB and the CEB by age at survey date. The third test (Section 4) compares the ages at which children lose a parent. The age distribution in the virtual population is compared with the distribution observed in the Survey of Income and Program Participation (SIPP) of 2021 (https://www.census.gov/programs-surveys/sipp.html). For comparison, two virtual populations are produced based on period data of 2010.

In the virtual population, death and childbirth are competing events. Death interrupts the fertility career. The reference populations are not at risk of death. In the standard fertility table mortality is absent and all individuals complete the reproductive period. In a retrospective survey, such as the CPS, respondents have survived until survey date. The observation window is from birth to the age at survey. A comparison of the virtual population and the reference population requires similar observation windows. That is accomplished by imposing the censoring schemes used in the reference population onto the virtual population. To compare a virtual population with the HFD fertility table, mortality is disregarded. To enable a comparison with observations in retrospective surveys, observations on members of the virtual population are censored at ages drawn from the age distribution of respondents in the retrospective surveys. The illustrate the effect of mortality on life course measures, Appendix A discusses the impact of mortality on childlessness.

Four external datasets are used. The first is the period HMD life table (fltper_1x1) of the USA 2021 and in particular the age distribution at death (dltd_prop). The second is the HFD period fertility table (pft) of the USA 2018. The third set of data consists of tabulated data from the Fertility Supplement of the CPS of June 2018 (United States Census Bureau 2022). The fourth dataset is the SIPP 2021 (the Stata file pu2021.dta (United States Census Bureau 2023)).

2 Generate a virtual population

The following R scripts produces virtual populations under the scenarios mentioned in the introduction. In scenario 1, the virtual population is based on period rates. In scenario 2, cohort rates are used. In each scenario mortality may be present or absent (mort is TRUE or FALSE). The user should provide the username (e-mail address) and the passwords received at registration with the HMD and HFD.

```
countrycode <- "USA"
ncohort <- 2000
refyear <- 2021
cohort <- NULL
ngen <- 2
```

```
dLH <- tryCatch(VirtualPop::BuildViP(user,pw_HMD,pw_HFD,
                              countrycode=countrycode,
                              cohort=NULL,  # period data
                              refyear=refyear,
```

```
                             ncohort=ncohort,
                             ngen=2,
                             mort=TRUE),
      error=function(cond) {warning(cond);
        message(paste0("Error reading HFD data.",
        "Download data and read data locally (see Tutorial Section 4.2)."))})
#> Extract data from HMD and HFD for USA
```

The virtual population is stored in the object dLH. The virtual population of the United States based on period rates of 2021 is included in the VirtualPop package. The following code loads the dataset into the R workspace.

```
data(dLH,package="VirtualPop")
```

3 Ages at death

Consider the 2021 period death rates of the population of the United States, by single years of age and sex. The data are part of the period life tables, found at https://www.mortality.org/File/GetDocument/hmd.v6/USA/STATS/fltper_1x1.txt for females and at https://www.mortality.org/File/GetDocument/hmd.v6/USA/STATS/mltper_1x1.txt) for males[1]. The data for 2021 are included in VirtualPop as data object *rates*. Consider a virtual population of 10,000 individuals. To generate lifespans that are consistent with the empirical age-specific death rates, the highest age possible must be defined. The maximum age is set to be 120. The death rate for persons aged 110-120 applies to all survivors at ages above 110. The function Lifespan() of VirtualPop is used to simulate lifespans.

Figure 1 shows the age distribution at death, by sex. The histogram is overlaid with the distribution of ages at death in the period HMD life table of USA in 2021 (dashed line in black). The distribution of simulated ages at death is close to the distribution of life-table ages.

```
#> Warning: A numeric ‘legend.position‘ argument in ‘theme()‘ was deprecated in ggplot2
#> 3.5.0.
#> i Please use the ‘legend.position.inside‘ argument of ‘theme()‘ instead.
#> This warning is displayed once every 8 hours.
#> Call ‘lifecycle::last_lifecycle_warnings()‘ to see where this warning was
#> generated.
```

The mean and the variability (standard deviation) of ages at death are respectively 73.67 years for males and 17.96 and 79.41 and 16.12 years for females. The HMD shows a life

[1] It is necessary to first log in to access the data in the HMD (for details see VirtualPop Tutorial https://cran.r-project.org/web/packages/VirtualPop/vignettes/Tutorial.html).

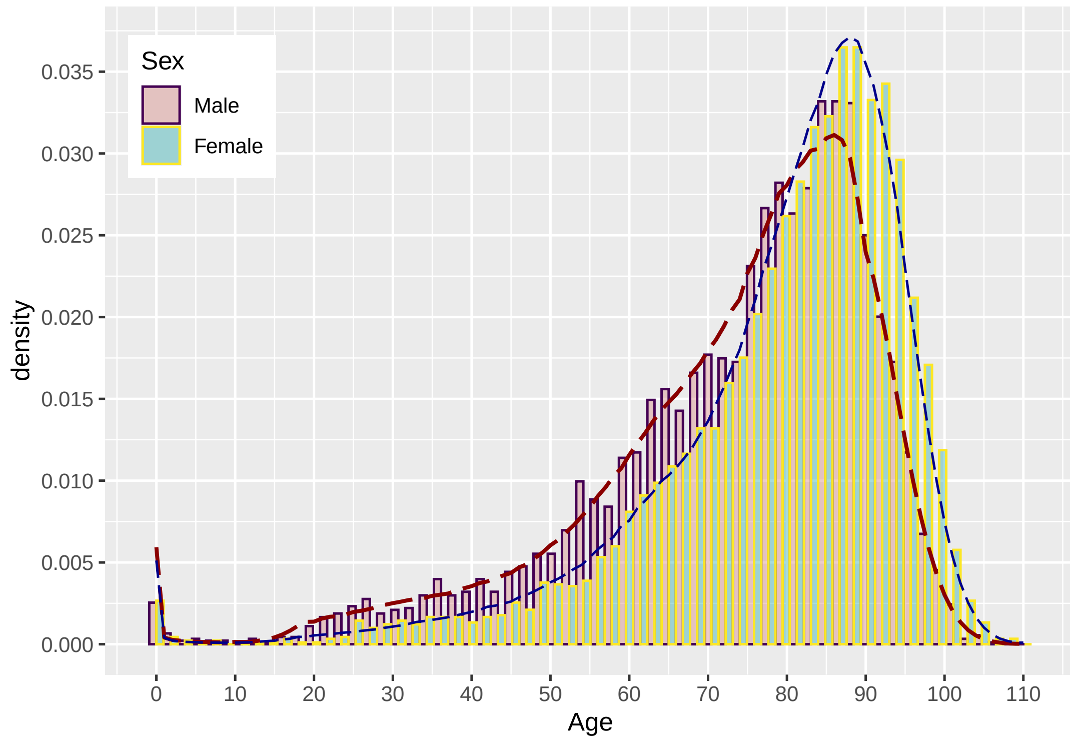

Figure 1: Ages at death in the virtual population, USA, 2021

expectancy of 73.62 for males and 79.37 for females. The U.S. National Center for Health Statistics published life expectancies of 73.2 and 79.1, respectively (Health Statistics 2022). Differences are due to method and chance. In the conventional life table, used in the HMD, the survival function is assumed to be a piecewise linear function (Wilmoth et al. 2021, 36). In the simulation it is a piecewise exponential function. A brief discussion of the difference is included in Appendix A.

4 Number of children ever born

4.1 Women with children by number of children ever born

In this section, the number of children in the virtual population is compared with the figures reported in the period fertility table of 2018 and the U.S. Current Population Survey (CPS) of June 2018. To enable a comparison, a virtual population of the USA is generated from period fertility rates of 2018 (by age and parity) without mortality. The following code produces a virtual population with and without death as a competing risk.

```
ncohort <- 10000
dLH <- VirtualPop::BuildViP(user,pw_HMD,pw_HFD,
                             country="USA",
                             refyear=2018,
```

Table 1: Distribution of women with children by number of children ever born (CEB)

CEB	ViP2018m	ViP2018nm	ft2018	CPS2018
1	0.32	0.30	0.31	0.30
2	0.39	0.40	0.39	0.39
3	0.18	0.19	0.19	0.19
4	0.07	0.06	0.04	0.08
5	0.05	0.05	0.07	0.03

Source: Period fertility table 2018 (HFD) and
CPS2018
https://www.census.gov/data/tables/2018/demo/fertility/women-fertility.html

```
                                 ncohort=ncohort,
                                 ngen=2)
dLHnm <- VirtualPop::BuildViP(user,pw_HMD,pw_HFD,
                                 country="USA",
                                 refyear=2018,
                                 ncohort=ncohort,
                                 ngen=2,mort=FALSE)
```

In the absence of mortality, a woman in the virtual population (generation 1) has 1.763 children, on average, lower than the total fertility rate (TFR) of 1.727 reported in the period fertility table (2018; TFR in 2021: 1.662). The TFR of 2018 reported by the National Center for Health Statistics was 1.730 (2021: 1.664) (Osterman et al. 2023, 13). The proportion of women remaining childless is 19.19 percent, a little higher than the 19.76 percent in the period fertility table (2018; 2021: 22.90 percent). In the presence of mortality, women in the virtual population have 1.744 children, on average, and 19.59 percent remain childless.

Table 1 shows the distribution of women with children by the number of children they have. The figures are based on (a) the virtual population in the presence of mortality and the absence of censoring (ViP2018m), the virtual population in the absence of both mortality and censoring (ViP2018nm), (c) the period fertility table 2018 of the HFD (ft2018) and (d) the June 2018 CPS survey (CPS2018). The distribution of the number of women by CEB in the virtual population is very close to that in the fertility table and the CPS. The difference can be attributed to the effect of mortality and the method used to compute probabilities from rates (see Appendix A).

The distribution of women with children by number of children ever born is similar to that recorded in the Current Population Survey (CPS) 2018. The result is unexpected because the CPS records the number of children ever born between birth and the age at survey date (June 2018), while the virtual population covers the births between birth and death. Censoring is introduced in the next subsection.

4.2 Women by age and number of children ever born

In this subsection the distribution of number of women in the virtual population by age and number of children ever born is compared with the distribution observed in the CPS of June 2018. To obtain comparable figures, the virtual population omits the competing risk of death, and the CPS censoring scheme is imposed onto the virtual population. The CPS 2018 includes 76,413 female respondents, 13.5 percent are between ages 15 and 20, 13.9 percent between ages 20 and 25, etc. The age of interview is the age at censoring. The same age distribution at censoring is imposed onto the virtual population by assigning an age at censoring to individuals. The 5-year age group at censoring is drawn from the age distribution of CPS respondents (multinomial distribution). After assigning an age group, an exact ages at censoring is assigned by assuming a uniform age distribution within a 5-year age interval. It is implemented by sampling a uniform distribution with minimum value 0 and maximum value 5. The sampled value is added to the minimum age of the selected age group yielding the exact age at censoring. The calendar date of censoring is obtained by adding the age at censoring to the date of birth.

Table 2 shows the age distribution at censoring in the virtual population and the age distribution of respondents in the CPS survey.

Table 2: Age distribution at censoring in CPS and virtual population

	VirtualPop	CPS
<20	13.39	13.47
20-24	13.89	13.88
25-29	15.17	15.02
30-34	14.06	14.25
35-39	14.10	14.04
40-44	13.22	12.95
>=45	16.17	16.39

Table 3 shows the CPS data on distribution of female respondents of a given age at survey date by number of children ever born (CEB). The ages are given for 5-year age groups from 15 to 50. Of those aged 15-19, 96.9 percent have no children at survey date, 2.1 percent have 1 child and 0.8 percent 2 children. Of those 45-50 at survey, 15.4 percent are childless. More than one third (35.5 percent) have two children. Table 4 shows the distribution of women in the virtual population by number of children ever born, by age group at censoring.

Table 3: Distribution of women by number of children ever born, by age (CPS June 2018)

AgeGroup	Number_of_children_ever_born_ViP							
	nfemales	0	1	2	3	4	5-6	7-8
<20	10294	96.9	2.1	0.8	0.1	0.0	0.1	0.0
20-24	10607	78.6	14.0	6.0	1.0	0.3	0.2	0.0
25-29	11476	54.2	20.4	16.2	6.5	2.1	0.5	0.1
30-34	10889	33.6	22.3	24.6	12.8	4.4	1.9	0.3
35-39	10727	20.0	19.2	32.6	17.4	7.3	3.2	0.4
40-44	9896	15.0	18.7	34.6	18.6	8.7	3.8	0.7
>=45	12524	15.4	19.8	35.4	17.3	7.4	3.6	1.2
Total	76413	44.2	16.8	21.7	10.7	4.3	1.9	0.4

Source: https://www.census.gov/data/tables/2018/demo/fertility/women-fertility.html#par_list_57

Table 4: Distribution of women by number of children ever born, by age (virtual population)

AgeGroup	Number_of_children_ever_born_CPS							
	nfemales	0	1	2	3	4	5	6
<20	635	97.80	2.05	0.16	0.00	0.00	0.00	0.00
20-24	674	81.16	14.54	4.15	0.15	0.00	0.00	0.00
25-29	748	63.77	24.20	9.49	2.41	0.13	0.00	0.00
30-34	678	40.27	32.74	19.62	5.60	1.62	0.15	0.00
35-39	670	28.66	27.76	27.16	10.45	3.88	1.64	0.45
40-44	643	22.71	27.99	29.24	13.37	4.04	1.24	1.40
>=45	783	24.52	25.80	27.84	13.15	4.60	2.94	1.15
Total	4831	50.67	22.40	16.99	6.54	2.07	0.89	0.43

The pattern exhibited by the virtual population is similar to that observed in the CPS. The simulated and observed distributions differ for two reasons. First and foremost, the numbers recorded in the CPS are collected retrospectively and are the result of demographic rates that vary in time and between cohorts. In the CPS, young respondents have different age- and parity-specific rates than old respondents when they were young because junior and senior respondents experience the first stage of the reproductive career in different historical contexts. In the virtual population, the age- and parity-specific fertility rates are based on data collected during a single reference year (2018). The second reason is the effect of sampling. A comparison of Tables 3 and 4 reveals some major characteristics of fertility change in recent decades. First, the proportion childless increased. Second, the age profile of fertility became narrower because more woman have their first child between ages 25 and 34. Third, the two-child norm became less manifest.

Women in the virtual population and respondents in the CPS have a comparable number of children. The relative closeness of the figures demonstrates the power of simulation and the computational approach. It also justifies the use of virtual populations to gain insight in demographic processes.

5 At what age do children lose a parent?

In this section, the age distribution of children at the death of a parent in the virtual population is compared with the distribution recorded in the Survey of Income and Program Participation (SIPP) of 2021. In the SIPP, respondents were asked a series of questions regarding parental mortality, including whether their biological parents were still alive at the time of the survey, and, if not, the respondent's age at which they died (Scherer, Berchick, and Kreider 2021). The United States Census Bureau (2023) reports the age distribution of individuals at mother's death (in 5-year age groups) based on the SIPP 2021 data. For the comparison I retrieved the microdata from the SIPP website (https://www.census.gov/programs-surveys/sipp.html) and computed the ages of children at mother's death. The ages at which SIPP respondents lost their mother are shown in Figure 3. The age distribution is based on 16,487 observations. Of those who lost their mother, 20 percent did not report their age at mother's death. In this section, two scenarios are considered. In the first, the virtual population is based on period mortality and fertility rates of the Unites States in 2010. In the second, cohort rates are used (1964 birth cohort). Data of 2010 are used instead of data of 2021 to exclude changes in fertility during the past decade since these changes did not affect most of the respondents in the SIPP. The birth cohort 1964 is used because

To enable a comparison of ages at mother's death, the observation window used in the SIPP (observation starts at birth and ends at survey date) is imposed onto the virtual population. The procedure consists of three steps. First, the age distribution of respondents in the SIPP is determined. Second, members of the virtual population are allocated ages at censoring by sampling the age distribution of SIPP respondents. Third, the IDs of individuals in the virtual population who are alive at censoring and experienced the death of their mother by that age are retrieved and the ages at their mother's death are computed. The age distribution is compared with the ages reported by SIPP respondents. Differences are attributed to different observation windows, differences in mortality and differences in ages at childbearing.

This section describes the procedure followed.

5.1 The age distribution of SIPP respondents

The code to download the data is shown in the rmarkdown (Rmd) file. SIPP respondents are born between 1931 and 2020. They are aged 0 to 90+. Male respondents are 42.61 years, on average, and female respondents 45.01 years. The standard deviations are 23.98 years and 24.11 years, respectively. The age distribution of respondents at survey date (2021) is shown in Figure 2.

Table 5: Mean age and standard deviation of respondents in SIPP

	meanAge	sd
Males	42.61	23.98
Females	45.06	24.11

Thirty eigth percent of the SIPP respondents lost their mother (before survey date) The percentage is a little higher among females (40 percent) than among males (36 percent). Note that, at survey date, female respondents are older than male respondents.

5.2 Allocate ages at censoring to individuals in the virtual population (period data)

```
countrycode <- "USA"
ncohort <- 2000
refyear <- 2010
cohort <- NULL
ngen <- 2
case <- ifelse(is.null(cohort), "cohort","period")

dLH <- tryCatch(VirtualPop::BuildViP(user,pw_HMD,pw_HFD,
                             countrycode=countrycode,
                             cohort=NULL,  # period data
                             refyear=refyear,
                             ncohort=ncohort,
                             ngen=2,
                             mort=TRUE),
      error=function(cond) {warning(cond);
        message(paste0("Error reading HFD data.",
        "Download data and read data locally (see Tutorial Section 4.2)."))})
```

Members of the virtual population are assigned an age at censoring. The age is obtained by sampling the age distribution of SIPP respondents. The sampling is done separately for males and females. Individuals who died before the age at censoring are removed from the population. The ages at censoring are shown in Figure 2. The age distribution of members of the virtual population alive at censoring is the same as the age distribution of SIPP respondents, except for random variation.

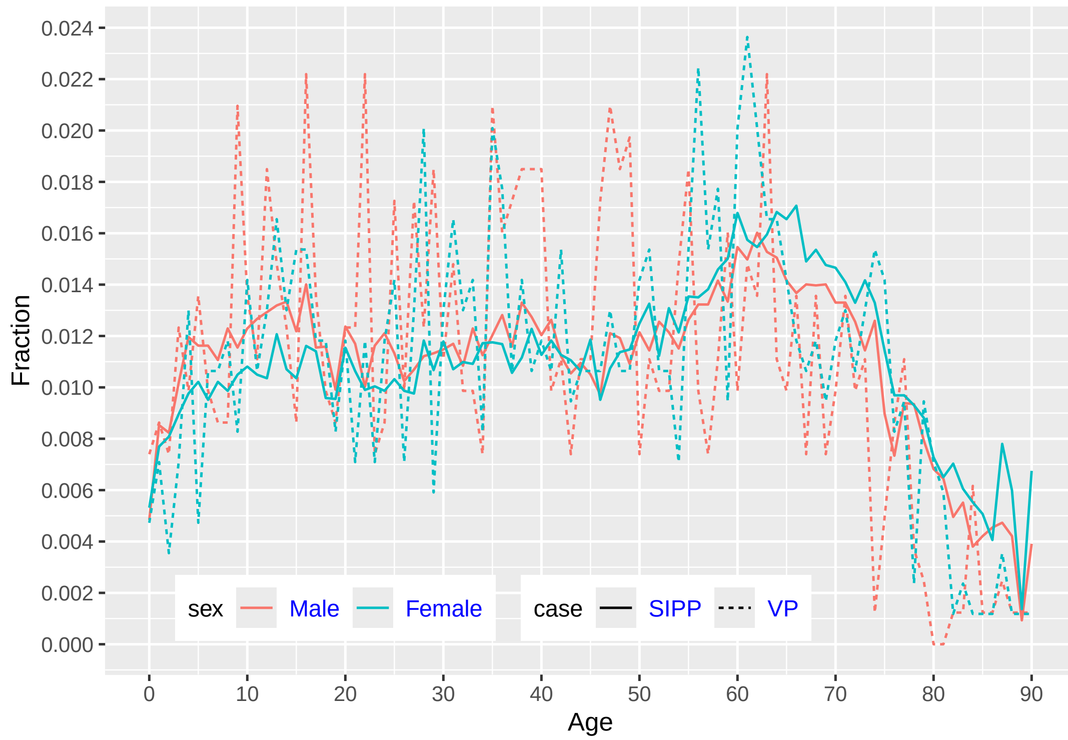

Figure 2: Ages of respondents in SIPP2021 and ages at censoring in virtual population USA, cohort 2010

5.3 Ages of children at mother's death (virtual population based on period data)

In a third step, individuals of the second generation of the virtual population are retrieved, provided they are alive at censoring date. If their mother died before the censoring date, the age of the individual at death of the mother is recorded. The age distribution of members of the second generation at death of their mother is shown in Figure 3 (green curve denoted ViP_C). Members of the virtual population lose their mother at younger ages than SIPP respondents (red curve). Part of the reason is that, due to postponement of childbearing in recent decades, women in the virtual population have their children at a higher age (2021 period rates) than women in SIPP. As a consequence, children are younger at mother's death. If censoring is omitted and maternal deaths at higher ages are included, the age distribution shifts to the right (curve ViP_D). The distribution of ages observed in SIPP lie between the two, as expected. Figure 4 shows the age distributions of children at father's death. The figure is shown for illustrative purpose only. The ages of children at father's death depend in part on the age differences between males and females at couple formation. The simple partner search model implemented in VirtualPop does not restrict the age difference between partners except that both should be members of the same generation.

To illustrate the effect of calendar year selected on the age distribution of children at mother's death, Figure 5 shows the age distribution in a virtual population produced from 2021 rates.

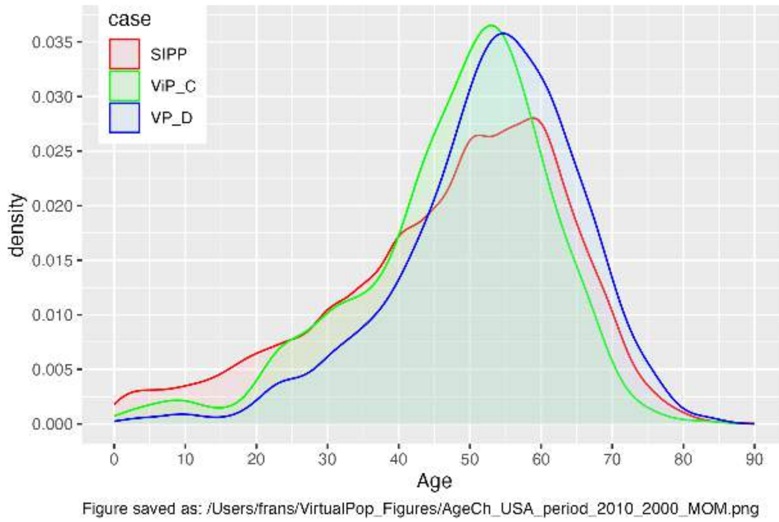

Figure 3: Age distribution of children at mother's death, Virtual Population USA, period 2010 and SIPP2021

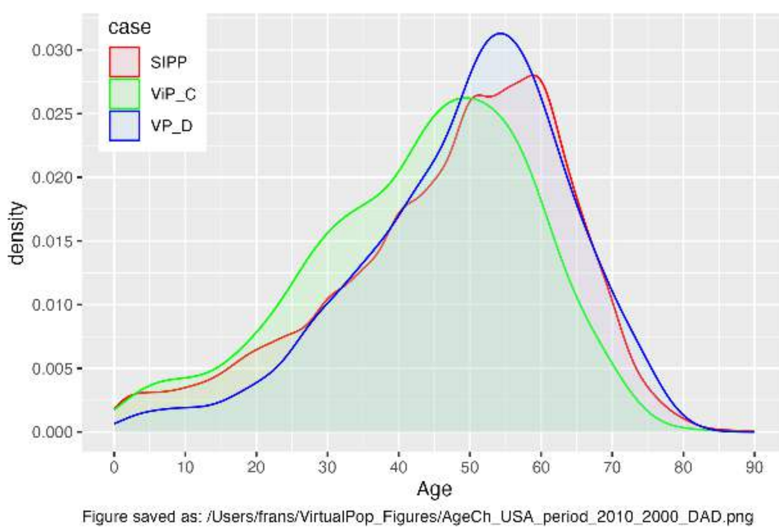

Figure 4: Age distribution of children at father's death, Virtual Population USA, period 2010 and SIPP2021

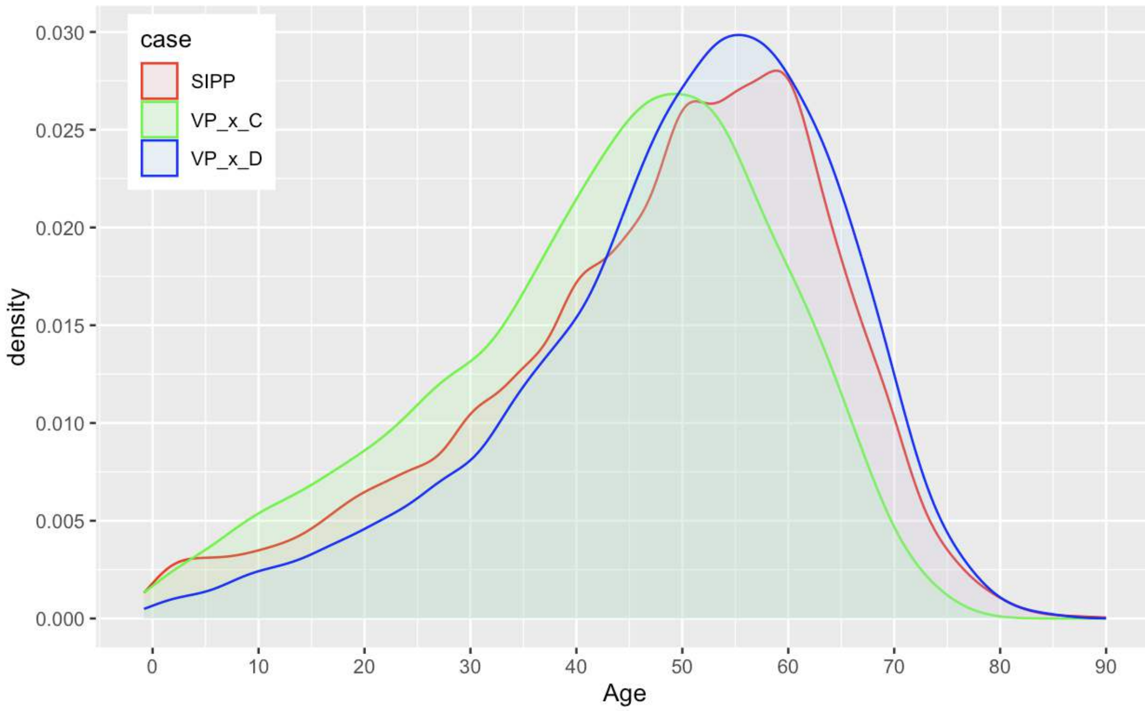

Figure 5: Age distribution of children at death of mother. SIPP2021 and virtual population USA 2021

5.4 Ages of children at mother's death (virtual population based on cohort data)

Figures 6 and 7 show the age distribution of children at mother's death in a virtual population when cohort mortality and fertility rates (1964 birth cohort) are used. The pattern is similar to that observed when period rates are used.

```
countrycode <- "USA"
ncohort <- 2000
cohort <- 1964
refyear <-  NULL
ngen <- 2
dLH <- tryCatch(VirtualPop::BuildViP(user,pw_HMD,pw_HFD,
                             countrycode=countrycode,
                             cohort=cohort,
                             refyear=refyear,
                             ncohort=ncohort,
                             ngen=ngen,
                             mort=TRUE),
      error=function(cond) {warning(cond);
        message(paste0("Error reading HFD data.",
        "Download data and read data locally (see Tutorial Section 4.2)."))})
```

```
dLH <- AllocateCensoring(dLH)
p <- PlotAgeCensoring(dLH,ageDistrSIPP)
filename <- PlotSave(dLH)
#> Warning: Removed 2 rows containing non-finite outside the scale range
#> (`stat_density()`).
#> Warning: Removed 158 rows containing non-finite outside the scale range
#> (`stat_density()`).
names(filename) <- c("filenameM","filenameD")
case <- attr(dLH,"type")
```

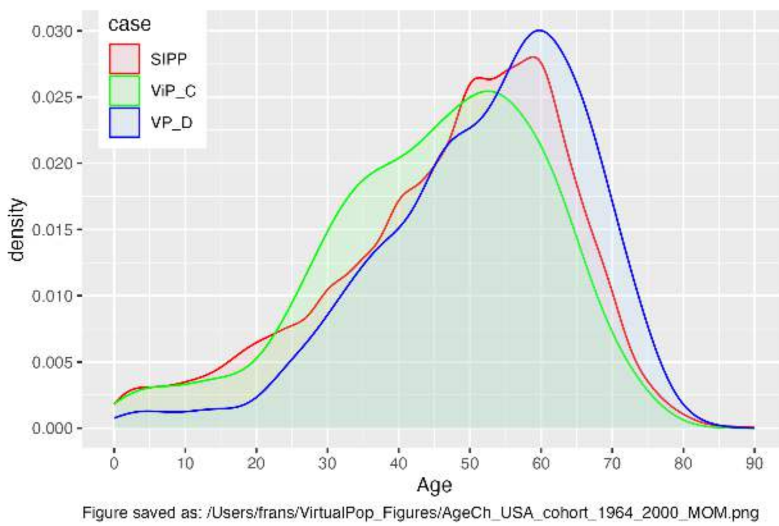

Figure 6: Age distribution of children at mother's death, Virtual Population USA, cohort 1964 and SIPP2021

6 Conclusion

The validation of the virtual population generated from mortality rates by age and sex, and fertility rates by age and parity leads to three major conclusions. First, the virtual population is an accurate picture of the real population observed during the same period. Second, a comparison of simulated and real populations requires comparable observation windows. Retrospective survey data combine different periods and cohorts. Virtual populations produced using data from a single calendar year or a single birth cohort can differ significantly from the picture obtained by observations covering several periods and cohorts. Third, differences between a virtual population and a real population should have a sound explanation in order to justify the conclusion that the virtual population gives an accurate picture of the reference population. Because of the need for satisfactory explanations of differences, the validation of the virtual population can be a powerful learning process.

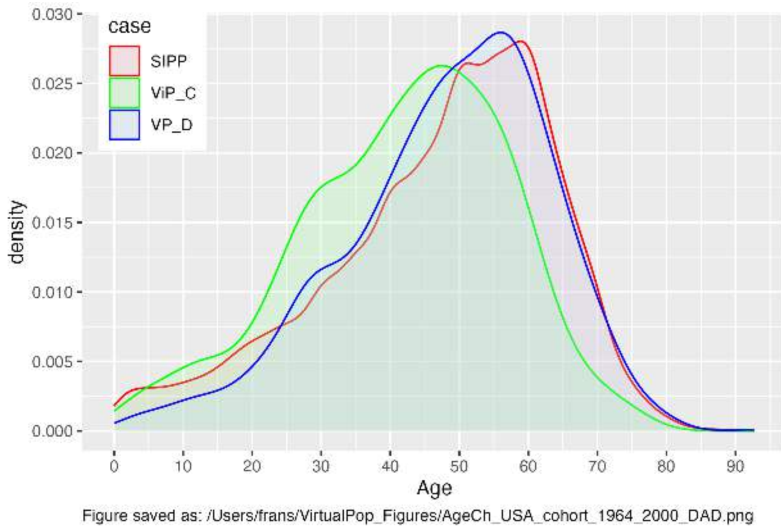

Figure 7: Age distribution of children at father's death, Virtual Population USA, cohort 1964 and SIPP2021

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Appendix A. Effect of mortality on childlessness

The virtual population experiences births and deaths. Premature deaths interrupt fertility careers and contribute to the proportion of a birth cohort remaining without children. In the virtual population, childlessness is in part due to premature death. The difference in childlessness between a virtual population and the corresponding period fertility table of the HFD is the subject of this appendix. Fertility tables disregard mortality. The percentage childless is therefore conditional on survival until the end of the reproductive period. According to the 2021 period life table in the HMD, 7.92 percent of newborn girls die before the age of 55. The percentage in the virtual population is similar (8.38 percent). 52.94 percent of the deceased have no children. That explains the higher childlessness in the virtual population.

A minor part of the difference in childlessness and the period fertility table can be attributed to differences in method. The period fertility table assumes a piecewise-linear survival function within age groups, while the simulation assumes a piecewise-exponential survival function. The linear model gives a percentage childless of 22.9 percent and the exponential model gives 22.92 percent, both in the absence of mortality. The piecewise-linear model is common in demography, while the piecewise-exponential model is common in biostatistics. Consider the fertility rate of childless women aged 32. The rate is 0.10119. The probability of having a first child within a year is m/(1+0.5m)=0.10119/(1+0.5*0.10119)=0.09632. In the exponential model, the probability is 1-exp[-m]=1-exp[-0.10119]=0.09624. An exponential survival function with constant rate implies a lower transition probability than a linear survival function with uniform distribution of events. The cumulative effect over all ages is a higher childlessness in the piecewise exponential model than in the piecewise linear model.

Figure 5 shows the proportion childless in the presence and in the absence of mortality. The difference is very small. The figure also shows the age-specific death rates of the USA used in the construction of the virtual population.

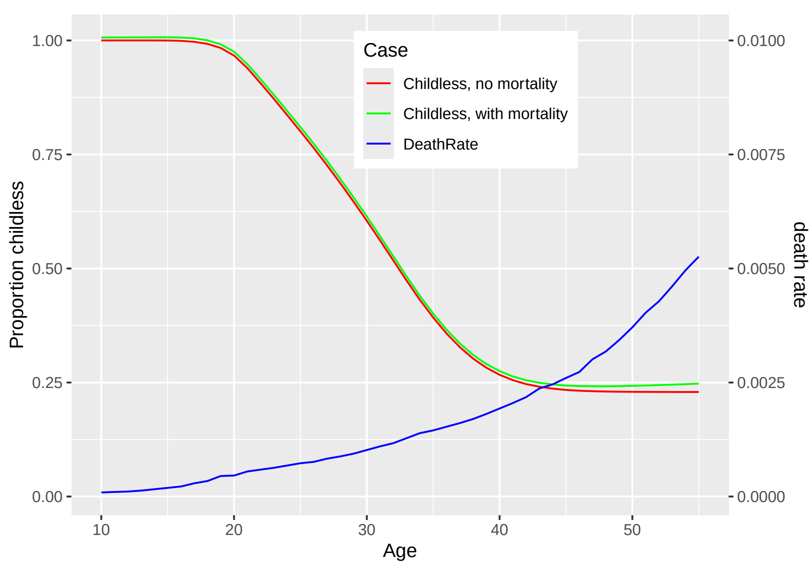

Figure 8: Proportion childless in presence and in absence of mortality, and death rates, USA,